Quantum Shot Noise: From Schottky to Bell

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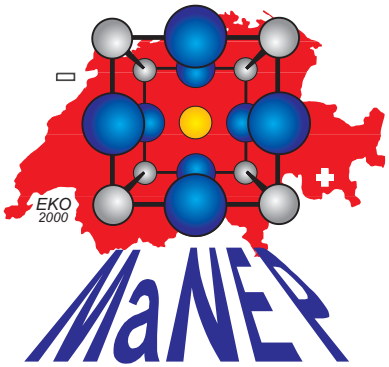
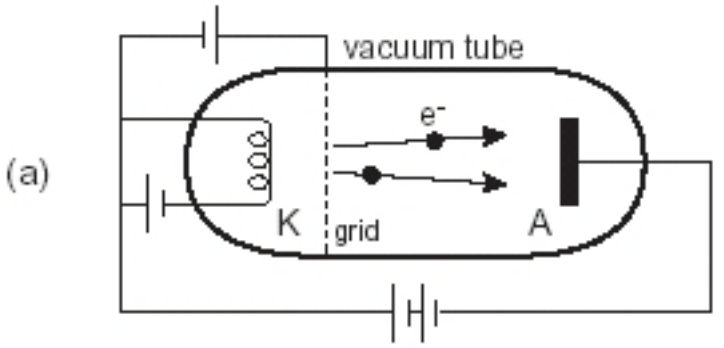


From Schottky to Bell

Classical shot noise:

W. Schottky, Ann. Phys. (Leipzig) 57, 541 (1918)

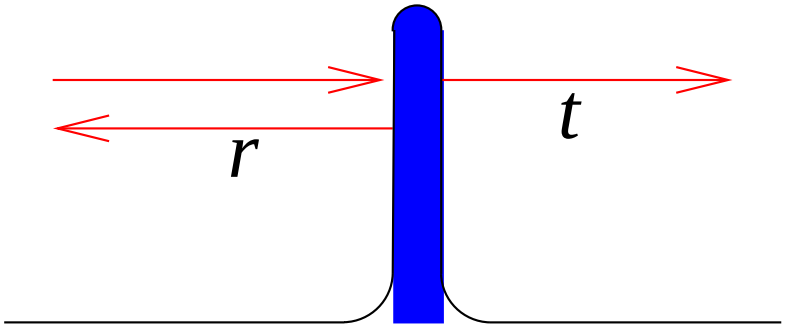
$$\langle (\Delta I)^2 \rangle_\nu = 2e\langle I \rangle$$



Quantum shot noise:

Khlus (1987), Lesovik (1989), Yurke and Kochanski (1989), Buttiker (1990)

$$|\Psi>_{inc} = e^{ikx}$$
$$|\Psi>_{ref} = re^{-ikx}$$
$$|\Psi>_{tra} = te^{ikx}$$



$\Rightarrow$

$$\langle (\Delta n_T)^2 \rangle = T(1-T) \quad \Rightarrow \quad \langle (\Delta I)^2 \rangle_\nu = 2e\langle I \rangle (1-T)$$

