π states and textural effects in superfluid ^{3}He weak links

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http://boojum.hut.fi/research/theory/jos.html

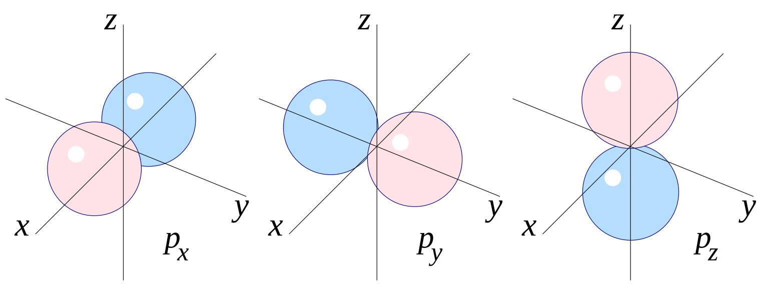
Outline

- weak links in ^{3}He-B
- theoretical models for "π states"
  - quasiclassical pinhole model
  - GL calculation of a large aperture
  - textural effects in aperture arrays
- summary

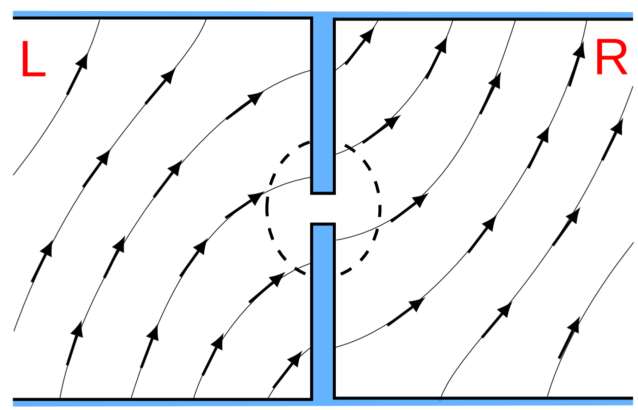
Weak link in superfluid ^{3}He-B

- triplet pairing
  $\Delta^t_\mu = A_{\mu i}\hat{k}_i,\ \mu, i = x, y, z$
- order parameter

$$\begin{matrix} -|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \rightarrow \\ \mathrm{i}|\uparrow\uparrow\rangle + \mathrm{i}|\downarrow\downarrow\rangle \rightarrow \\ |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \rightarrow \end{matrix} \quad \begin{bmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{bmatrix}$$

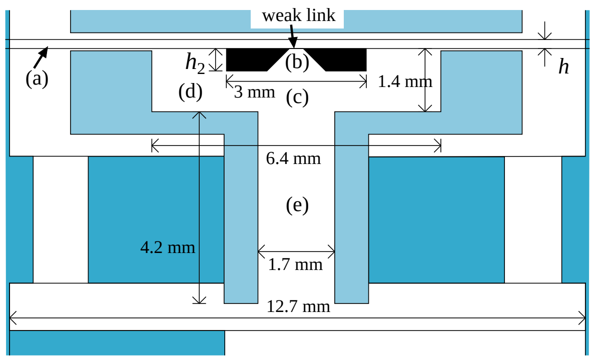


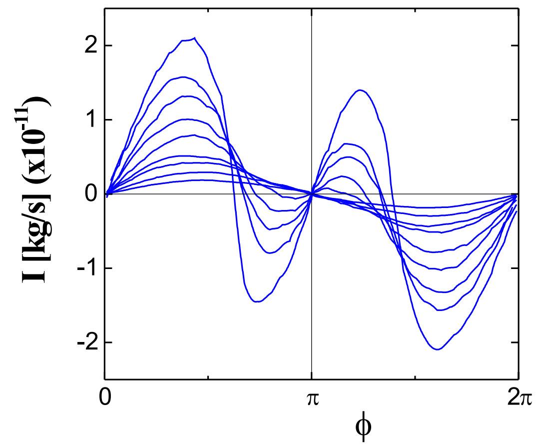
- $A_{\mu i} = \Delta R_{\mu i}(\hat{\mathbf{n}}, \theta_0)\exp(\mathrm{i}\phi)$
- $\hat{\mathbf{n}}$ forms textures



