Vortices at the A-B phase boundary in superfluid ^{3}He

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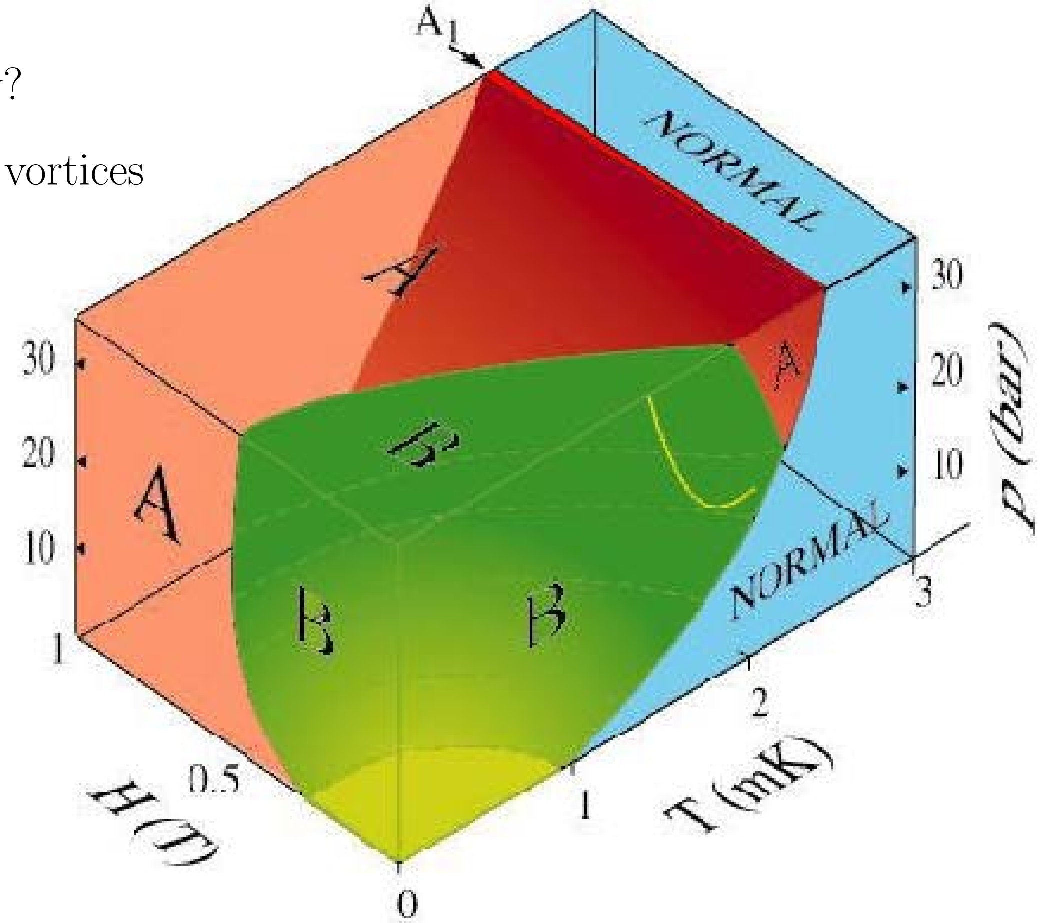
Outline:

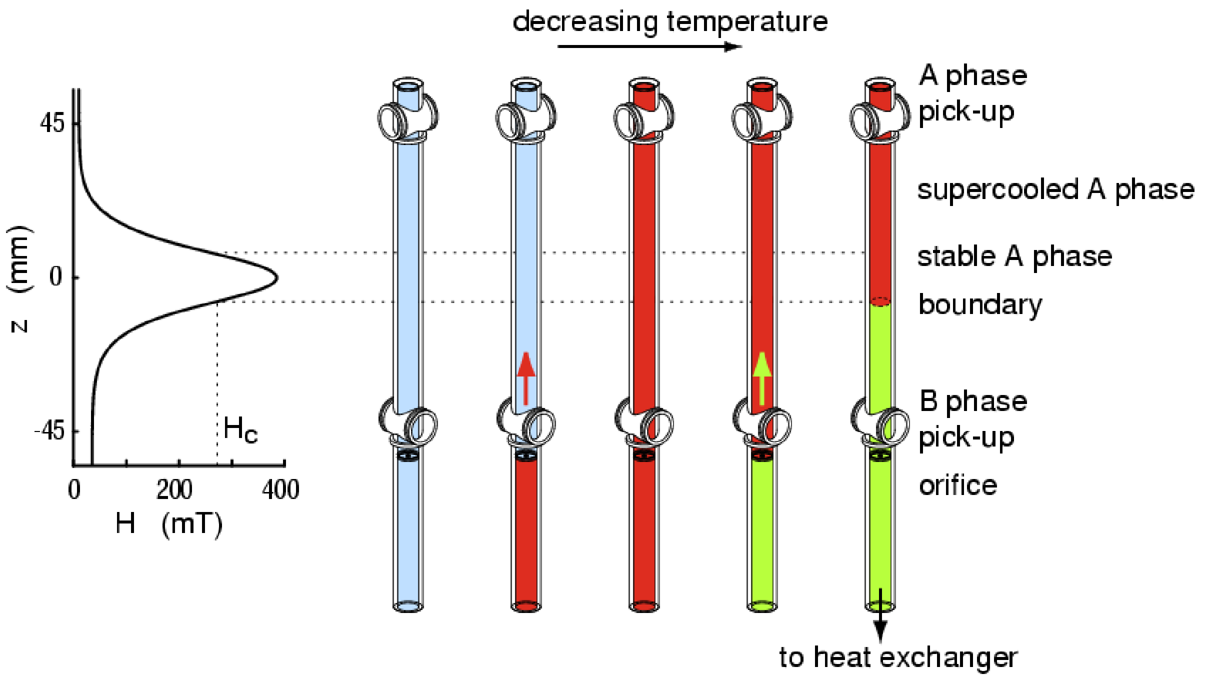
1. Introduction
2. Experimental setup
3. Hydrostatic theory in ^{3}He
4. A-B phase boundary
5. Results
6. Summary

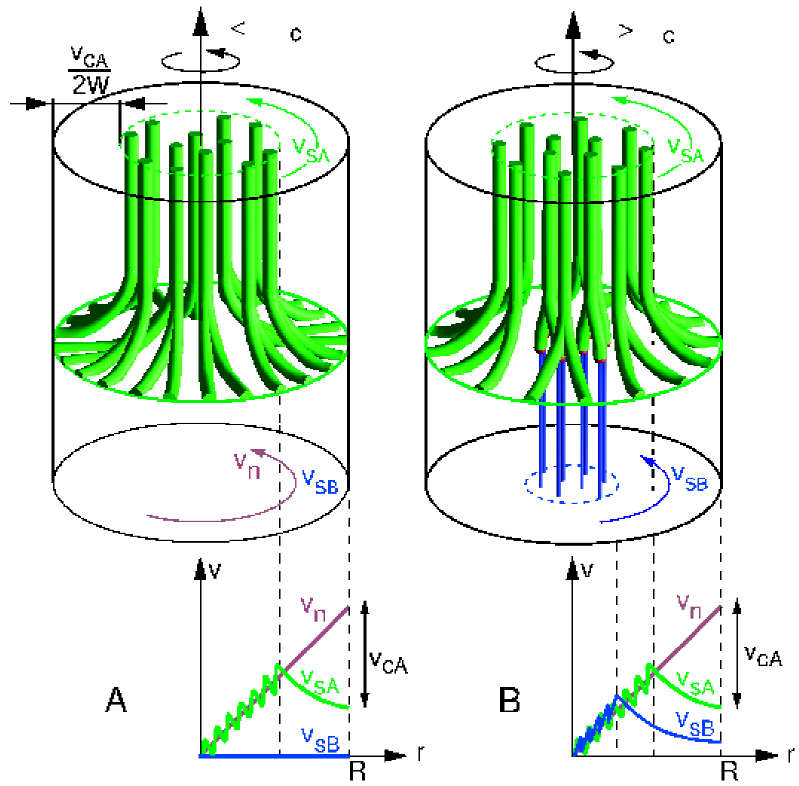
Introduction:

