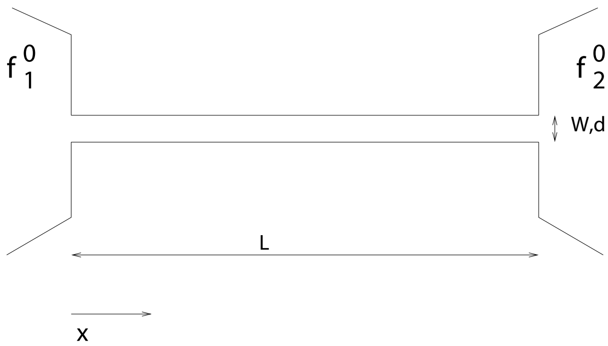
Exercise 4

(On Mon Oct 17th at 12:15, in F2. Participants should present their solutions on the blackboard.)

In each exercise, you are requested to (i) solve for the distribution function in a wire between two reservoirs (where the distribution function has the Fermi function form with chemical potentials μ_1 and μ_2 and temperatures T_1 and T_2) in a stationary limit (time derivative of the distribution function equal to zero) and to (ii) compute the current(s) flowing in the sample due to the applied voltage $eV = \mu_2 - \mu_1$ and/or the temperature difference $T_2 - T_1$. This means that we may assume the reservoirs as boundary conditions to the Boltzmann equation. You may assume a constant density of states ($\nu(E) = 1$) and a diffusion constant $D(E) = D =$ const. throughout.

Note that even if you could not solve the previous problem, you may try to solve the next one.

Each problem considers an essentially quasi-one-dimensional wire (see figure). In the diffusive limit (exercises 2-3), the distribution function changes only along the wire in, say, x-direction, and there are no changes in the perpendicular directions. The ballistic limit has a slightly different ideology and you may try to solve it after the other parts.



4.1 Consider a ballistic wire (no scattering) of width W, thickness d and length L between two reservoirs (see figure). Find the stationary-state distribution function $f(\mathbf{r}, \hat{p}; E)$ of electrons inside the wire. *Hint: define "left-movers" and "right-movers".*

4.2 Consider a similar situation with a diffusive wire, but no inelastic scattering: $l_{el} \ll L \ll l_{ee}, l_{eph}$.

(a) Solve the diffusive-limit Boltzmann equation with boundary conditions given as in the figure to find out the stationary (time independent) distribution function $f(x, E)$.

(b) Then, calculate the charge current flowing in the system for arbitrary potentials μ_1 and μ_2 and temperatures T_1 and T_2.

(c) Finally, calculate the heat current flowing in the structure for $\mu_1 = \mu_2 = \mu$ and temperatures T_1 and T_2.

Hint: use the formulae given below.

4.3 Now assume $L \gg l_{ee}$ but $L \ll l_{eph}$ and otherwise the same system. In this case, the space dependence of the distribution function can be described through the form $f(x, E) = f^0(E; \mu(x), T(x))$, where $f^0(E; \mu, T)$ is a Fermi function with potential $\mu(x)$ and temperature $T(x)$.

(a) Plugging this into the diffusive-limit Boltzmann equation, find the kinetic equations for $\mu(x)$ and $T(x)$. *Hint: you can get rid of the energy dependence by integrating over the energies in the same way as one integrated over the momentum directions in the derivation of the diffusive limit - use the formulae given below.*

(b) Once you have the kinetic equations, find the potential/temperature profile $\mu(x)$ and $T(x)$ and the corresponding currents (again for the heat current, it is enough to treat only the case $\mu_1 = \mu_2$).

The case of a macroscopic wire, $L \gg l_{eph}$ reduces just to finding the potential profile for a given voltage. In this case, temperature difference would require maintaining a temperature difference also for the phonons and would then go as the quasiequilibrium limit.

Hint: You may have use for the following formulae:

$$\int_{-\infty}^{\infty} (f^0(E;\mu_1,T_1) - f^0(E;\mu_2,T_2))dE = \mu_1 - \mu_2 \tag{1}$$

$$\int_{-\infty}^{\infty} E(f^0(E;\mu_1,T_1) - f^0(E;\mu_2,T_2))dE = \frac{\pi^2}{6}k_B^2(T_1^2 - T_2^2) + \frac{1}{2}(\mu_1^2 - \mu_2^2) \tag{2}$$

$$\int_{-\infty}^{\infty} dE \partial_\mu f^0(E;\mu,T) = 1 \tag{3}$$

$$\int_{-\infty}^{\infty} dE \partial_T f^0(E;\mu,T) = 0 \tag{4}$$

$$\int_{-\infty}^{\infty} dE E \partial_\mu f^0(E;\mu,T) = \mu \tag{5}$$

$$\int_{-\infty}^{\infty} dE E \partial_T f^0(E;\mu,T) = \frac{k_B^2 T \pi^2}{3} \tag{6}$$

Here $f^0(E;\mu,T)$ are Fermi functions.