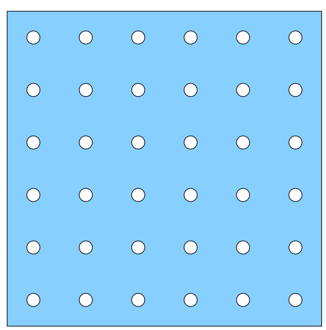
- $\phi = \phi^R - \phi^L$, $\psi_{ij} = R^L_{\mu i} R^R_{\mu j}$
  coupling $F_J(\phi, \psi_{ij})$

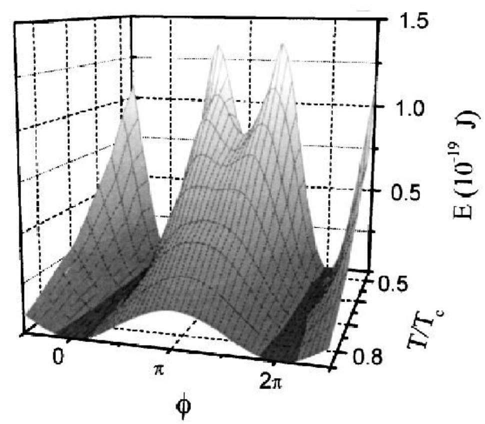
Berkeley experiments [Marchenkov et al., PRL **83**, 3860 (1999)]





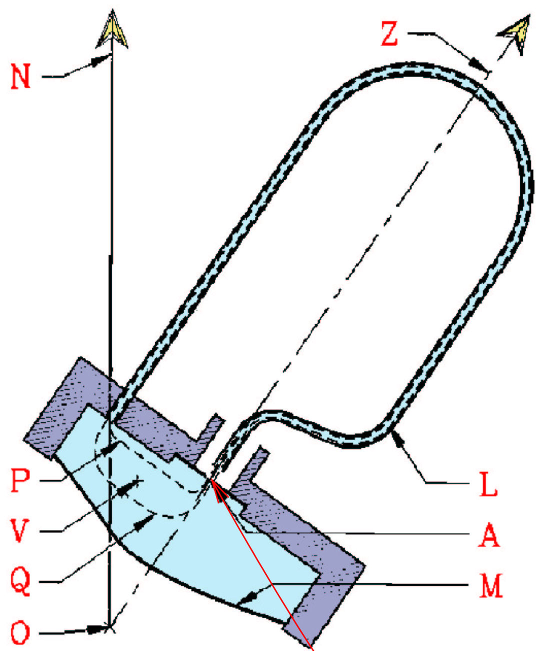
aperture array



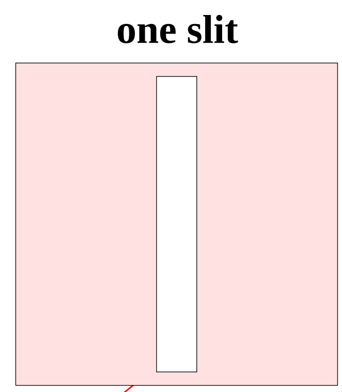


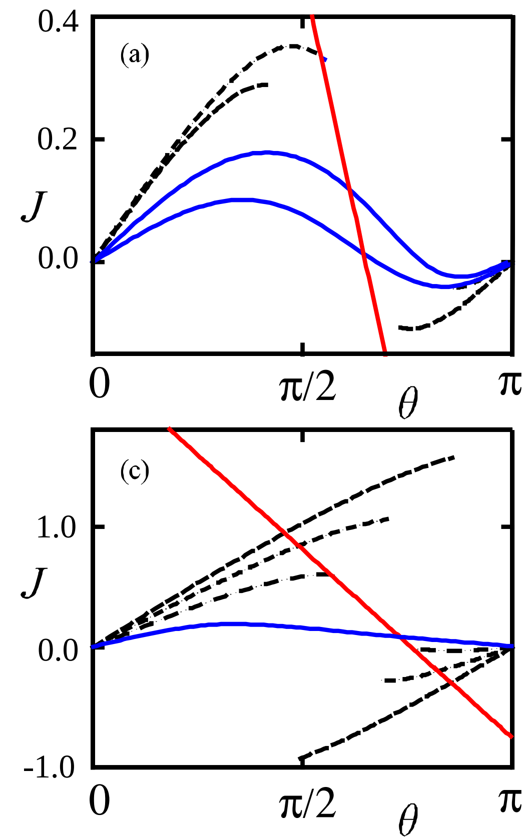
- two CPR states: “H” and “L”