- two main phases: A and B phase
- several textures, especially in the A phase
- new experiments with the phase boundary
- what is the effect of the A-B phase boundary?
- we have calculated the effect on the A phase vortices
- two different textures obtained



Experimental setup:





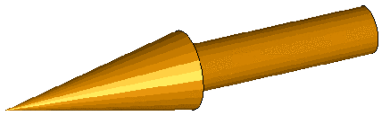
Hydrostatic theory in ^{3}He:

- p-wave spin triplet $\Longrightarrow$ order parameter $A_{\mu i}$ is a 3×3 matrix

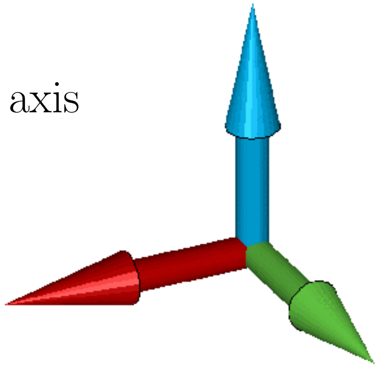
A phase order parameter:

$$A_{\mu j} = \Delta_A \hat{d}_\mu (\hat{m}_j + i\hat{n}_j), \text{ where } \hat{\mathbf{m}} \perp \hat{\mathbf{n}}$$

- vector $\hat{\mathbf{d}}$ defines the axis along which the spin of the Cooper pair vanishes.



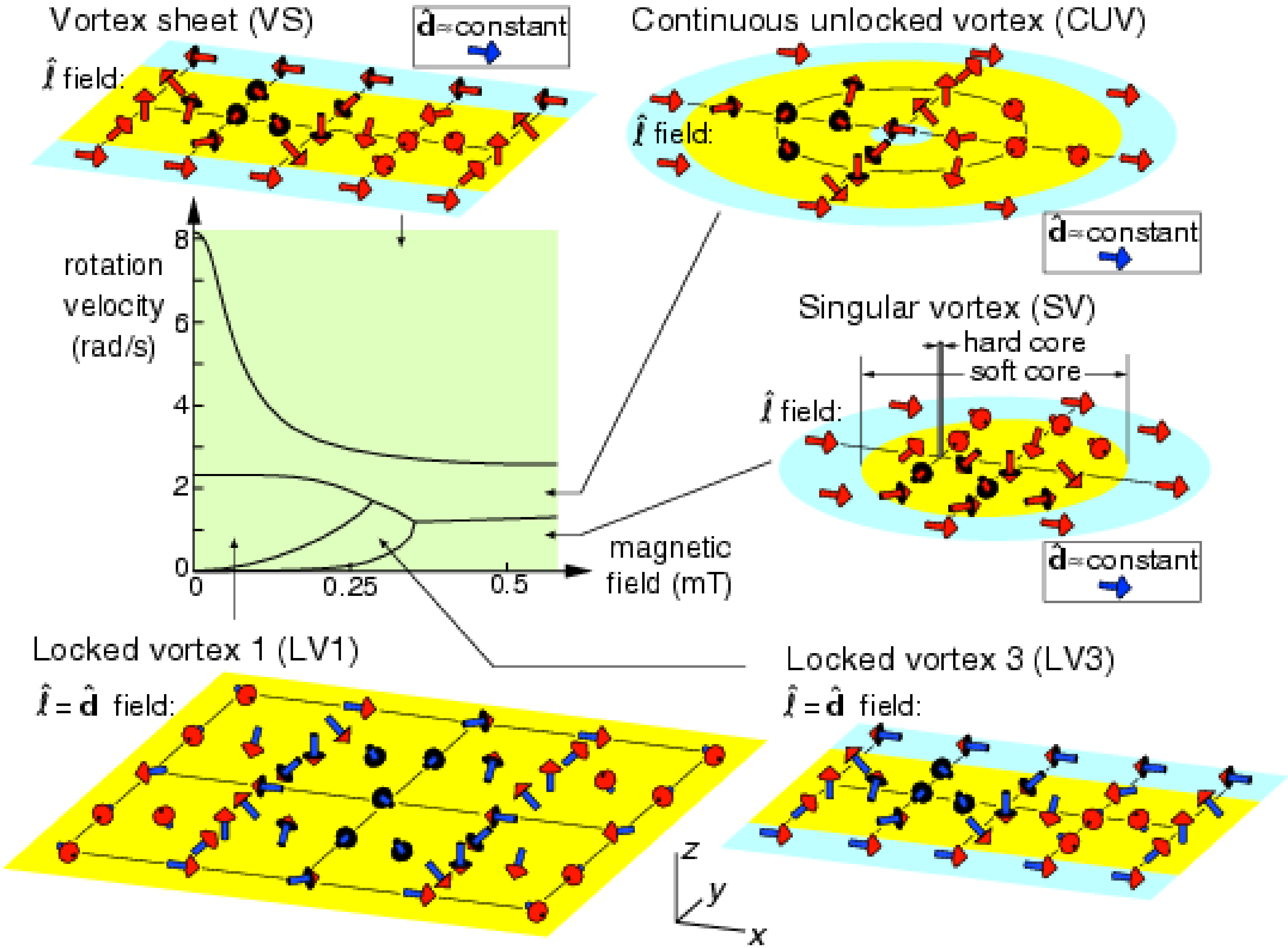
- vector $\hat{\mathbf{l}} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$ gives the orbital angular momentum axis



- superfluid velocity:

$$\mathbf{v}_{sA} = \frac{\hbar}{2m_3} \sum_i \hat{m}_i \nabla \hat{n}_i$$

Different vortex textures in ^{3}He-A:



Hydrostatic free energy:

$$F = \int d^3r(f_\mathrm{d} + f_\mathrm{h} + f_\mathrm{g})$$

- dipole term:

$$f_\mathrm{d} = -\frac{1}{2}\lambda_\mathrm{d}(\hat{\mathbf{d}} \cdot \hat{\mathbf{l}})^2,$$

- magnetic anisotropy term:

$$f_\mathrm{h} = \frac{1}{2}\lambda_\mathrm{h}(\hat{\mathbf{d}} \cdot \mathbf{H})^2 \ ,$$

- kinetic terms + gradient energy density ($v_n = 0$):

$$\begin{aligned} 2f_\mathrm{g} &= \rho_\perp \mathbf{v}_\mathrm{sA}^2 + (\rho_\parallel - \rho_\perp)(\hat{\mathbf{l}} \cdot \mathbf{v}_\mathrm{sA})^2 + 2C\mathbf{v}_\mathrm{sA} \cdot \boldsymbol{\nabla} \times \hat{\mathbf{l}} - 2C_0(\hat{\mathbf{l}} \cdot \mathbf{v}_\mathrm{sA})(\hat{\mathbf{l}} \cdot \boldsymbol{\nabla} \times \hat{\mathbf{l}}) \\ &+ K_\mathrm{s}(\boldsymbol{\nabla} \cdot \hat{\mathbf{l}})^2 + K_\mathrm{t}(\hat{\mathbf{l}} \cdot \boldsymbol{\nabla} \times \hat{\mathbf{l}})^2 + K_\mathrm{b}|\hat{\mathbf{l}} \times (\boldsymbol{\nabla} \times \hat{\mathbf{l}})|^2 + K_5|(\hat{\mathbf{l}} \cdot \boldsymbol{\nabla})\hat{\mathbf{d}}|^2 + K_6 \sum_{i,j}[(\hat{\mathbf{l}} \times \boldsymbol{\nabla})_i \hat{\mathbf{d}}_j]^2. \end{aligned}$$

