

# Superfluid density for $^3\text{He}$ in aerogel assuming inhomogeneous scattering

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## Abstract

When aerogel is filled with  $^3\text{He}$  the thin aerogel strands work as impurities and scatter the helium quasiparticles. These impurities cause a suppression in critical temperature, pair potential and superfluid density. The experimentally measured critical temperatures and superfluid densities for  $^3\text{He}$  in aerogel can be fairly well explained by using an isotropic inhomogeneous scattering model where the impurity density is allowed to depend on location.

*Keywords:* Superfluid  $^3\text{He}$ ; aerogel; superfluid density

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## 1. Introduction

Several measurements in the last few years have shown that the superfluid transition occurs not only in pure  $^3\text{He}$  but also in 98% porous aerogel [1–4]. The thin randomly oriented aerogel strands scatter  $^3\text{He}$  quasiparticles and result in a suppression of critical temperature, pair potential and superfluid density. Here we concentrate on the isotropic inhomogeneous scattering model (IISM) since the homogeneous model (HSM), where the impurities are thought to be evenly distributed in helium, is found to give roughly twice as large superfluid density as the experiments [5,6]. The critical temperatures and pair potentials given by the IISM are already calculated in Ref. [5]. Here we use the same model to calculate the superfluid density.

## 2. Model

In the IISM we assume that aerogel forms a periodic lattice and that the unit cell may be approximated by a sphere of radius  $R$ . The boundary condition is that a quasiparticle escaping from the sphere will be returned at the diametrically opposite point without changing its momentum. Inside the sphere the impurity density  $n(r)$  is allowed to depend only on radial coordinate  $r$ . The forms used in the calculations are:

$$n(r) = A \left[ \left( \frac{r}{R} \right)^j - \frac{j}{j+2} \left( \frac{r}{R} \right)^{j+2} \right] \quad (1)$$

$$n(r) = A \cos^j \left( \frac{\pi r}{2R} \right), \quad (2)$$

where the constant  $A$  defines the averaged mean free path inside the sphere by  $\ell_{\text{ave}}^{-1} = \langle \ell^{-1} \rangle = \sigma \langle n(r) \rangle$ . Here  $\sigma$  is the transport cross section and the notation  $\langle \cdot \rangle$  means average over the sphere.

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The first form for the impurity density has its highest value on the shell and it builds a lattice that can support itself. In this form the current has to flow through high impurity regions somewhere. The second form has its maximum value at the center and it allows the current to flow continuously through the low density regions resulting a higher superfluid density than the first form.

In our model we take into account only non-magnetic  $s$ -wave scattering with random phase shifts  $\delta_0$ , i.e.  $\sin^2 \delta_0 = \frac{1}{2}$ . The calculations are done using the quasiclassical theory which is described in Ref. [7].

### 3. Results

Since the structure of aerogel is expected to be independent of the pressure and the Fermi wave number  $k_F$  changes less than 10% in the pressure range considered, the ideal case would be where same  $R$  and  $\ell_{ave}$  could be used at all pressures and temperatures. By using the impurity profile of Eq. (1) the critical temperatures can be fitted well using this assumption but the resulting superfluid densities  $\rho_s$  are not in good agreement with measurements. Especially near  $T_c$  the model gives far too small  $\rho_s$ . By using the impurity profile of Eq. (2) one obtains a better fit for the superfluid density, but at the same time the fit for critical temperature becomes somewhat poorer, see Fig. 1.

For better fits one must limit to pressures above 10 bars or allow the mean free path to depend on pressure. The required variation in  $\ell$  as a function of pressure is more than by factor 2. The most likely conclusion is that the IISM scattering profile containing only one length scale is too limited if detailed correspondence between theory and experiments is required. A less likely alternative is that there really is a strong pressure dependence in the mean free path. The quality of the fit depends slightly on the sample but for all samples the inhomogeneous model gives considerably better fit than the homogeneous model.

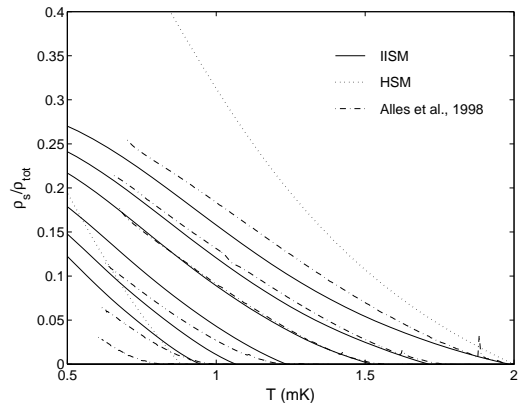


Fig. 1. Superfluid density using scattering profile of Eq. (2) and  $j = 3$  together with  $\ell_{ave} = 75$  nm and  $R = 60$  nm. From left to right, the pressures are 5.8, 7.4, 10.0, 14.9, 19.7, and, 29.3 bar for both experiments (dash-dotted lines) and theory (solid lines). Experimental values are from Ref. [4]. The dotted lines are from HSM at pressures 5.8 and 29.3 bar with  $\ell = 244$  nm.

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