Entangled States

J. Bell, Physics 1, 195 (1964)

Content of lecture

Scattering theory conductance and noise

Two-particle Aharonov-Bohm effect

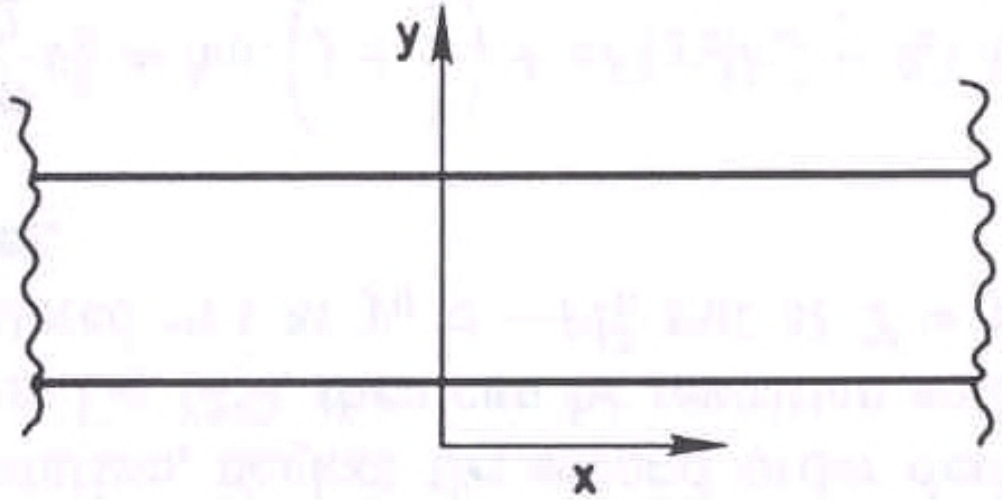
Orbital quasi-particle entanglement

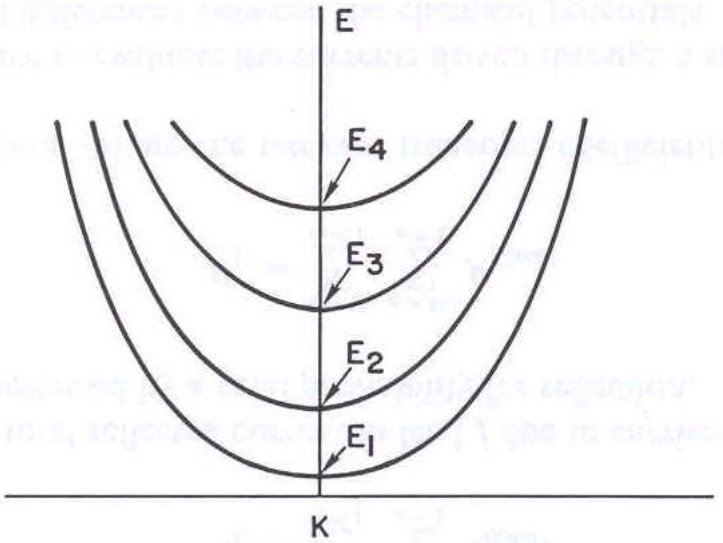
Entanglement test: Bell Inequality

Dynamic generation of entangled states

Scattering Theory of Electron Conduction and Noise

Scattering channels





asymptotic perfect translation invariant potential

$$V(x,y) = V(y) \implies$$

seprable wave function

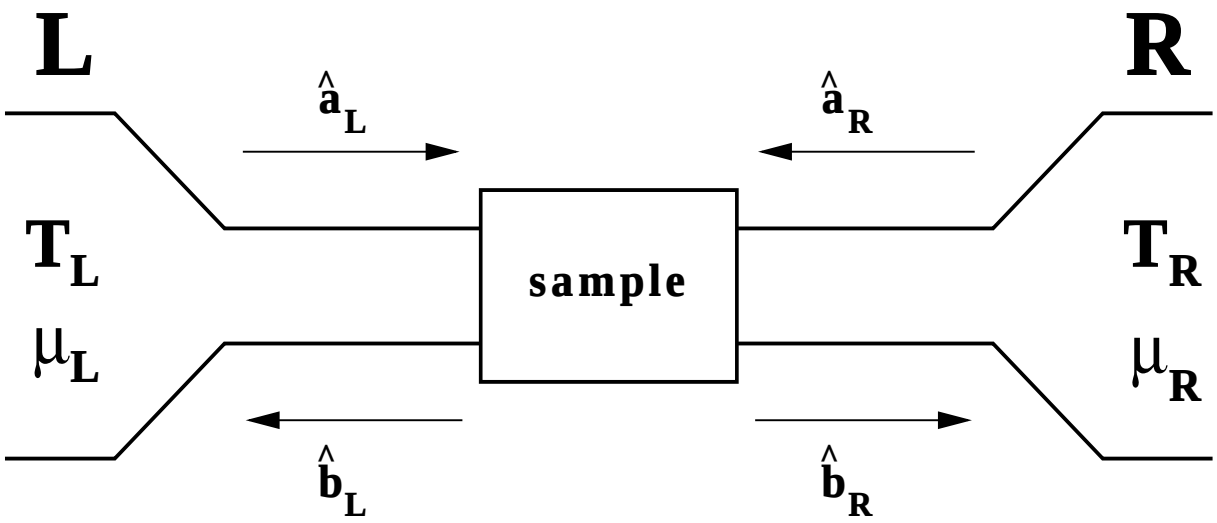
$$\phi_n^{\pm}(\mathbf{r},E) = e^{\pm ik_n(E)x}\,\chi_n(y)$$

energy of transverse motion E_n channel threshold

energy for transverse and longitudnial motion

$$E = E_n + \hbar^2 k^2/2m \quad \Leftrightarrow$$ scattering channel

The scattering matrix and the Fermi-field



$$b_{\alpha n} = \sum_{\beta m} s_{\alpha\beta nm} a_{\beta m} \qquad \mathbf{b}_\alpha = \sum_\beta \mathbf{s}_{\alpha\beta} \mathbf{a}_\beta$$

Fermi-field in contact α

$$\hat{\Psi}(\mathbf{r},t) = \sum_n \int \frac{dE_{\alpha n}\, \chi_{\alpha n}(y)}{[h v_{\alpha n}]^{1/2}} [e^{ikx} \hat{a}_{\alpha n} + e^{-ikx} \hat{b}_{\alpha n}] e^{-iE_{\alpha n} t/\hbar}$$

Elimination of $\hat{b}_{\alpha n}$ $\Longrightarrow$ Equilibrium. Stat. Mech.

The current matrix

Buttiker, PRB 46, 12485 (1992)

$$j(r,t) = -\frac{e\hbar}{2mi}[\Psi^\dagger(r)\nabla\Psi(r) - \Psi(r)\nabla\Psi^\dagger(r)]$$

$$I_\alpha(t) = \int dx_\alpha dy_\alpha j(x_\alpha, y_\alpha, z_\alpha, t)$$

Remark: particle current versus total current

$$I_\alpha(t) = \frac{e}{h}\int dE'dE[a^\dagger_\alpha(E')a_\alpha(E) - b^\dagger_\alpha(E')b_\alpha(E)]e^{i(E'-E)t/\hbar}$$

$$I_\alpha(t) = \frac{e}{h}\int dE'dE\sum_{\beta,\gamma} a^\dagger_\beta(E')A_{\beta\gamma}(\alpha, E', E)a_\gamma(E)e^{i(E'-E)t/\hbar}$$

$$A_{\beta\gamma}(\alpha, E', E) = 1_\alpha\delta_{\alpha\beta}\delta_{\alpha\gamma} - s^\dagger_{\alpha\beta}(E')s_{\alpha\gamma}(E)$$

