Exercises E.2.1, E.2.2 from Datta's book.

3. In the lectures the Landau levels were solved for an ideal conductor. In the presence of impurities the Landau levels will be broadened. Show this by calculating in the first order perturbation theory the effect of a single circularly symmetric impurity potential

$$U(r, \phi) = \begin{cases} U_0 & \text{for } r < r_0 \\ 0 & \text{for } r > r_0 \end{cases}$$

on the energies in the lowest Landau level. Calculate the energies by assuming r_0 small compared to the cyclotron radius of lowest Landau level, $r_0 \ll \sqrt{\hbar/m\omega_c}$. (Note that because all the states in the same Landau level have initially equal energies, we have to deal with degenerate perturbation theory in general. However, for a proper choice of basis functions the perturbation forms a diagonal matrix $\langle\psi_i|U|\psi_j\rangle = c_i\delta_{ij}$, and one can effectively use simple (nondegenerate) perturbation theory. In the present case this is achieved by using the symmetric gauge centered at the same position as the impurity, and using circularly symmetric eigenstates, which were studied in exercise 5 of set 1.)

4. (a) Use the Landauer formula to derive the contact resistance (Sharvin resistance)

$$\frac{1}{R_c} = \frac{e^2 k_F^2 S}{4\pi^2\hbar} \tag{1}$$

for a channel between two bulk (3D) conductors. Assume that the transverse cross section S of the channel is not too small: $S \gg k_f^{-2}$. (Hint: consider first a rectangular channel. How this case can be generalized to different shapes?)

(b) The Sharvin resistance can also be derived semiclassically by considering electrons as point particles that move with velocity v_f along straight trajectories except when hitting a wall. In three dimensions their density of states is $N_v = mk_f/\pi^2\hbar^2$ (at the Fermi surface, including 2 for spin). Derive the Sharvin resistance (1) in this way. In order to calculate contact resistance only, it has to be assumed that the reflections of the electrons at the channel walls are specular (mirror like). (Hint: start from similar formula as in exercise 3 of set 2 but in 3D).