Tfy-44.140 Quantum computing 4. Exercise Spring 2007 12.4.2007

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1. CNOT

Exercise 4.2 in the lecture notes.

2. Gates of a quantum computer

a) An arbitrary unitary 2×2 matrix can be written in the form ($|a|^2 + |b|^2 = 1$)

$$U = e^{i\theta/2} \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}.$$

Use this fact to show that σ_x is the unique 2×2 matrix that satisfies $U|0\rangle = |1\rangle$ and $U|1\rangle = |0\rangle$.

b) Let $U_{\text{XOR}} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X$. Show that

$$U_{\text{XOR}}|x\rangle \otimes |y\rangle \equiv U_{\text{XOR}}|x, y\rangle = |x, x \oplus y\rangle.$$

c) Write down the unitary matrix V that yields

$$V|x, y\rangle = |x \oplus y, y\rangle, \quad x, y \in \{0, 1\}.$$

3. Hadamard gate

Exercise 4.5 in the lecture notes.

4. Elementary gates

a) Let

$$U_{\text{OR}} = |00\rangle\langle 11| \otimes X + |01\rangle\langle 10| \otimes X + |10\rangle\langle 01| \otimes X + |11\rangle\langle 00| \otimes I,$$

Show that U_{OR} satisfies

$$U_{\text{OR}}|x, y, 0\rangle = |\neg x, \neg y, x\check{y}\rangle, \quad x, y \in \{0, 1\}.$$

b) Show that the NAND gate can be obtained from the CCNOT gate.

c) Define

$$U_{\text{SWAP}} = |00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01| + |11\rangle\langle 11|.$$

Show that U_{SWAP} can be written as

$$\begin{aligned} U_{\text{SWAP}} &= (|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X)(I \otimes |0\rangle\langle 0| + X \otimes |1\rangle\langle 1|) \\ &\quad (|0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes X). \end{aligned}$$

d) Show that the NOT, AND and OR gates are obtained from a single Fredkin gate.