$\Longrightarrow$ dc-transport, ac-transport, noise

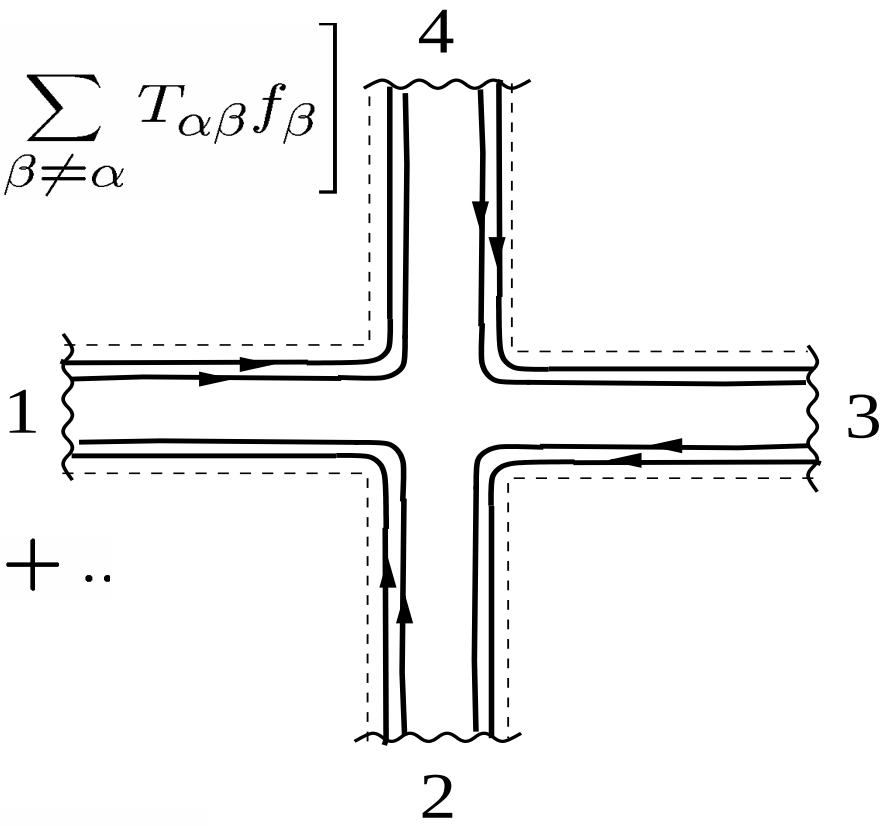
Conductance matrix

Butttiker, PRL 57, 1761 (1986), IBM J. Res. Developm. 32, 317 (1988)

$$I_\alpha = \frac{e}{h}\int dE \left[(N_\alpha - R_{\alpha\alpha}) f_\alpha - \sum_{\beta\neq\alpha} T_{\alpha\beta} f_\beta \right]$$

$$R_{\alpha\alpha} = Tr(s^\dagger_{\alpha\alpha} s_{\alpha\alpha})$$

$$T_{\alpha\beta} = Tr(s^\dagger_{\alpha\beta} s_{\alpha\beta})$$



$$f_\alpha(\mu_\alpha) = f(\mu_0) - (df/dE) eV_\alpha + ..$$

$$\Longrightarrow \qquad I_\alpha = \frac{e}{h}\sum_\beta G_{\alpha\beta} V_\beta$$

$$G_{\alpha\alpha} = \frac{e^2}{h}\int dE(-df/dE)(N_\alpha - R_{\alpha\alpha})$$

$$G_{\alpha\beta} = -\frac{e^2}{h}\int dE(-df/dE) T_{\alpha\beta}$$

$$\sum_\alpha G_{\alpha\beta} = 0$$

$$\sum_\beta G_{\alpha\beta} = 0$$

Shot-noise: two-terminal

$$S_{\alpha\beta} = 2\frac{e^2}{h}\sum_{\gamma\delta}\int dE Tr[A_{\gamma\delta}(\alpha)A_{\delta\gamma}(\beta)]f_\gamma(E)(1-f_\delta(E))$$

Consider kT = 0, V>0, and a two-terminal conductor:

$$S = S_{11} = -S_{12} = -S_{21} = S_{22}\,;$$

Quantum partition noise

$$S = 2\frac{e^2}{h}|eV|\,Tr[tt^\dagger rr^\dagger] = 2\frac{e^2}{h}|eV|\,\sum_n T_n(1-T_n)$$

If all $T_n \ll 1$ $\Longrightarrow$

$$S = 2e(\frac{e^2}{h}\sum_n T_n)|V| = 2e\,|I|$$

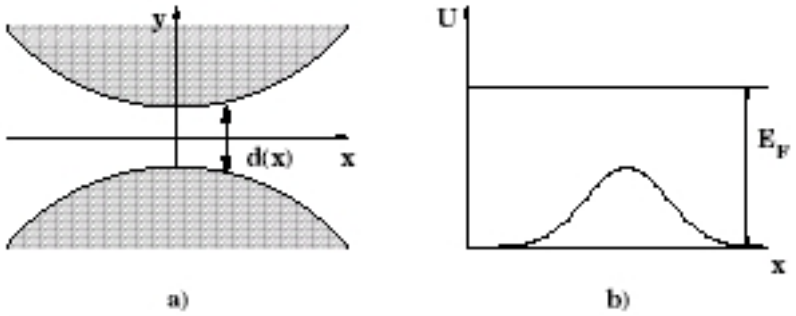
Schottky (Poisson)

Fano factor

$$F = \frac{S}{S_P} = \frac{\sum_n T_n(1-T_n)}{\sum_n T_n}$$

Khlus (1987)
Lesovik (1989)
Buttiker (1990)