Paris experiments [Avenel et al., Physica B **280**, 130 (2000)]

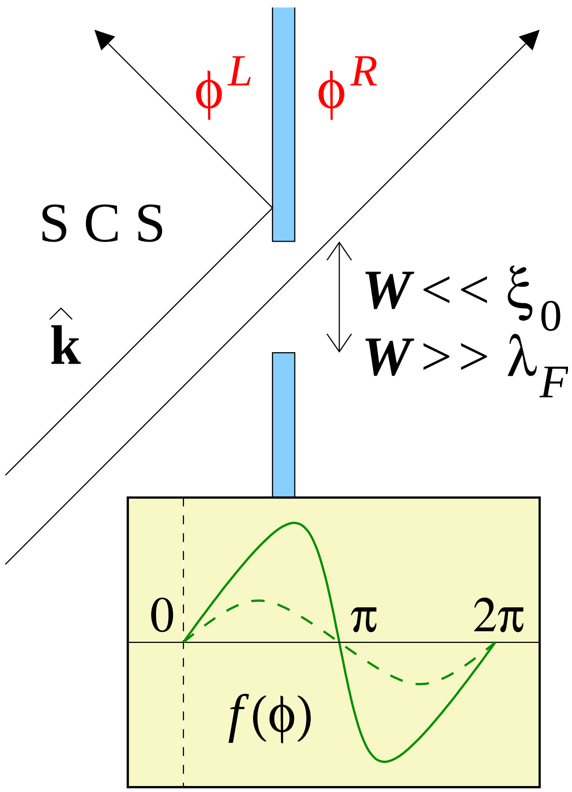


- many CPR's





Josephson currents in a small weak link



- singlet s wave pinhole:
  [Kulik & Omel'yanchuk (1977)]

$$J_s(\phi) = f(\phi)$$

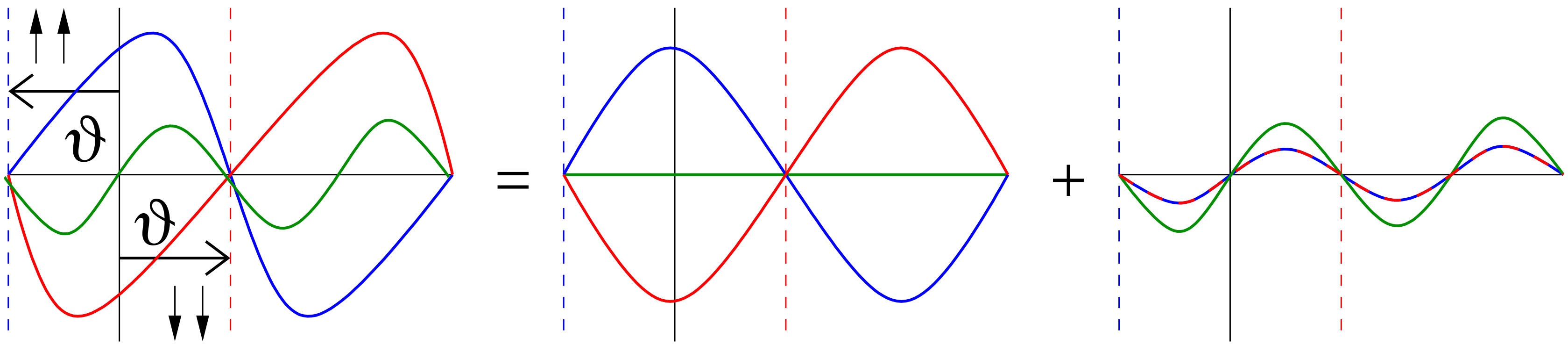
- triplet p wave: ↑↑ and ↓↓ condensates
  [Yip, PRL **83**, 3864 (1999)]

$$J_s(\phi) = \frac{1}{2}\left\langle f(\phi + \vartheta_{\hat{\mathbf{k}}}) + f(\phi - \vartheta_{\hat{\mathbf{k}}})\right\rangle_{\hat{\mathbf{k}}}$$

$$\cos\vartheta_{\hat{\mathbf{k}}} = [R^L_{\mu i} R^R_{\mu j}]\hat{k}_i \hat{k}_j$$

