Exercise 9

(On Mon Nov 28th at 12:15, in F2. Participants should present their solutions on the blackboard.)

8.1 Wiener-Khintchine theorem:
Assume we measure random noise signal $\delta x(t)$ in a stationary system over a long time $t \to \infty$. As $\delta x(t)$ is a real quantity, its Fourier transform satisfies $\delta x(-\omega) = \delta x^*(\omega)$. Then one may write

$$\delta x(t) = \int_0^\infty \frac{d\omega}{2\pi}[\delta x(\omega)e^{-i\omega t} + \delta x^*(\omega)e^{i\omega t}]. \tag{1}$$

Now consider a spectrum analyzer which measures this signal. It contains a narrow-band band-pass filter which only leaves the (angular) frequencies $\omega \in [\omega_0 - \Delta\omega/2, \omega_0 + \Delta\omega/2]$, and an output detector that measures the mean square of this signal (corresponding to the signal power). Show that the averaged squared signal from the filtered band equals the spectral density of noise, $S(\omega_0)$ times the band width $\Delta f = \Delta\omega/2\pi$ (assuming that $S(\omega_0)$ does not essentially change within this band), i.e.,

$$\langle \delta x^f(t) \delta x^f(t) \rangle = S(\omega_0)\frac{\Delta\omega}{2\pi} = S(\omega_0)\Delta f. \tag{2}$$

Here $\delta x^f(t)$ is the signal after the band-pass filtering,

$$\delta x^f(t) = \int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} \frac{d\omega}{2\pi}[\delta x(\omega)e^{-i\omega t} + \delta x^*(\omega)e^{i\omega t}]. \tag{3}$$

Hint: Write down the spectral density of noise for two times, t and t', and use the fact that in a stationary setup, it only depends on $t - t'$.

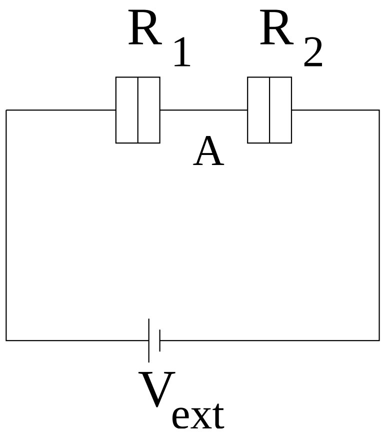
8.2 Prove Eq. (8.31),

$$\langle \{\delta \hat{I}_\alpha(\omega), \delta \hat{I}_\beta(\omega')\} \rangle = 2\pi\delta(\omega + \omega')S_{\alpha\beta}(\omega) \tag{8.31}$$

8.3 Prove Eqs. (8.37) and (8.38),

$$S = \frac{4e^2}{h}\sum_n \int \mathrm{d}E \left\{T_n(E)[f_L(1-f_L) + f_R(1-f_L)] + T_n(E)[1-T_n(E)](f_L - f_R)^2\right\}, \tag{8.37}$$

$$S = \frac{4e^2}{h}\left[2k_BT\sum_n T_n^2 + eV\coth(\frac{eV}{2k_BT})\sum_n T_n(1-T_n)\right] \tag{8.38}$$



8.4 Consider two tunnel barriers, of resistances R_1 and R_2 in series, connected to an ideal voltage source (see figure above) with voltage V_{ext}. Assume that the current noises through R_1 and R_2 are uncorrelated, such that the noise powers from these two sources can be simply added to obtain the total current noise in the system. Then, we can analyze the noise due to R_1 treating R_2 simply as a resistor and vice versa. The current fluctuations in the resistor R_1 produce voltage fluctuations $\Delta V(\omega)$ at the point A between the barriers. These result in additional fluctuations $\Delta V(\omega)/R_1$. Thus, the total current fluctuations due to the fluctuations in R_1 are

$$\Delta I(\omega) = \Delta V(\omega)/R_1 + \delta I(\omega), \tag{4}$$

where $\delta I(\omega)$ is the intrinsic (shot or thermal) noise over R_1. $\Delta V(\omega)$ may be easily calculated by requiring the fluctuations in the total voltage over the system to vanish, i.e.,

$$\Delta V(\omega) + R_2 \Delta I(\omega) = 0. \tag{5}$$

a) By using the fact $S_{int}(\omega) = \langle \delta I(\omega) \delta I(-\omega) \rangle$, find the total current noise $S_I = \langle \Delta I(\omega) \Delta I(-\omega) \rangle$ (using an incoherent sum due to fluctuations in R_1 and those in R_2).

b) Assuming $R_1 = R_2$, show that adding tunnel barriers but keeping the same total resistance decreases the shot noise (noise at $T = 0$, i.e., $S_{int} = 2e\langle I \rangle$), but the thermal noise $S_{int}^i = 4k_B T/R_i$ remains unaffected.

c) Calculate the voltage noise $S_V = \langle \Delta V \Delta V \rangle$ at point A.