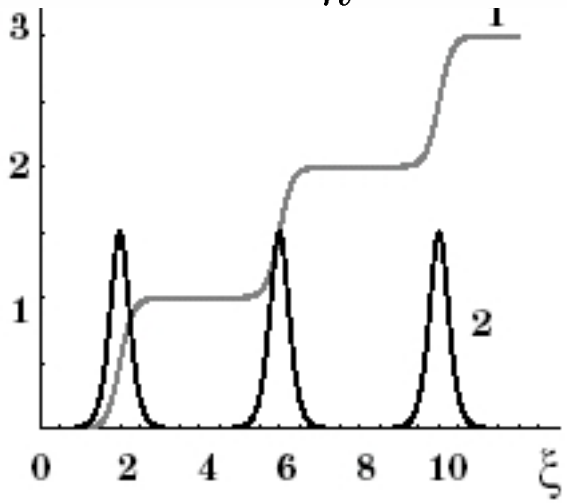
Shot-noise: Qunatum point contact

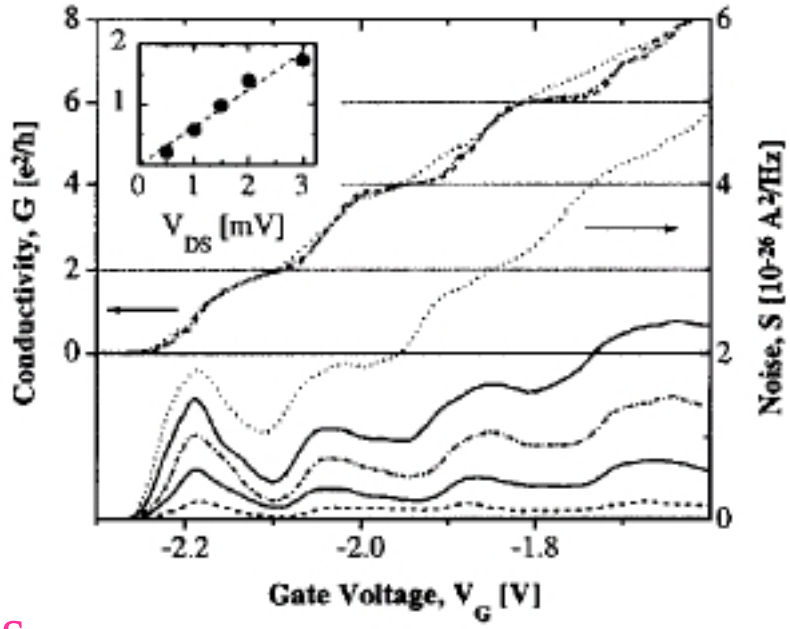


A. Kumar, L. Saminadayar, D. C. Glattli, Y. Jin, B. Etienne, PRL 76, 2778 (1996)

M. I. Reznikov, M. Heiblum, H. Shtrikman, D. Mahalu, PRL 75, 3340 (1996)

$$S = 2\frac{e^2}{h}|eV| \sum_n T_n(1-T_n)$$





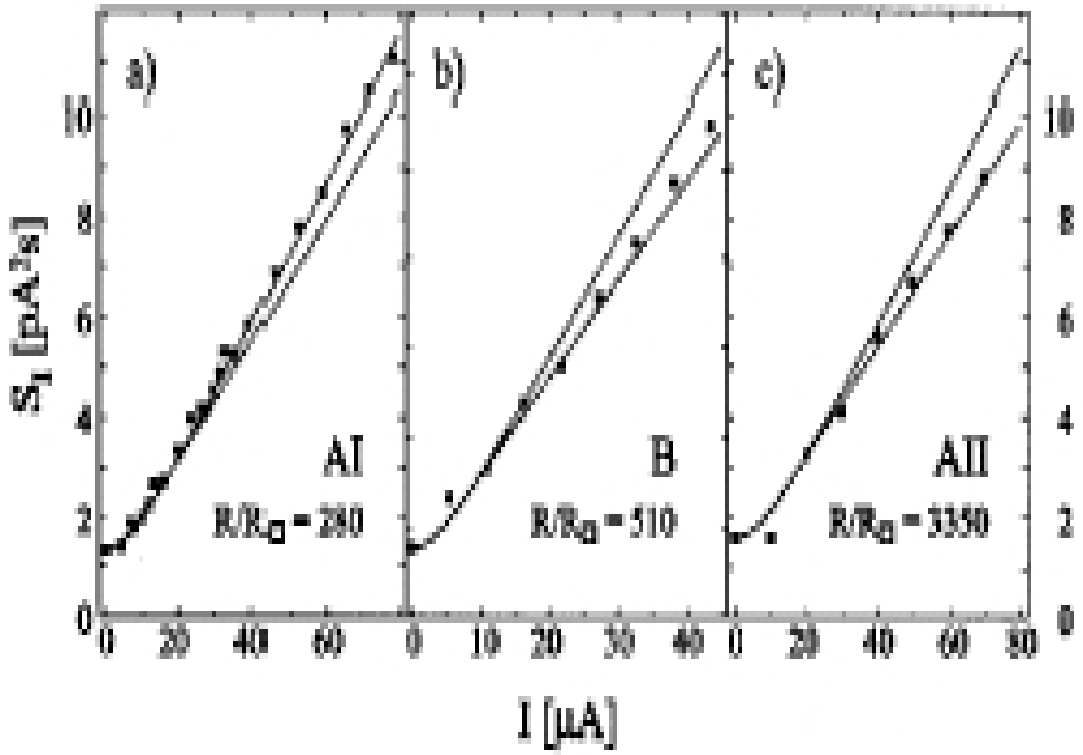
Ideally only one channel contributes

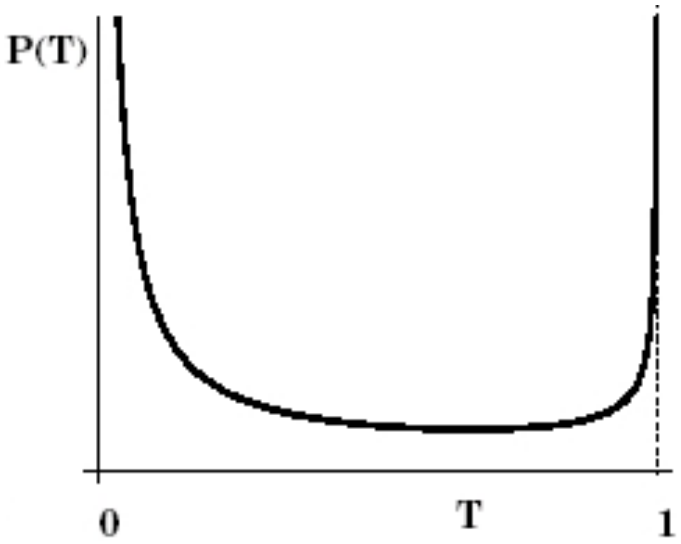
Shot-noise: Metallic diffusive wire

Beenakker and Buttiker, PRB 46, 1889 (1992)

$$G = \frac{e^2}{h}\sum_n T_n$$

$$S = 2\frac{e^2}{h}|eV|\sum_n T_n(1-T_n)$$





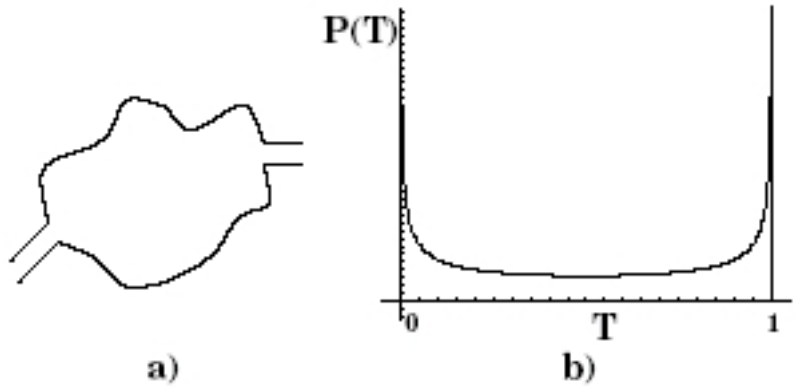
$$p(T) = \frac{l_e}{2L}\frac{1}{T\sqrt{(1-T)}}$$

$$\langle G\rangle = \frac{e^2}{h}N\frac{l_e}{L}$$

$$\langle S\rangle = \frac{1}{3}2e|I|$$