π periodicity and "π states"

- for each $\hat{\mathbf{k}}$, expand $f(\phi \pm \vartheta) = \sum_{n=1}^{\infty} f^{(n)} \sin[n(\phi \pm \vartheta)]$

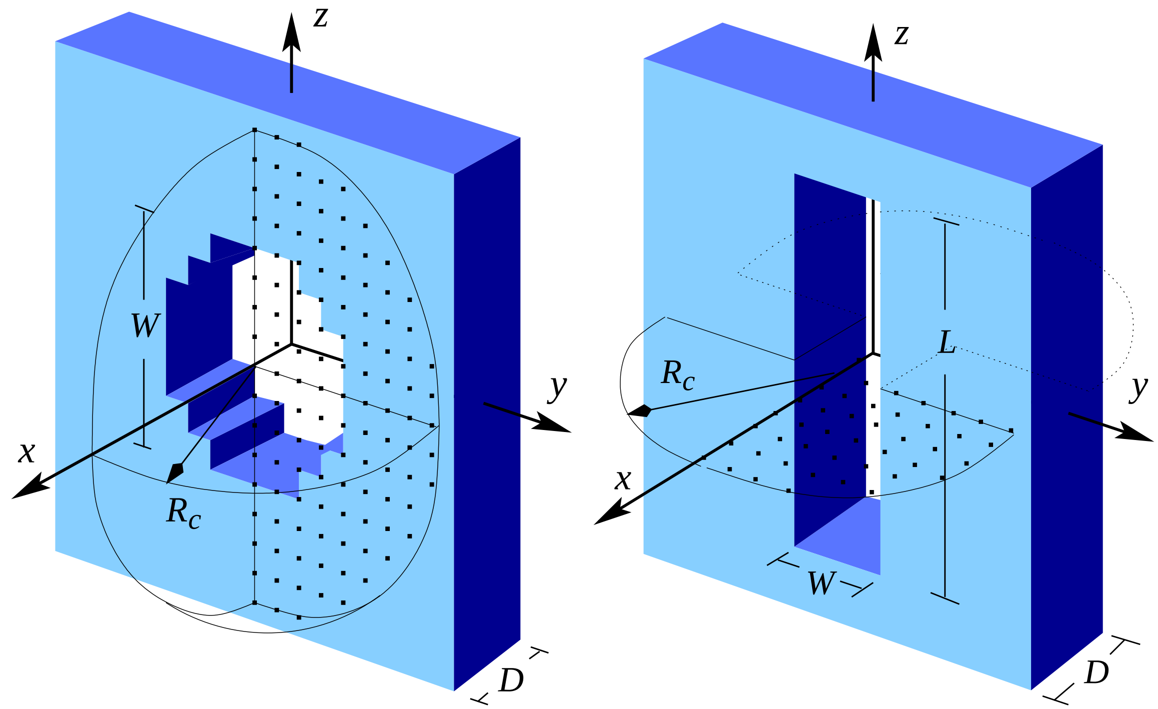


$n = 1$ $n = 2$

- average over $\hat{\mathbf{k}}$ to get $J_s(\phi) = \sum_{n=1}^{\infty} J_c^{(n)} \sin(n\phi)$
- $\hat{\mathbf{n}}^{L,R}$ and thus $J_c^{(n)}$ controllable with magnetic field: $F_{SH} \propto -(\mathbf{H} \cdot R\hat{\mathbf{s}})^2$

GL simulation of large aperture

- for $T \ll T_c$ holes with $W \approx 100$ nm not good "pinholes"
- self-consistent calculation for 3D hole or 2D slit on ξ_{GL} scale