Characteristic scales:

- dipole length:

$$\xi_\mathrm{d} = (\hbar/2m_3)\sqrt{\rho_\parallel/\lambda_\mathrm{d}} \approx 10\ \mu\mathrm{m}$$

- dipole field:

$$H_\mathrm{d} = \sqrt{\lambda_\mathrm{d}/\lambda_\mathrm{h}} \approx 2\ \mathrm{mT}$$

- dipole velocity:

$$v_\mathrm{d} = \sqrt{\lambda_\mathrm{d}/\rho_\parallel} \approx 1\ \mathrm{mm/s}$$

B phase order parameter:

$$A_{\mu j} = \Delta_{\rm B} R_{\mu j}(\hat{\mathbf{n}}, \theta) e^{{\rm i}\phi},$$

- $R_{\mu j}$ rotation matrix around $\hat{\mathbf{n}}$ with angle θ
- superfluid velocity:

$$\mathbf{v}_{\rm sB} = \frac{\hbar}{2m_3}\nabla\phi$$

Hydrostatic free energy:

- kinetic term ($v_n = 0$):

$$f_{\rm K} = \frac{1}{2}\rho_{\rm s} v_{\rm sB}^2,$$

- dipole term:

$$f_{\rm D} = \lambda_{\rm D}(R_{ii}R_{jj} + R_{ij}R_{ji}) \qquad \Longrightarrow \theta = 104^\circ$$

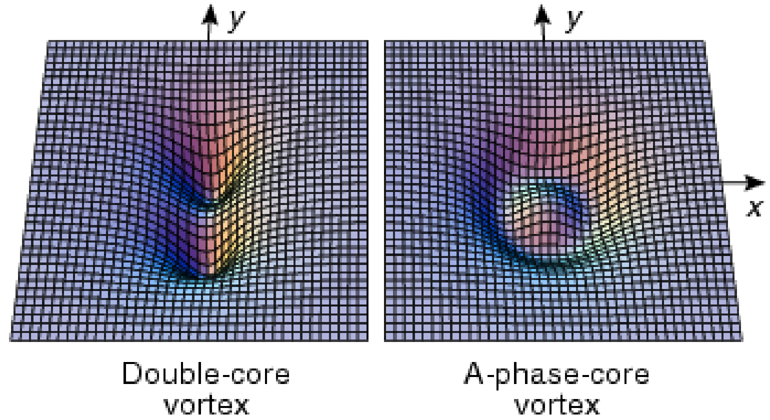
- gradient term:

$$f_{\rm G} = \lambda_{\rm G1}\partial_i R_{\alpha {\rm i}}\partial_j R_{\alpha {\rm j}} + \lambda_{\rm G2}\partial_i R_{\alpha {\rm j}}\partial_i R_{\alpha {\rm j}} \quad \Longrightarrow R_{\mu i} = {\rm const}$$

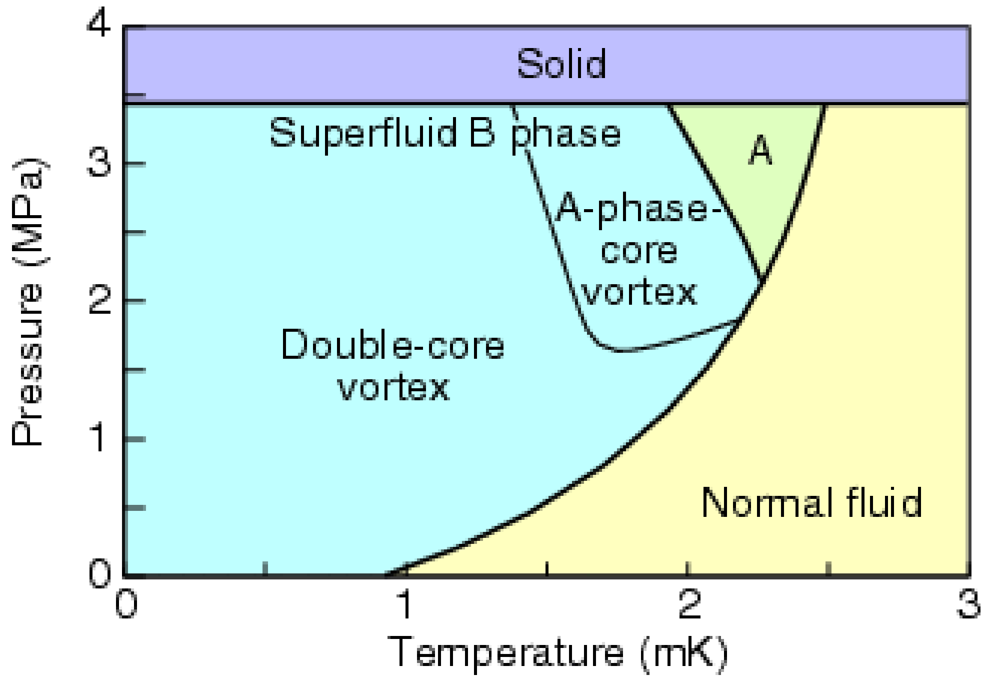
- plus some other terms

Vortices in the B phase:

$\sum_{\mu i} |A_{\mu i}|^2$



- singular (in the scale of ξ_d)
- larger critical velocity ($v_{cB} \gg v_{cA}$)
- carry one quantum of circulation

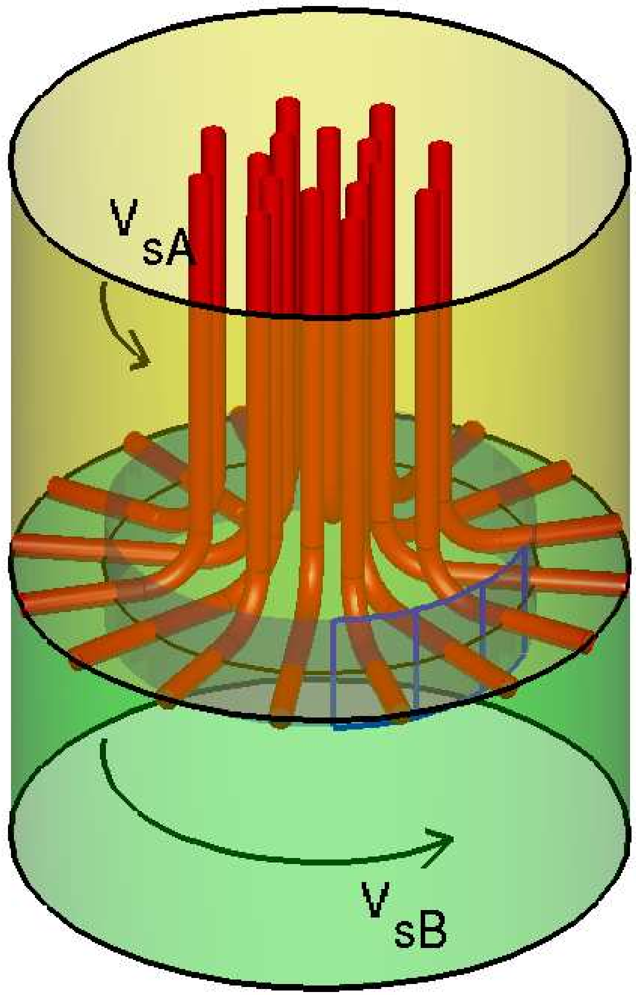


A-B phase boundary:

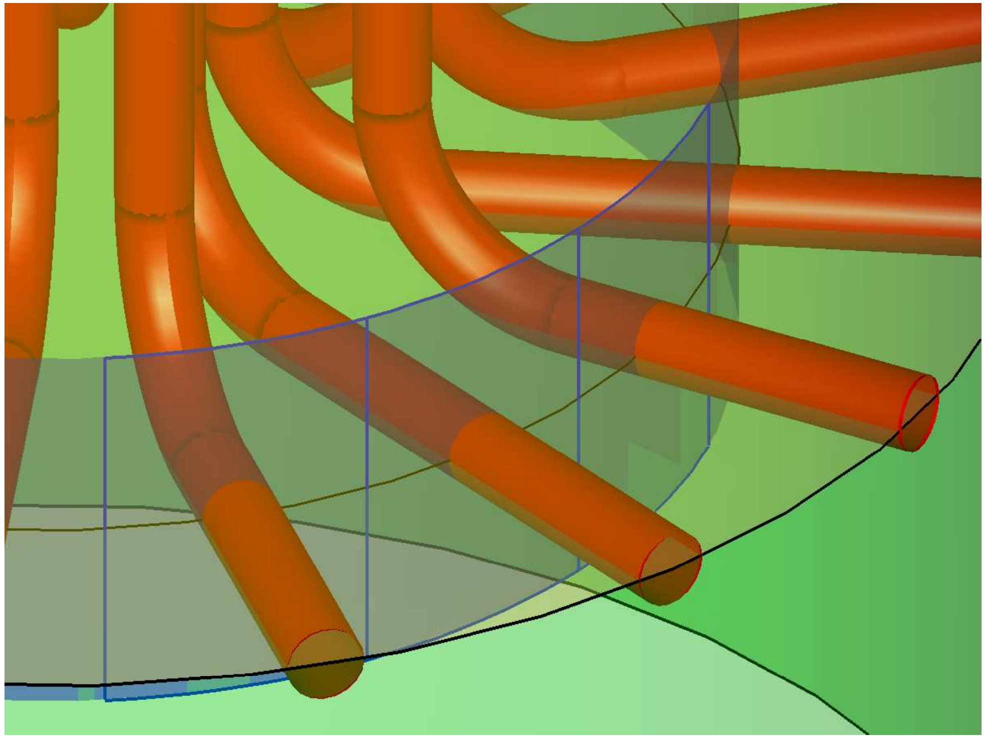
- requirements at the A-B boundary (normal $\hat{\mathbf{s}} = \hat{\mathbf{z}} \parallel \mathbf{\Omega}$):

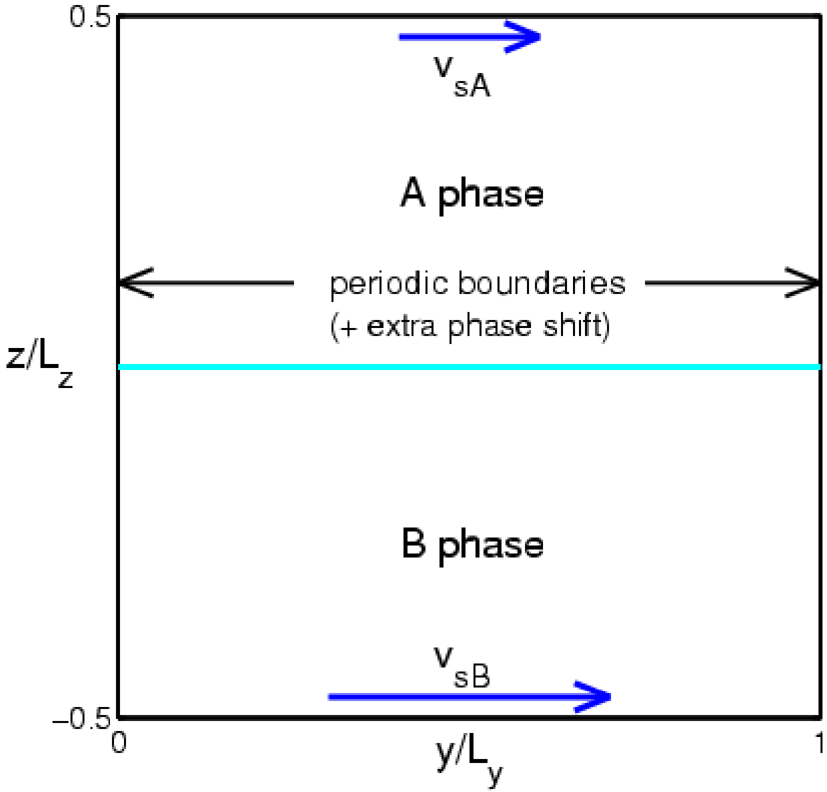
$$\begin{aligned} \hat{\mathbf{d}} &= \overleftrightarrow{R} \cdot \hat{\mathbf{s}} \\ (\hat{\mathbf{m}} + \mathrm{i}\hat{\mathbf{n}}) \cdot \hat{\mathbf{s}} &= e^{\mathrm{i}\phi} \\ \hat{\mathbf{l}} \cdot \hat{\mathbf{s}} &= 0 \end{aligned}$$

coordinate system where $v_{\mathrm{n}} = 0$



Simplified model:

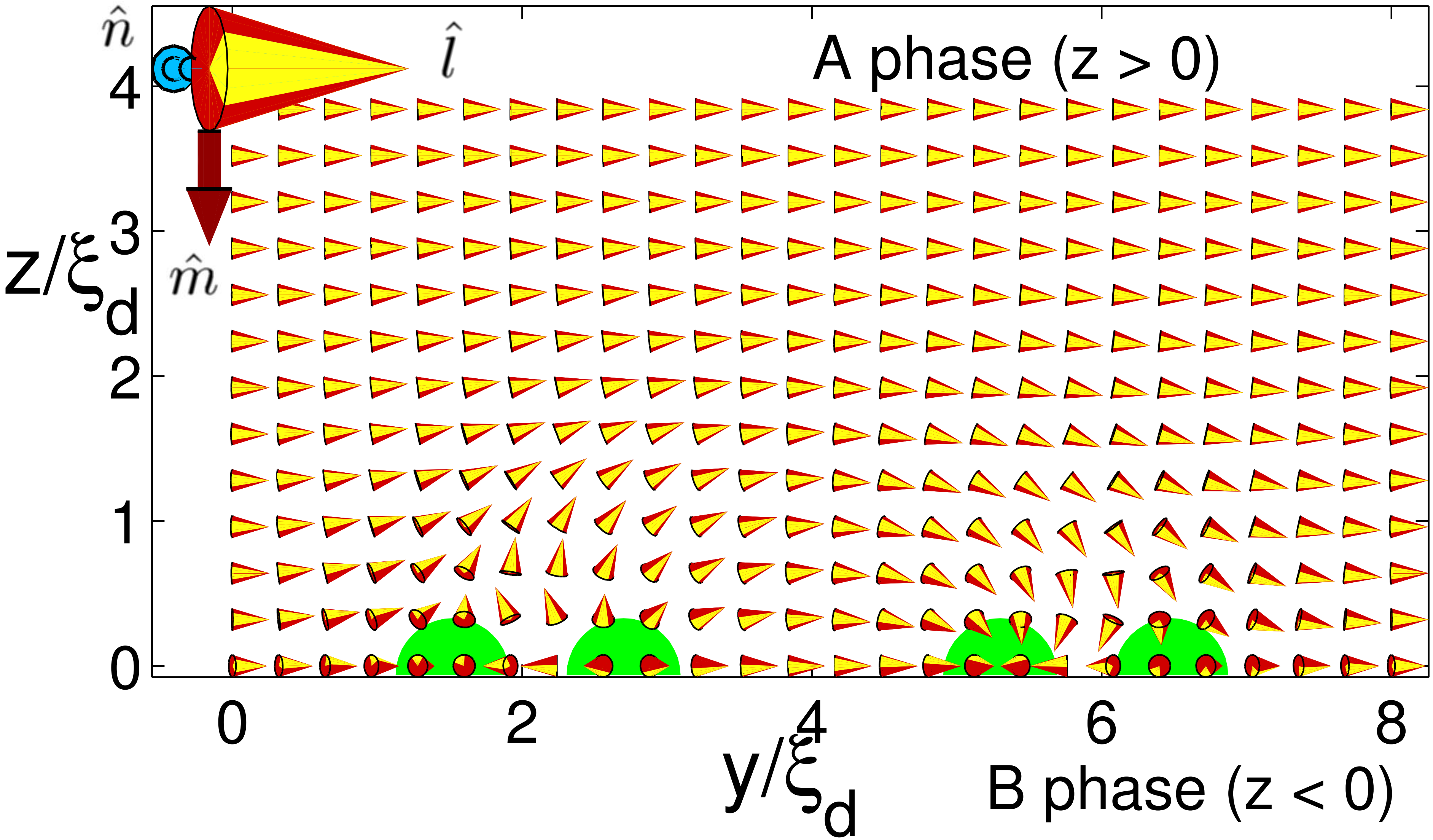




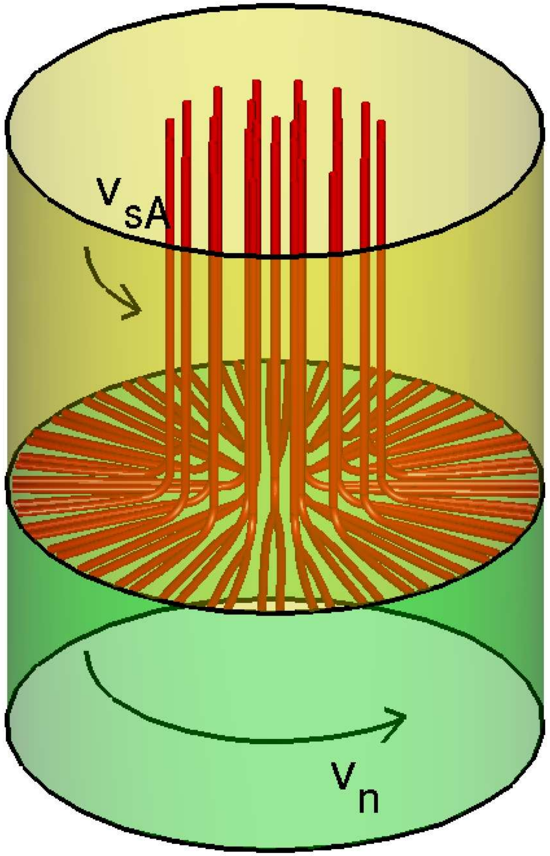
Results:

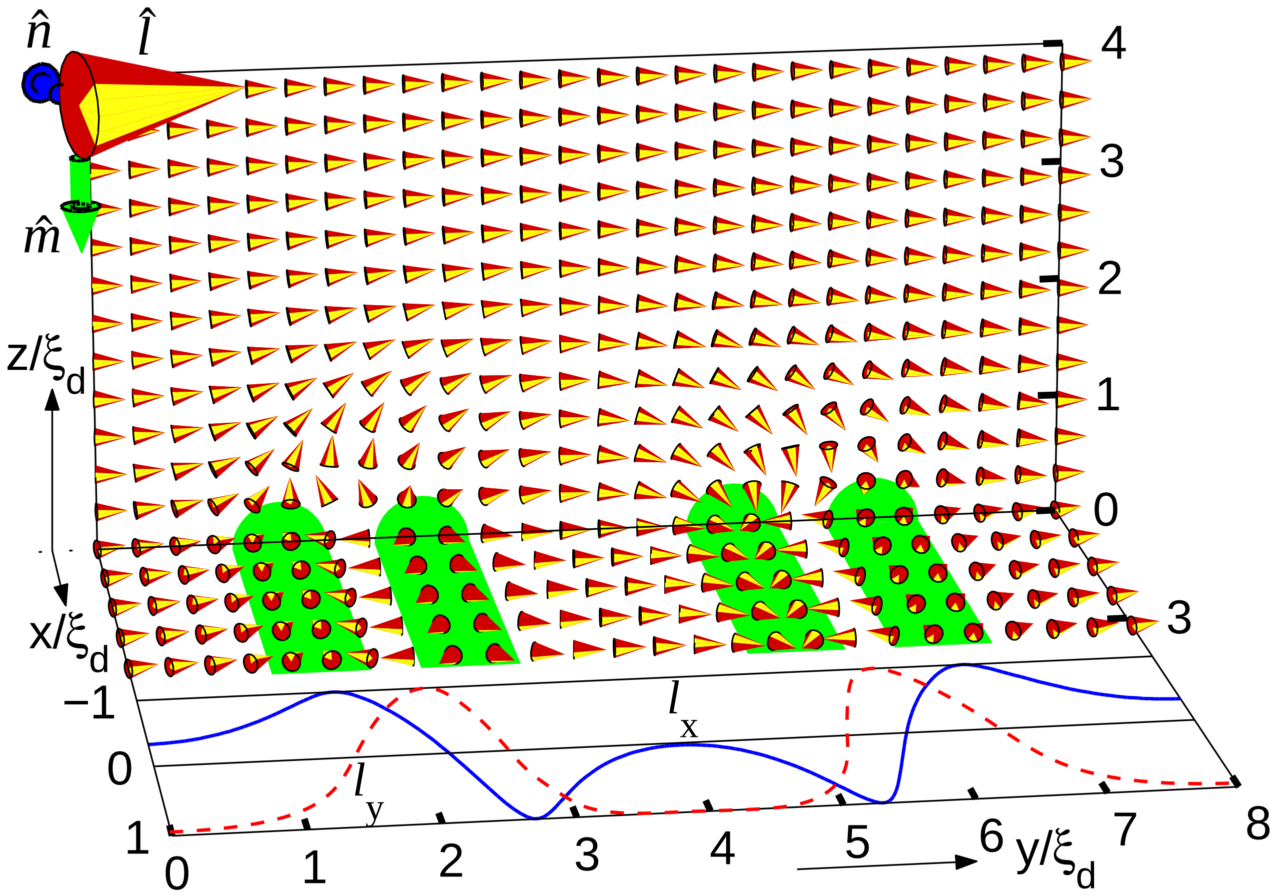
- calculations done for $v_{\mathrm{sA}} = 0$ deep in the A phase
- assumed GL-region ($T \approx T_{\mathrm{c}}$)
- minimization using conjugate gradient method
- two different textures obtained
  - texture depends on the rotation velocity (density of the vortices at the boundary)
- both textures have half-quantum vortex cores at the phase boundary
- $\hat{\mathbf{d}} \approx \hat{\mathbf{x}}$ everywhere
- independent of the initial ansatz

Low density texture (low rotation velocity):