Shot-noise: Chaotic cavity

Jalabert, Pichard and Beenakker, Europhys. Lett. 27, 255 (1994)



$$G = \frac{e^2}{h}\sum_n T_n$$

$$S = 2\frac{e^2}{h}|eV|\sum_n T_n(1-T_n)$$

$$p(T) = \frac{1}{\pi}\frac{1}{\sqrt{T(1-T)}}$$

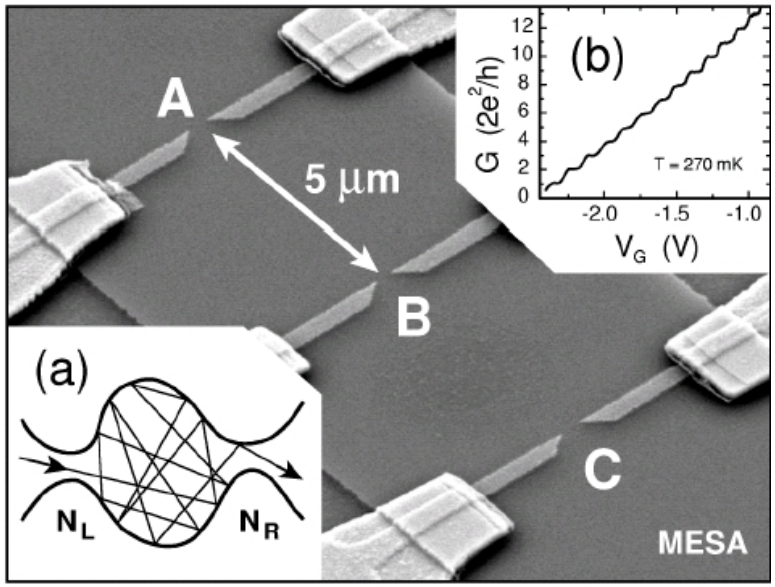
for symmetric cavity with $N_1 = N_2 \gg 1$

$$\Longrightarrow$$

$$\langle G\rangle = \frac{e^2}{h}\frac{N}{4}$$

$$\langle S\rangle = \frac{1}{4}2e|I|$$

Oberholzer et al., PRL 86, 2114 (2001)



Shot-noise: correlations

$$S_{\alpha\beta} = 2\frac{e^2}{h}\sum_{\gamma\delta}\int dE Tr[A_{\gamma\delta}(\alpha)A_{\delta\gamma}(\beta)]f_\gamma(E)(1-f_\delta(E))$$

Consider multi-terminal conductor at kT = 0,
M source contacts with distribution f at voltage eV
All other contacts grounded with distribution f_0