Ginzburg-Landau energy density

$$
\begin{aligned}
f(A, \nabla A) = \\
&- \alpha \operatorname{Tr}(AA^{\mathrm{T}*}) + \beta_1 |\operatorname{Tr}(AA^{\mathrm{T}})|^2 + \beta_2 [\operatorname{Tr}(AA^{\mathrm{T}*})]^2 \\
&+ \beta_3 \operatorname{Tr}(AA^{\mathrm{T}}A^*A^{\mathrm{T}*}) + \beta_4 \operatorname{Tr}(AA^{\mathrm{T}*}AA^{\mathrm{T}*}) + \beta_5 \operatorname{Tr}(AA^{\mathrm{T}*}A^*A^{\mathrm{T}}) \\
&+ K \left[(\gamma - 2)\partial_i A^*_{\mu i} \partial_j A_{\mu j} + \partial_i A^*_{\mu j} \partial_i A_{\mu j} + \partial_i A^*_{\mu j} \partial_j A_{\mu i} \right]
\end{aligned}
$$

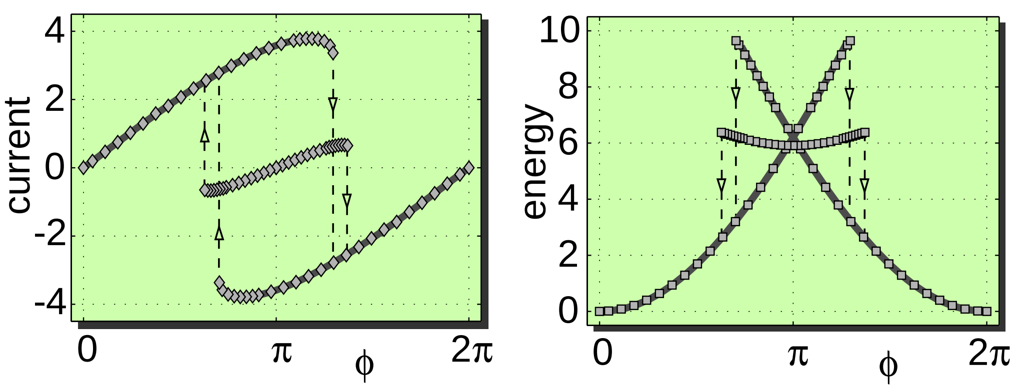
Conserved currents ($\xi_{GL}(T) \ll \xi_D$)

$$
\mathbf{j}_{\mathrm{s}} = \frac{2m_3}{\hbar} \left(+\mathrm{i} A_{\mu i} \frac{\partial f}{\partial \boldsymbol{\nabla} A_{\mu i}} + c.c. \right)
$$

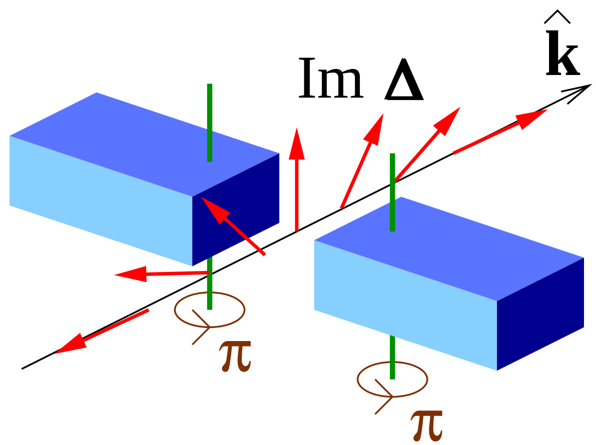
$$
\mathbf{j}^{\mathrm{spin}}_{\alpha} = +\epsilon_{\alpha\mu\nu} A_{\nu i} \frac{\partial f}{\partial \boldsymbol{\nabla} A_{\mu i}} + c.c., \qquad \alpha = x, y, z
$$

“π states” (3D hole)

- pinhole: “π states” only for $\hat{\mathbf{n}}^L \neq \hat{\mathbf{n}}^R$
- large hole: also for $\hat{\mathbf{n}}^L = \hat{\mathbf{n}}^R$ [Viljas & Thuneberg, PRL **83**, 3868 (1999)]



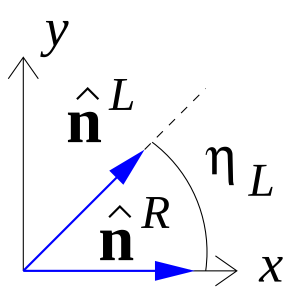
- π branch not accessible via phase slip?