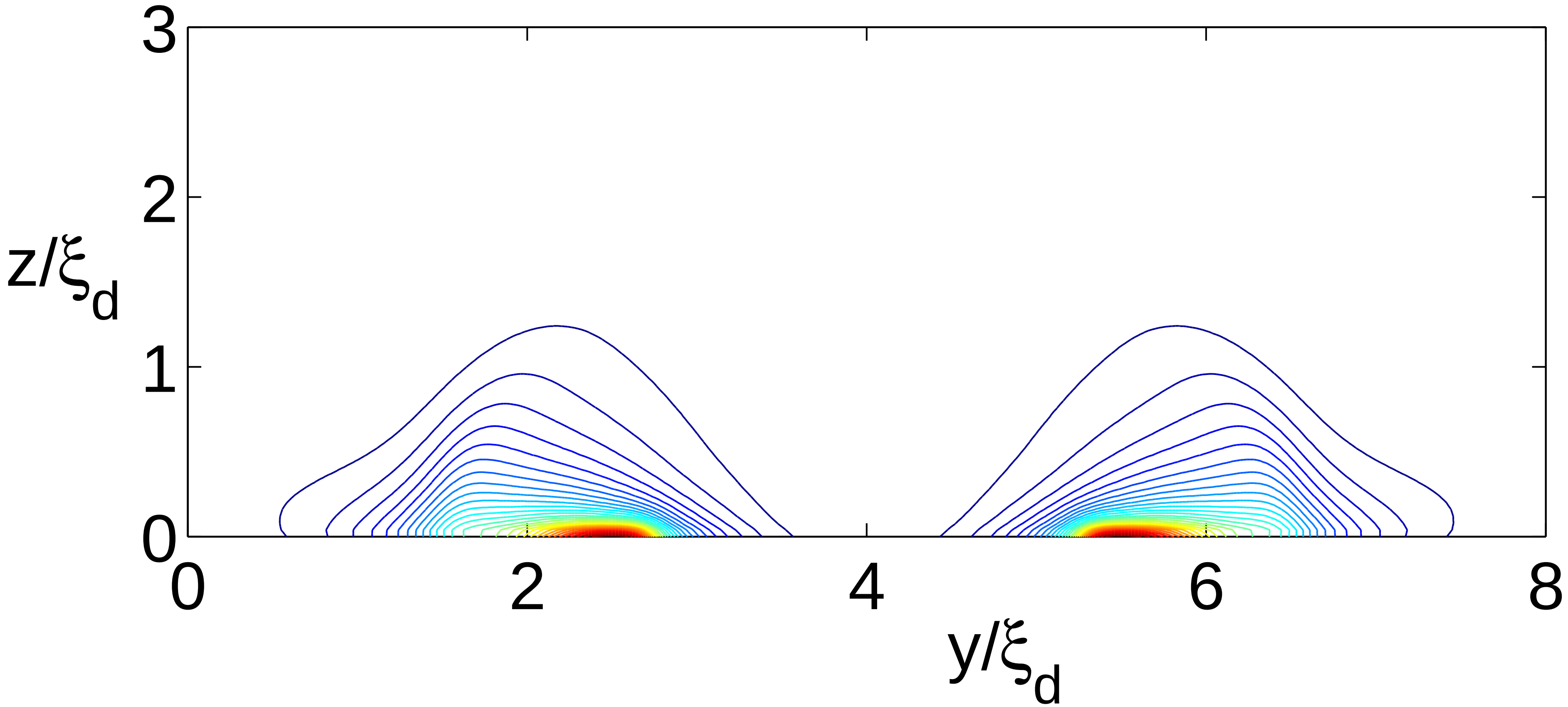


Low density texture (low rotation velocity):