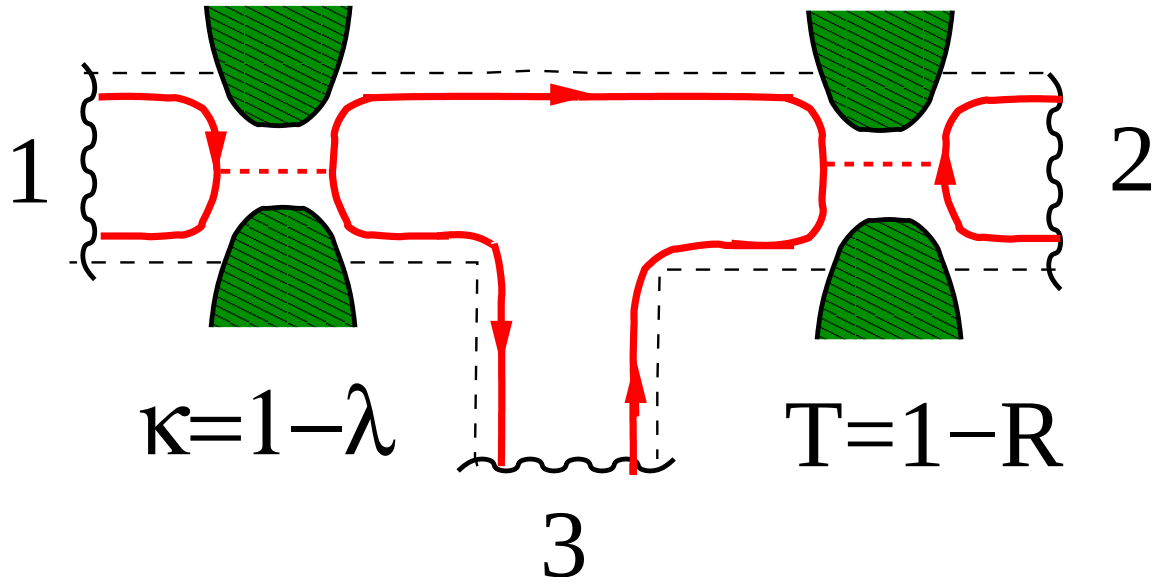
Correlation measured bewteen two grounded contacts:

$$S_{\alpha\beta} = -2\frac{e^2}{h}\int dE Tr[B^\dagger_{\alpha\beta}B_{\alpha\beta}]\ ;\quad B_{\alpha\beta} = \sum_{\gamma=1}^{M} s^\dagger_{\alpha\gamma}s_{\beta\gamma}(f-f_0)$$

M =1, partition noise
M =2, exchange effects,
two particle Aharonov-Bohm effect,
orbital entanglement, violation of Bell inequality

Current correlations, Oberholzer experiment

Oberholzer et al. Physica E6, 314 (2000)



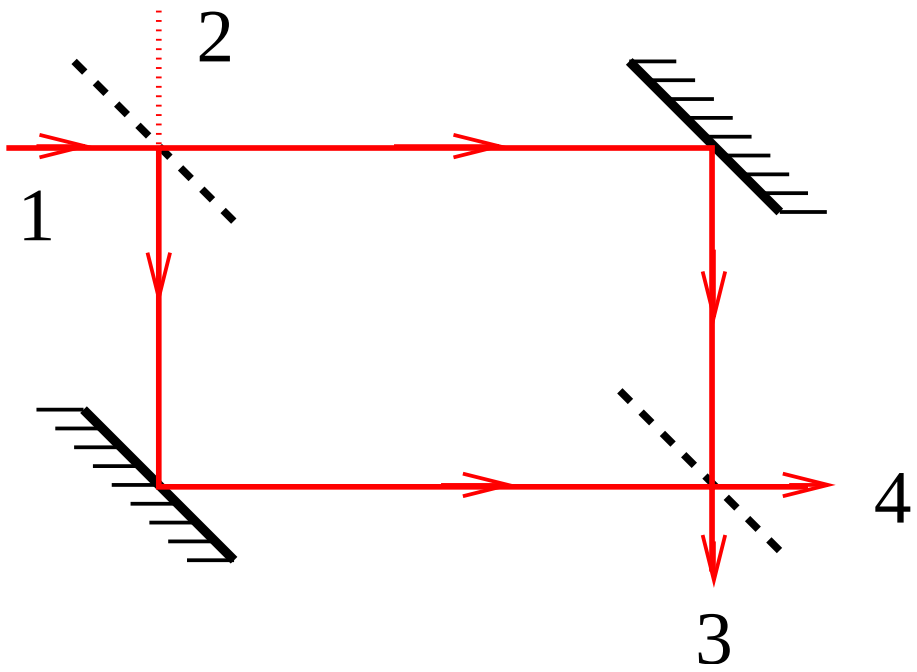
Bias configuration: $\mu_1 = \mu_0 + eV$, $\mu_2 = \mu_3 = \mu_0$

