Tfy-44.140 Quantum computing 3. Exercise Spring 2007 2.4.2007

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1. Bloch sphere

Find the density matrix of the pure state

$$|\psi(\theta, \phi)\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

and write it in the basis formed by the identity matrix and the Pauli matrices (Eq. 3.4 in the lecture notes).

2. Bell basis

The Bell basis given in Eq. 3.6 in the lecture notes is obtained from the ordinary basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ by a unitary transformation. Write down this transformation explicitly. Use this result to find the representation of the matrix CNOT in Bell's basis.

To remind: CNOT in the ordinary basis is

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

3. No-cloning theorem

Suppose U is a cloning unitary transformation such that

$$\begin{aligned} |\Psi\rangle &\equiv U|\psi\rangle|0\rangle = |\psi\rangle|\psi\rangle \\ |\Phi\rangle &\equiv U|\phi\rangle|0\rangle = |\phi\rangle|\phi\rangle \end{aligned}$$

for arbitrary $|\psi\rangle, |\phi\rangle$.

a) Write down $\langle\Psi|\Phi\rangle$ in all possible ways.

b) Show, by inspecting the result of a), that such U cannot exist.

4. Reversible calculation

Evaluation of arbitrary function $f(x)$ can be implemented reversibly, as the quantum computer concept requires, by a unitary transformation U:

$$U\sum_x |x\rangle_n|0\rangle_n = \sum_x |x\rangle_n|f(x)\rangle_n,$$

where $|x\rangle_n$ stands for the tensor product state $|b_{n-1}b_{n-2}\dots b_0\rangle$ with $x = b_{n-1}2^{n-1}+b_{n-2}2^{n-2}+\dots b_0$. That is, the first register is for the input while the second one is for the output. Here, both registers consists of n qubits.

Task: Find the explicit matrix representation for unitary matrix U, which implements the modular exponential function $f(x) = a^x \mod N$, for parameter values: $a = 3$, $N = 4$, and size of the quantum register $n = 2$.