Exercise 3

(On Mon Oct 10th at 12:15, in F2. Participants should present their solutions on the blackboard.)

3.1 Compare the transmission probabilities of a two-scatterer system obtained by summing the transmission probabilities for individual "Feynman paths" of the particle and the quantum-mechanical result obtained in Exercise 2.3. Show that the previous method results into a classical Ohm's law -type summing of the resistances.

3.2 Consider an Aharonov-Bohm ring, where both arms contain a single propagating mode. Assume that the scattering matrix for the three-way junction at both ends (positions "A" and "B" in Fig. 3.1) are described with the scattering matrix

$$S = \begin{pmatrix} c & \sqrt{\epsilon} & \sqrt{\epsilon} \\ \sqrt{\epsilon} & a & b \\ \sqrt{\epsilon} & b & a \end{pmatrix}, \tag{1}$$

where a, b, c, and ϵ are real numbers.

(a) Show that in order to ensure unitarity of S, we have to have

$$c = \pm\sqrt{1-2\epsilon}, \quad a = (1-c)/2, \quad b = (1+c)/2. \tag{2}$$

Thus the entire matrix is specified by the function ϵ.

(b) Assuming that the mode in the upper arm obtains a phase shift $\theta + \phi/2$ and that in the lower arm a phase shift $\theta - \phi/2$, where $\phi = \Phi/\Phi_0$ and $\theta = \sqrt{2mE}(\pi r/\hbar)$ is the dynamical phase (equal in both arms), calculate the total transmission probability for the ring as functions of ϵ, θ and ϕ. Plot the result for $\theta = 0, \pi/2 and \pi$, $\epsilon = 0.01$ as a function of ϕ.

3.3 Calculate the weak localization correction ΔG to the conductance of a quasi one-dimensional wire with length $L \gg \ell_\varphi$ and width $W \ll \ell_\varphi$. *Hint: divide the wire into L/ℓ_φ coherent sections. Then sum up the resistances (with the WL correction) of these sections. Calculate also the relative correction $\Delta G/G$. This is also the conductivity correction.*

3.4 Show that in the two-dimensional case $W, L \gg \ell_\varphi$, the conductivity is

$$\sigma_Q = \sigma_{CL} - \frac{2e^2}{\pi h}\ln(\ell_\varphi/\ell_{\mathrm{el}}). \tag{3}$$

Hint: show that the conductance of a cylindrical conductor is

$$G = \frac{\pi\sigma}{\ln(W_{\max}/W_{\min})}, \tag{4}$$

where $W_{\max}$ and $W_{\min}$ are the outer and inner radii of the cylinder. Now combine the conductances of such circular units with $W_{\max} = \ell_\varphi$ and $W_{\min} \sim \ell_{\mathrm{el}}$.