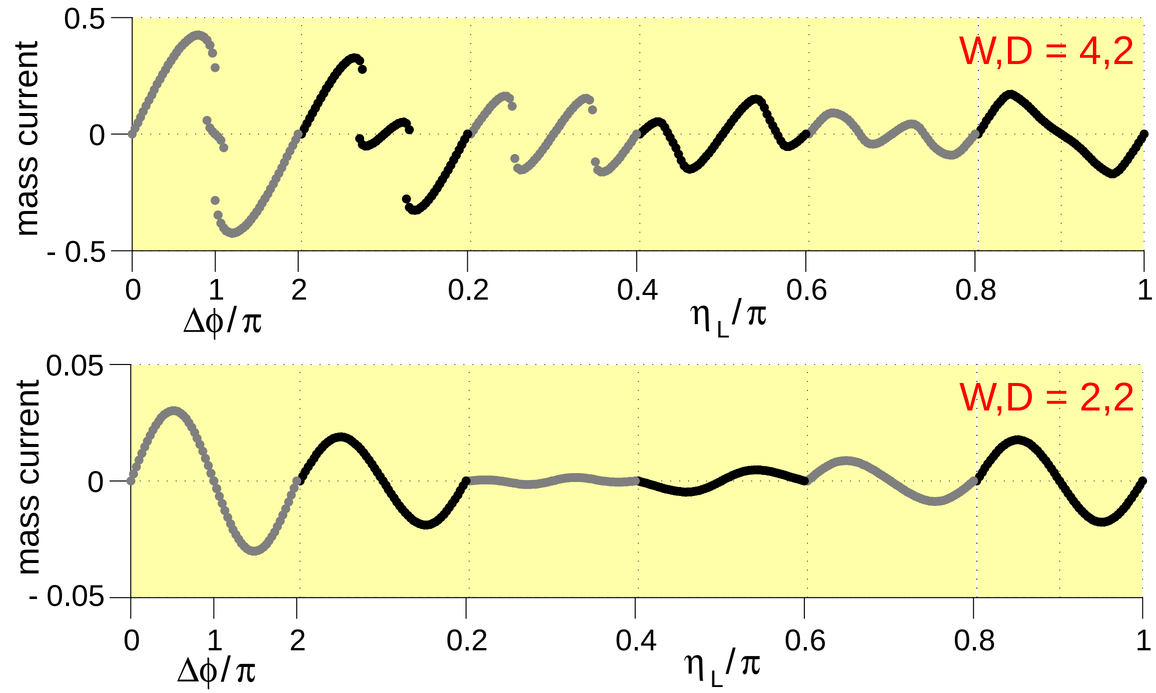
- order parameter on π branch: trapped double-core vortex