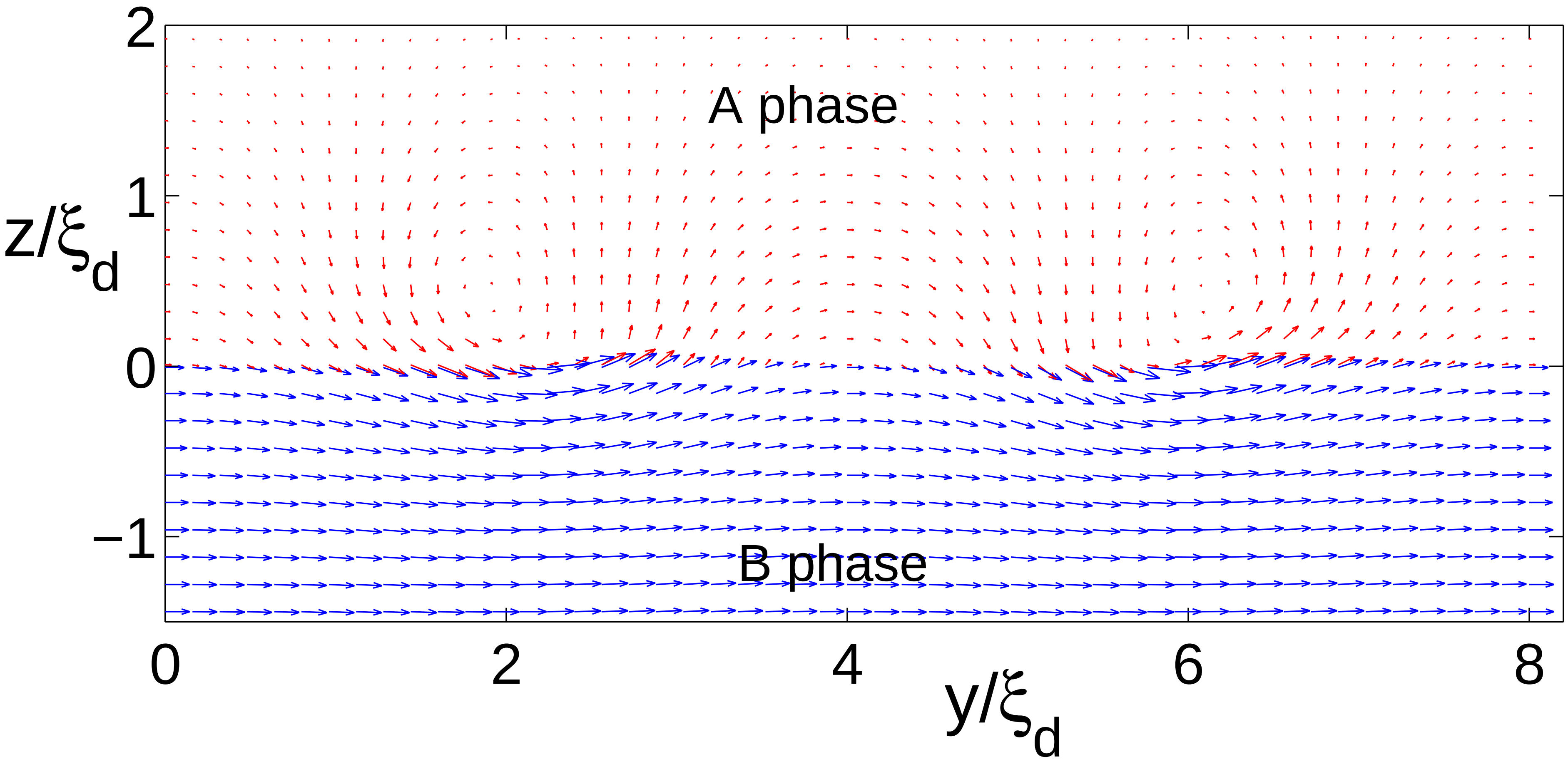




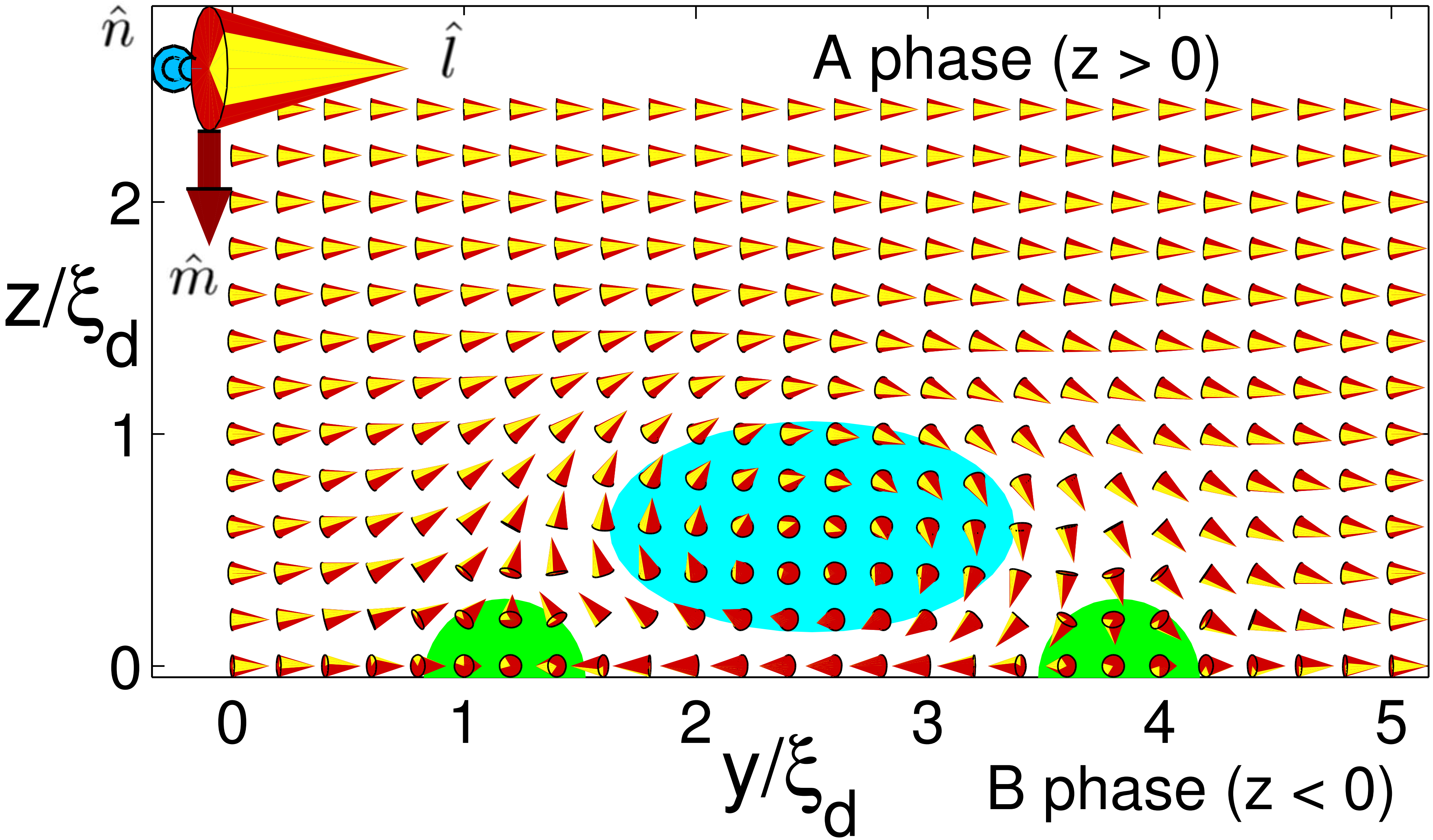
$\nabla \times \mathbf{v}_s$ for the low density texture:



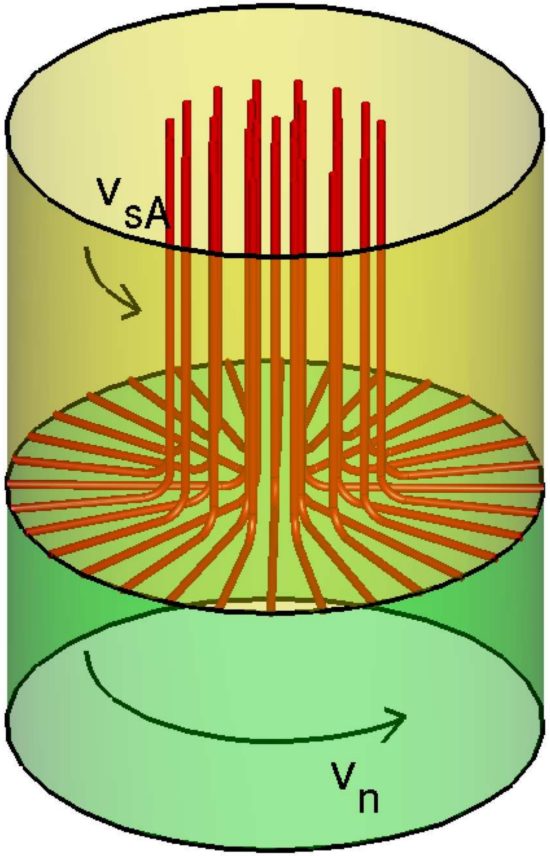
Superfluid current for the low density texture:

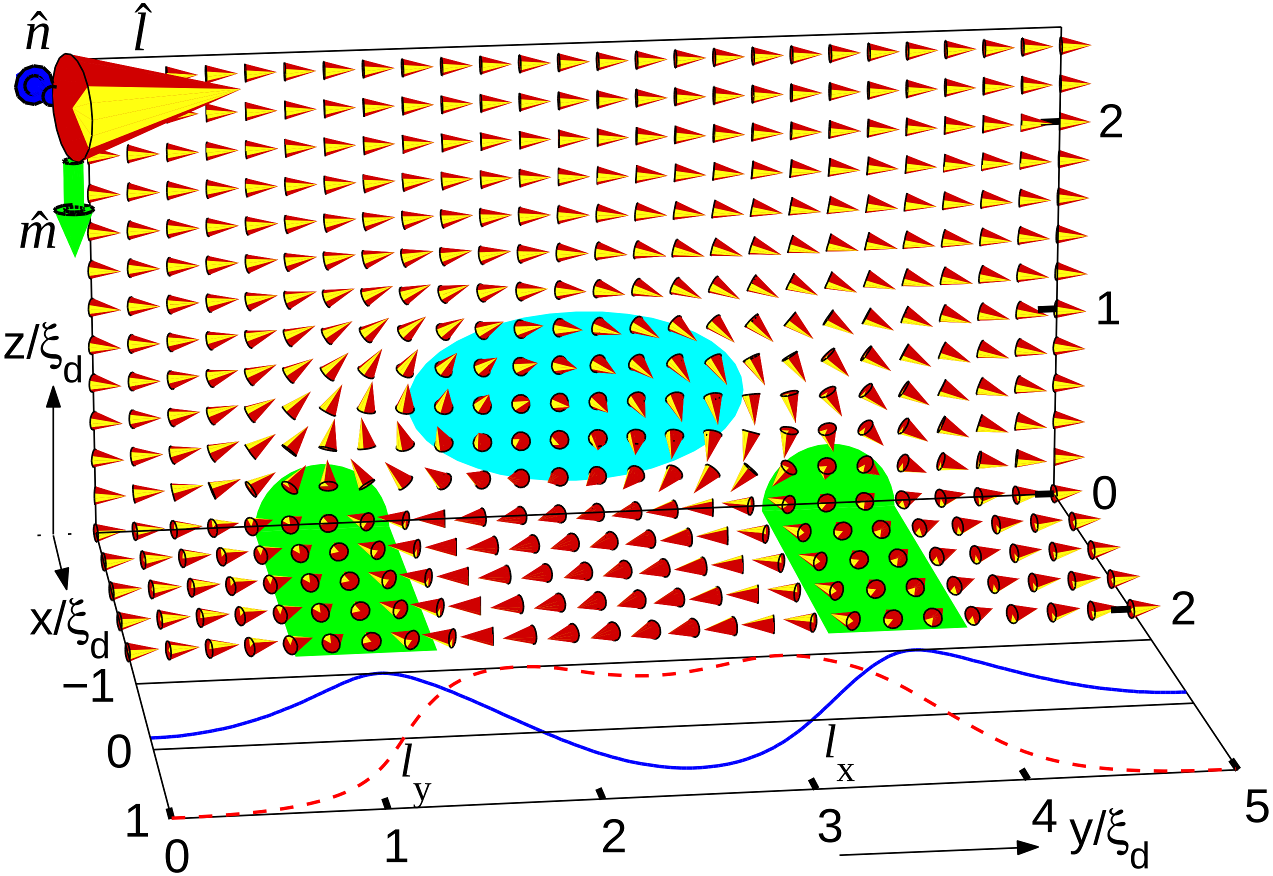


High density texture (high rotation velocity):