$$S_{23} = -2\frac{e^2}{h}|eV|\,\kappa^2 TR$$

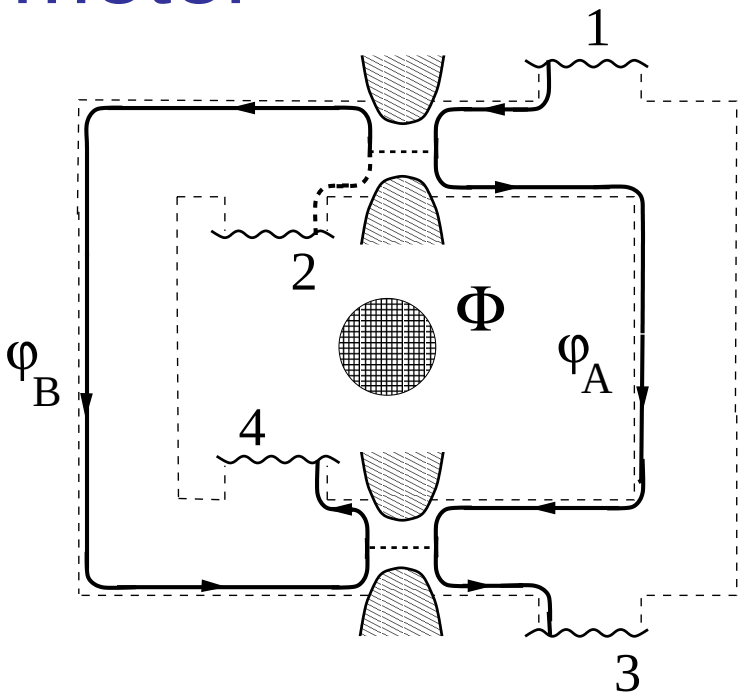
$$S_{22} = 2\frac{e^2}{h}|eV|\,\kappa T(1-\kappa T)$$

$$S_{33} = 2\frac{e^2}{h}|eV|\,\kappa R(1-\kappa R)$$

Optical and Electrical Mach-Zehnder-Interferometer



One particle Aharonov-Bohm effect



Ji et al, Nature 422, 415 (2003)

$$s_{31} = \frac{1}{2}\left[e^{i(\phi_A - \chi_1)} + e^{i(\phi_B - \chi_2)}\right]$$

$$\chi_1 - \chi_2 = 2\pi\Phi/\Phi_0$$

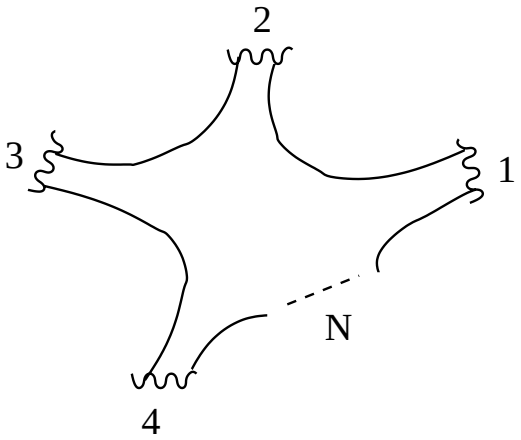
$$G_{31} = \frac{e^2}{2h}\left[1 + \cos(\phi_A - \phi_B - 2\pi\Phi/\Phi_0)\right]$$

$$S_{34} = -2\frac{e^2}{8h}|eV|\left[1 + \cos(2(\phi_A - \phi_B - 2\pi\Phi/\Phi_0))\right]$$

Two-particle Aharonov-Bohm effect

The quantum phase of current correlations

Mesoscopic conductor with N contacts



$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & \cdots & s_{1N} \\ s_{21} & s_{22} & s_{23} & \cdots & \\ s_{31} & s_{32} & s_{33} & & \\ \vdots & \vdots & & \ddots & \\ s_{N1} & & & & s_{NN} \end{pmatrix}$$

At kT = 0,

$$G_{\alpha\beta} = 2\frac{e^2}{h}\mathrm{Tr}\left[s^{\dagger}_{\alpha\beta}s_{\beta\alpha}\right]$$

$$S_{\alpha\beta} = 2\int dt\langle \Delta\hat{I}_{\alpha}(t)\Delta\hat{I}_{\beta}(0)\rangle$$

At kT = 0, M contacts with $f_\gamma = f$, N-M contacts at $f_\delta = f_0$

$$S_{\alpha\beta} = 2\frac{e^2}{h}\int dE\,\mathrm{Tr}\left[B^{\dagger}_{\alpha\beta}B_{\beta\alpha}\right], \qquad B_{\alpha\beta} = \sum_{\gamma=1}^{M} s_{\alpha\gamma}s^{\dagger}_{\beta\gamma}(f_\gamma - f_0)$$

M=1, partition noise

M > 1, relative phase of scattering matrix elements becomes important

Exchange interference effects: Buttiker, PRL 68, 843 (1992)

Physica B 175 (1991) 199–212
North-Holland

The quantum phase of flux correlations in waveguides

M. Büttiker
IBM T.J. Watson Research Center, Yorktown Heights, NY 10598, USA

The low frequency fluctuations of the flux at the ports of a wave guide structure connected to electron reservoirs or black body sources are investigated. At equilibrium the cross-correlation of the flux fluctuations are negative independent of whether the flux is subject to Fermi or Bose statistics. In the presence of transport the cross-correlations for a Fermi system remain negative whereas for a Bose system at certain ports the correlation function can become positive. We emphasize that waves emanating from different mutually incoherent electron reservoirs or black body radiators can nevertheless give rise to interference phenomena in the cross-correlation function. Specific examples are discussed which illustrate the appearance of phase sensitive cross-correlations generated by waves emanating from mutually incoherent reservoirs.

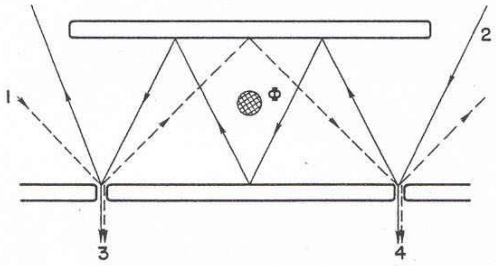


Fig. 3. Electron beams emanating from mutually incoherent electron reservoirs 1 and 2 with a magnetic flux tube Φ penetrating the fluctuation induced loop of fig. 2(a). The cross-correlation $\langle \Delta I_3 \, \Delta I_4 \rangle$ exhibits periodic oscillations in the flux with period $\Phi_0 = hc/e$ even though neither of the two electron beams encloses the flux.