Wide and narrow 2D slits

- $\hat{\mathbf{n}}^L \neq \hat{\mathbf{n}}^R$

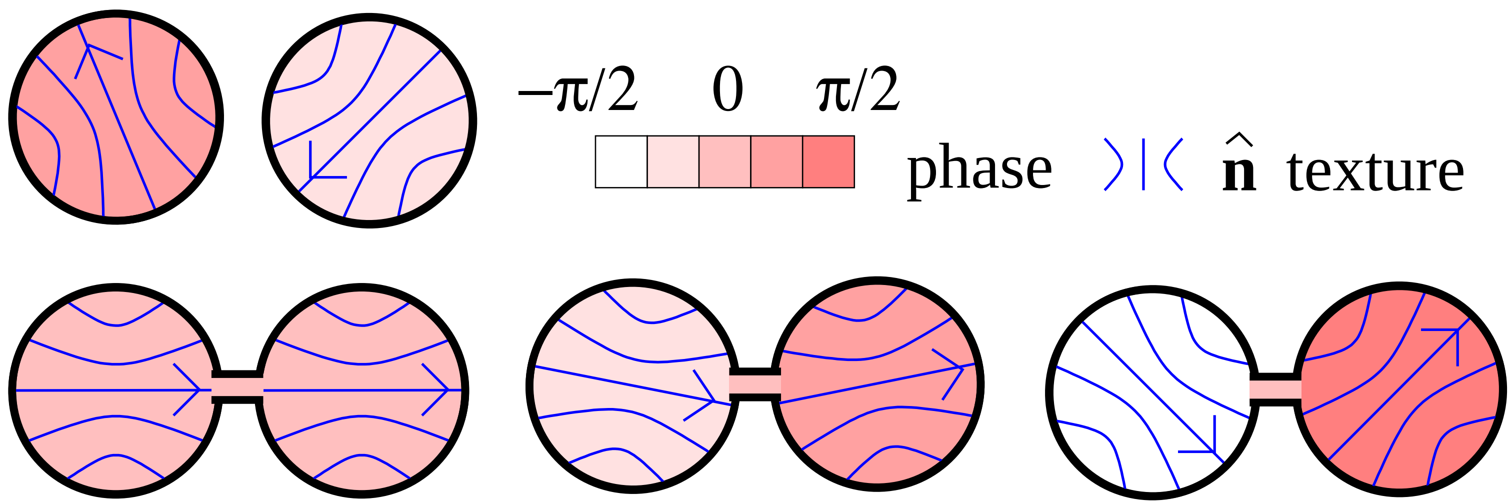


- continuous CPR vs. hysteretic CPR: $W_c/\xi_{GL} \approx 3$



“Anisotextural” effect

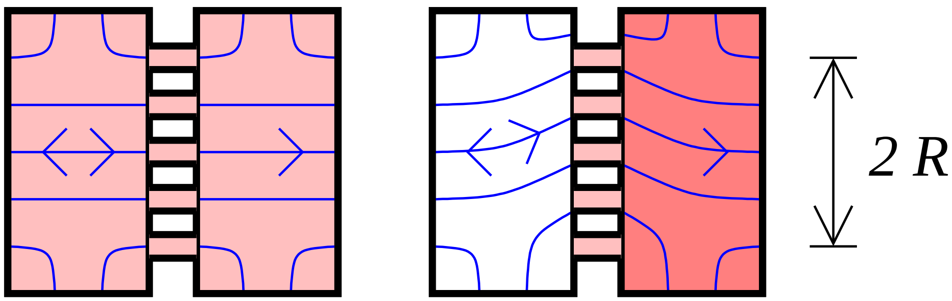
- length scale $> \xi_D \approx 10\mu\mathrm{m} \gg \xi_0$
- surface-dipole energy $\propto -(\hat{\mathbf{n}} \cdot \hat{\mathbf{s}})^2$ and coupling $F_J(\phi, \hat{\mathbf{n}}^L, \hat{\mathbf{n}}^R)$



