Tfy-44.140 Quantum computing 5. Exercise Spring 2007 16.4.2007

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1. Matrix decomposition

Find a quantum circuit which implements the unitary matrix

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 & 0 & c \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & b & 0 & 0 & 0 & 0 & d \end{pmatrix}.$$

You may use gates such as the negated control. Hint: Lecture notes starting from page 79.

2. Gate decomposition

Show that the circuit in Fig. 4.13 of the lecture notes is a controlled-U gate with three control bits, i.e., C^3U, where $U = V^2$. Sketch the C^4U and its corresponding decomposition.

3. Quantum Fourier Transform

(a) Design a gate that performs the quantum Fourier transform in the case of two qubits and has the SWAP gate in the very end of the implementation (Exercise 6.3). Verify that your scheme works by writing down the matrices explicitly.

(b) Now, based on the above, repeat the procedure for the case of three qubits. This time the gate to be relocated in the end of the circuit is the permutation gate. Write down the 8×8 unitary matrix that performs the operation. Hint: You should be able to do this without laborious matrix multiplications.