We study the spectrum of fermion states localized within the vortex core of a weak-coupling p-wave superfluid. The low energy spectrum consists of two anomalous branches that cross the Fermi level as functions of the impact parameter, and generate large density of states at the locations of the half cores of the vortex. Fermi liquid interactions significantly change the vortex structure, which leads to Lifshitz transition in the effective Fermi surface of the vortex core fermions. We apply the results to revise the interpretation of an experiment on rotational dynamics of vortices in superfluid $^3$He-B.

**Abstract**

We study the spectrum of fermion states localized within the vortex core of a weak-coupling p-wave superfluid. The low energy spectrum consists of two anomalous branches that cross the Fermi level as functions of the impact parameter, and generate large density of states at the locations of the half cores of the vortex. Fermi liquid interactions significantly change the vortex structure, which leads to Lifshitz transition in the effective Fermi surface of the vortex core fermions. We apply the results to revise the interpretation of an experiment on rotational dynamics of vortices in superfluid $^3$He-B.

**Double core vortex [1]**

- The stable vortex structure in a weak-coupling p-wave superfluid
- The stable vortex in superfluid $^3$He-B in large part of the phase diagram
- General order parameter in a vortex
  \[ A_{ij}(r, \varphi, z) \rightarrow_{r \rightarrow 0} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} e^{i\varphi} \]
- The order parameter on the $x$ axis of the double-core vortex
  - far left, $\beta = \pi$ center far right, $\beta = 0$
  \[ \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]
- On the $y$ axis the vortex has two half cores.

The distance between the half cores depends strongly on the temperature and pressure (via $P_f$).
- We calculate the structure using the weak coupling approximation [2].

**Quasiparticle excitations**

- There are quasiparticle states bound to the vortex core.
- We calculate the energies $\epsilon$ on the quasiparticle trajectories (trajectory angle $\theta$, impact parameter $b$ with respect to the vortex axis).

  - The trajectory precesses in time, $\theta(t)$. There are two limits.
  - The quasiparticle jumps periodically between the half cores (no transitions between the red and blue branches in figures above).
  - The quasiparticle remains localized in one of the half cores (Landau-Zener transition between red and blue branches at $\theta = \pi/2$ and $3\pi/2$).
  - The transition between the two limits is a Lifshitz transition in the effective Fermi surface of the fermions in the vortex core. Above the transition the half cores are expected to accommodate a Majorana fermion [3].
- The quasiparticle density of states at the $\epsilon = 0$

**Rotational friction**

- Oscillations of the core [4]: resonances at minigap values
  - We calculate the minigap $E_m$ and the friction coefficient $f = f_\text{LZ}(\epsilon_{\text{F}}) \epsilon_m^2$
    - below transition (avoided crossing): resonance at $\omega = 2E_m/h$
    - above transition (strong LZ tunneling): resonance at $\omega = E_m/h$
  These can be seen as Fourier components in $b(\theta)$. We calculate the same parameters. Our value of $f$ is three orders of magnitude larger than fitted in Ref. [5].

**Comparison to experiment [5]**

- Precessing magnetization $M$ twists the double-core vortex [5].
  A model for this was proposed in Ref. [5]. Comparison to experiment allowed to extract the friction coefficient $f$ and the twist rigidity $k$.
  - We calculate the same parameters. Our value of $f$ is three orders of magnitude larger than fitted in Ref. [5].
  - To remove the inconsistency we add to the model [5] an azimuthal shear $T_\alpha = k_\alpha (\alpha - \phi)^2$. This allows difference between rotation angles in the core ($\phi$) and far from to vortex axis ($\alpha$), where the dipole torque acts.
  - The new model gives the diffusion equation
    \[ \dot{\phi} = k_\alpha (\alpha - \phi) + P_{\phi} \]
    where $P_{\phi}$ is the absorbed power per vortex length.
    - In addition to removing the discrepancy, the new model gives the time scale $\tau_{\text{LZ}} \sim \frac{\epsilon}{\pi^2 k}$ of several minutes. This agrees with the observed slow relaxation, which remained unexplained in Refs. [5, 6].

**References**