- weak link couples phase and spin-orbit degrees of freedom

Aperture array

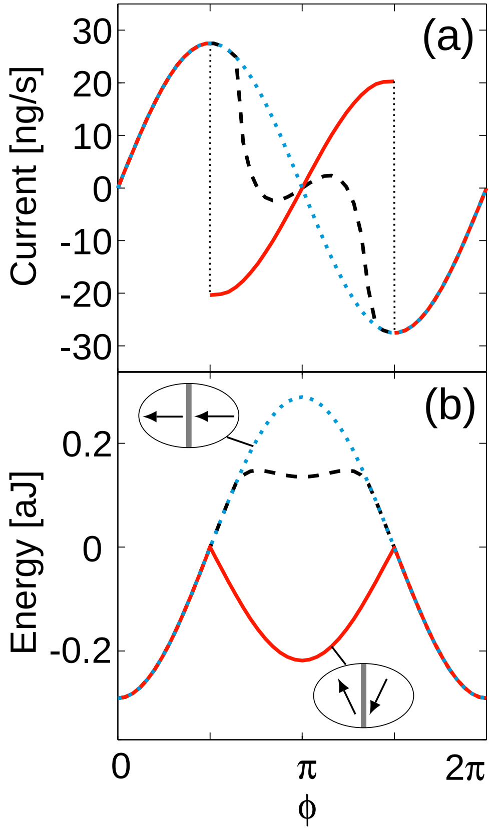
- a pinned textrure may **bend**



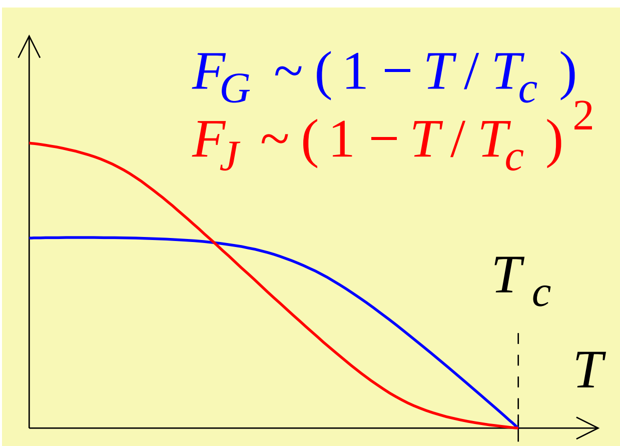
- minimize coupling + bending energy

$$F_J(\phi, \hat{\mathbf{n}}^L, \hat{\mathbf{n}}^R) = -E_c(\hat{\mathbf{n}}^L, \hat{\mathbf{n}}^R)\cos\phi$$
$$F_{\rm rig}(\hat{\mathbf{n}}^L, \hat{\mathbf{n}}^R) = F_G(\hat{\mathbf{n}}^L) + F_G(\hat{\mathbf{n}}^R)$$

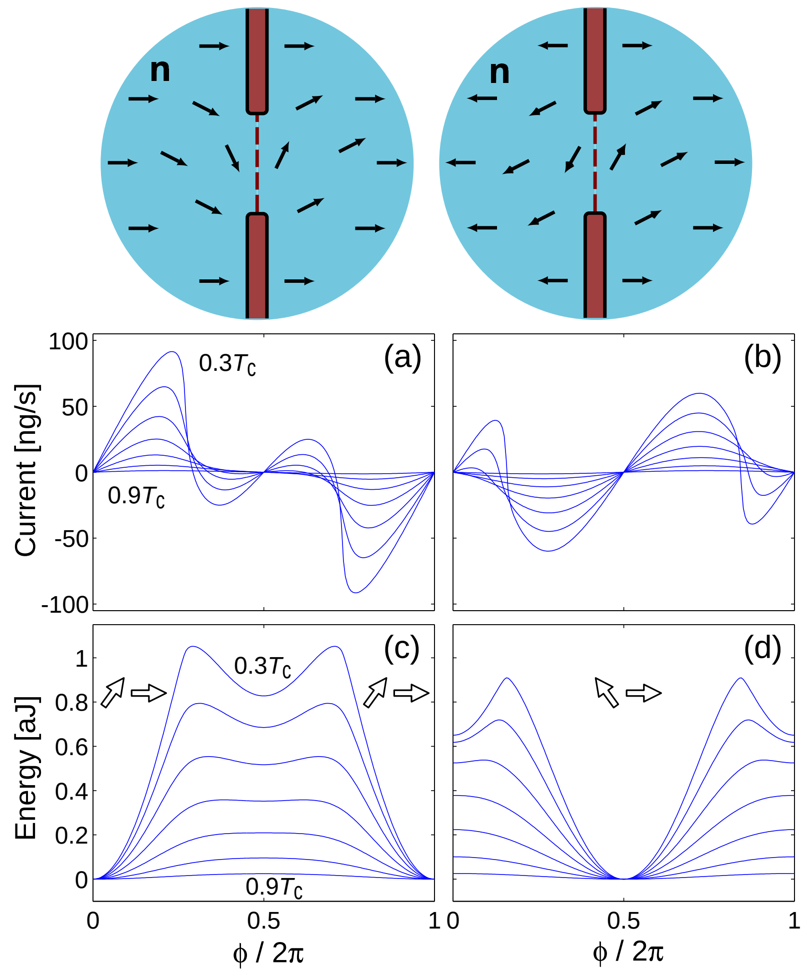
- transition at $\phi = \pm\pi/2$ where $\cos\phi = 0$



- estimates for pinholes in diffusive wall [Viljas & Thuneberg, PRB **65**, 064530 (2002)]
- $F_G \sim R$ and $F_J \sim \kappa R^2$



- $T = 0.4T_c$, $R\kappa \gtrsim 0.5\mu$m



Summary

- pinhole model reasonable for small apertures

$$F_J(\phi) = -E_c^{(1)} \cos\phi - E_c^{(2)} \cos 2\phi - \cdots$$

- modifications necessary for
  - large ($W, D \gtrsim \xi_{GL}$) apertures
  - hole arrays $\Longrightarrow$ anisotextural effects
- more experiments needed to identify these effects