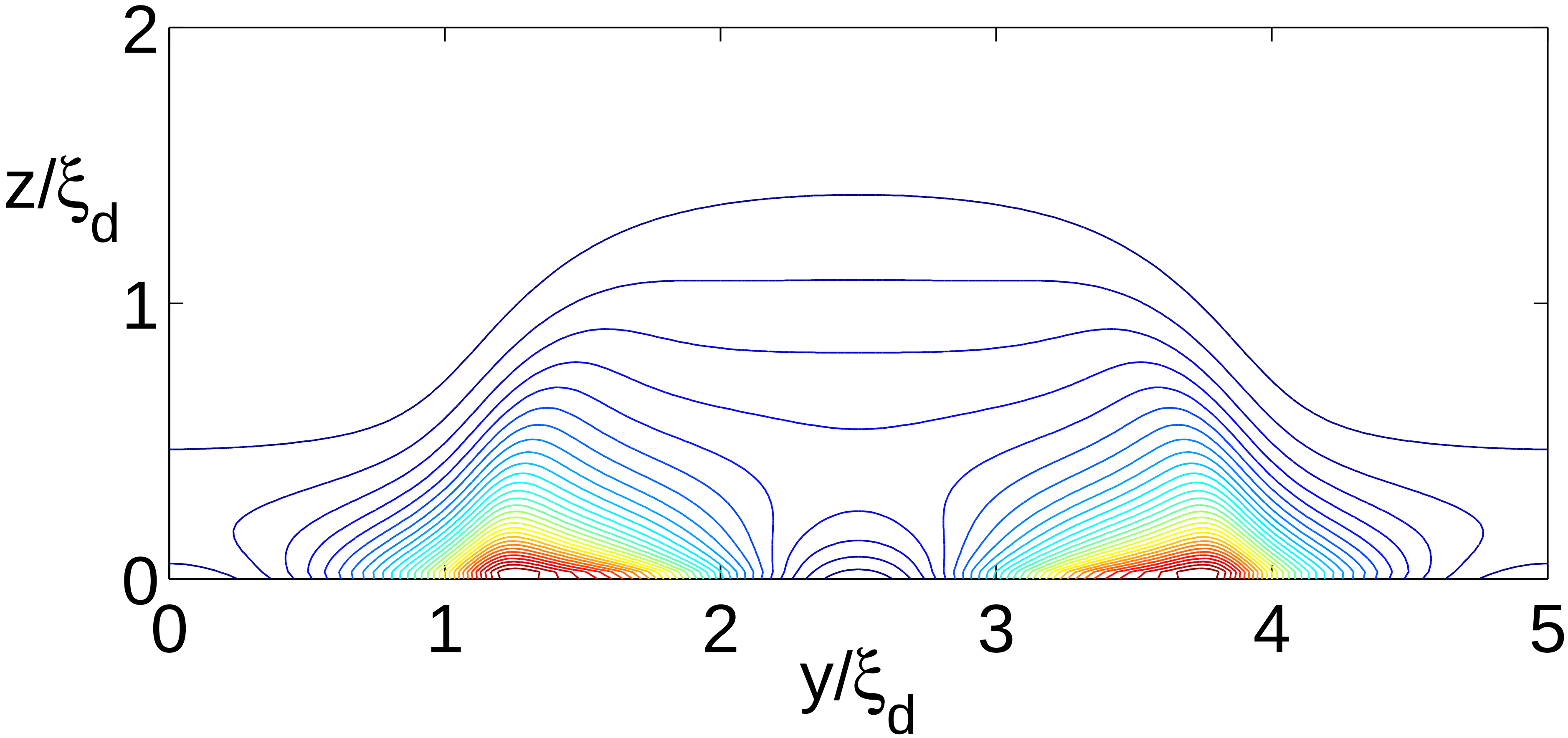


High density texture (high rotation velocity):

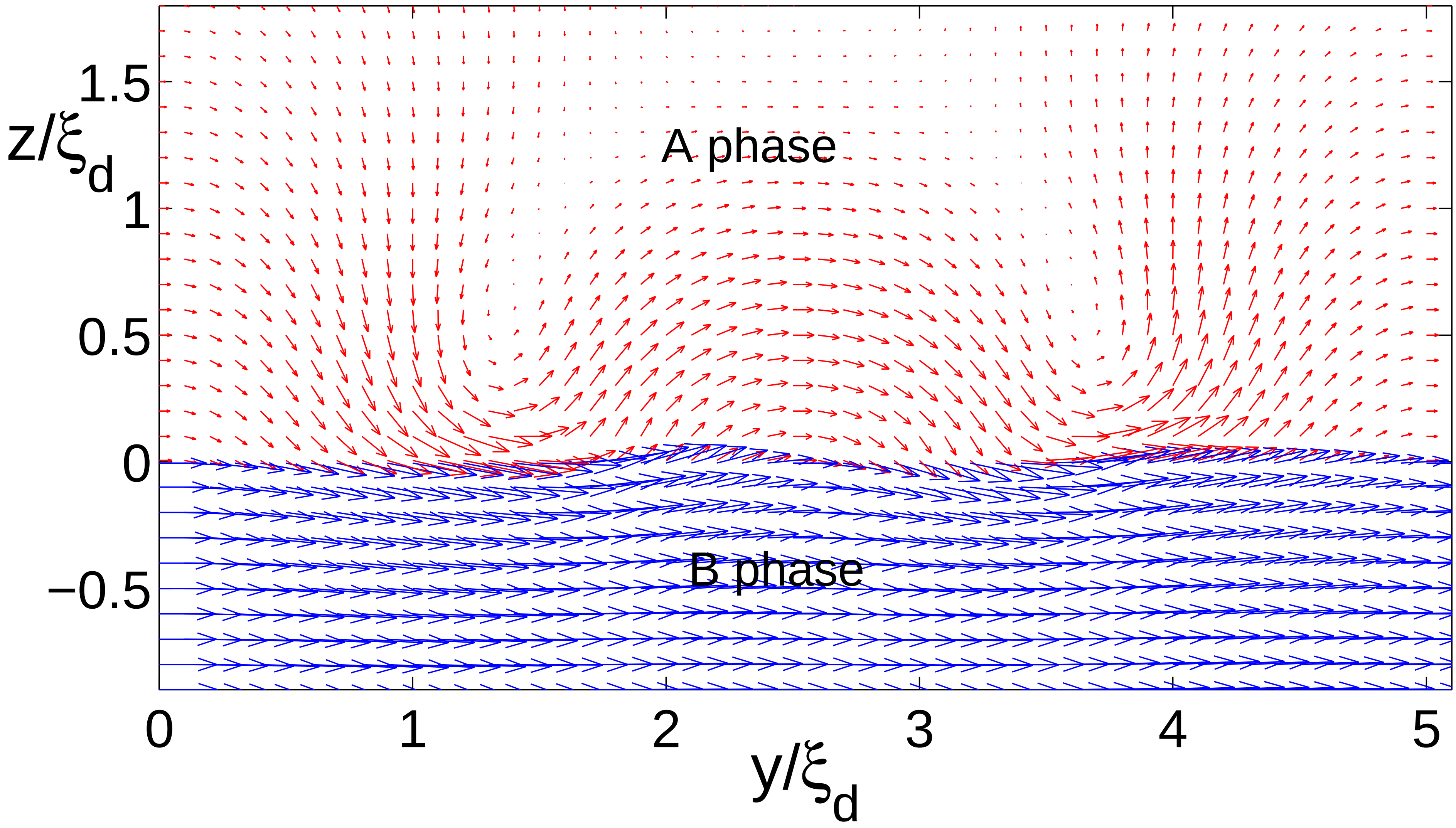




$\nabla \times \mathbf{v}_s$ for the high density texture:



Superfluid current for the high density texture:



Summary:

- calculated the vortex structure at the A-B boundary
- two different textures obtained (low density & high density)
  - half-quantum vortex cores

To be done:

- generalize to other pressures
- possible other textures??
- calculate the NMR spectrum
  - difficult to measure