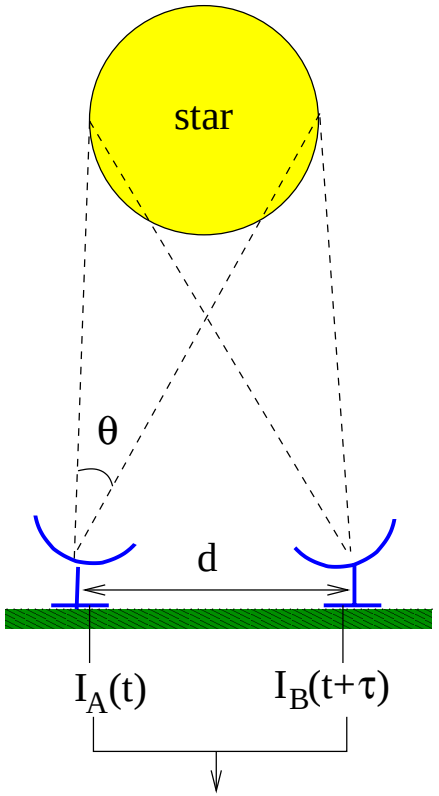
HBT-Intensity Interferometer

Hanbury Brown and Twiss, Nature 177, 27 (1956)

Interference not of amplitudes but of intensities
Optics: classical interpretation possible
Quantum mechanical explanation:
Purcell, Nature 178, 1449 (1956)
Indistinguishable particles:

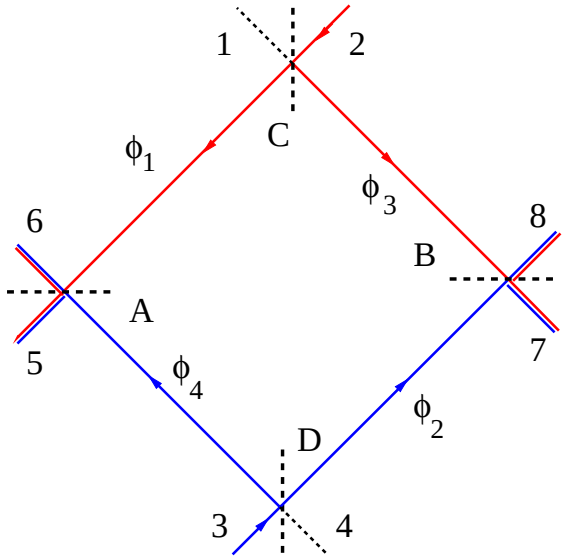
Statistics, exchange amplitudes

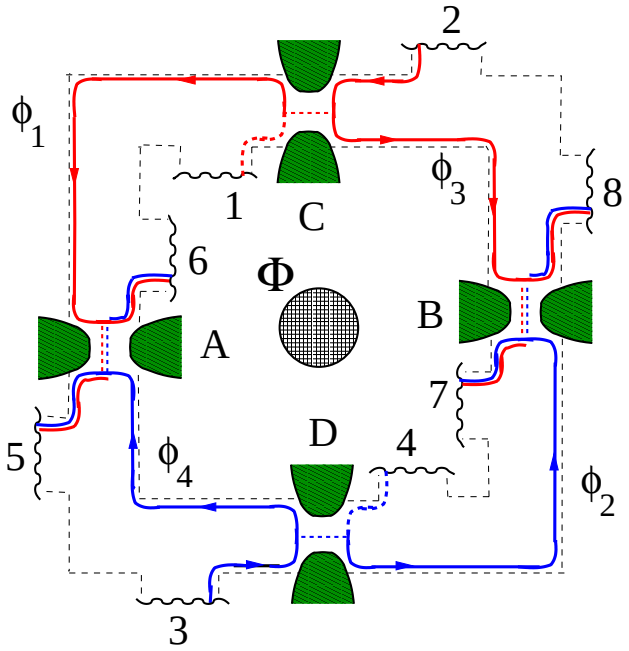
$$\int d\tau \langle \Delta I_A(t) \Delta I_B(t+\tau) \rangle = f\left(\frac{d\theta}{\lambda}\right)$$



Electrical HBT Interferometer

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)





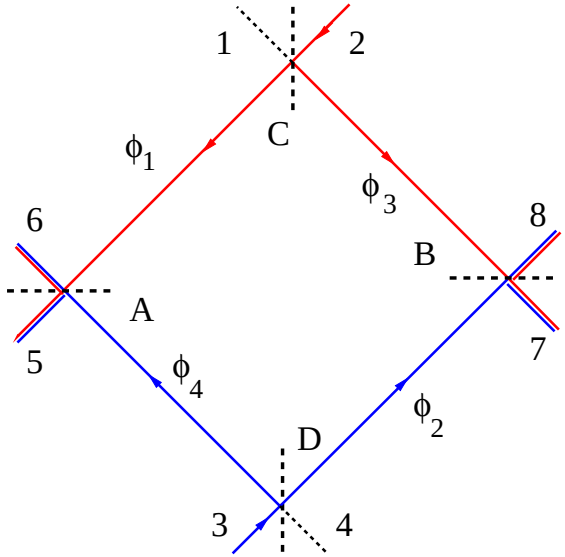
$$s_{52} = T_A^{1/2} e^{i(\phi_1+\chi_1)} T_C^{1/2}$$

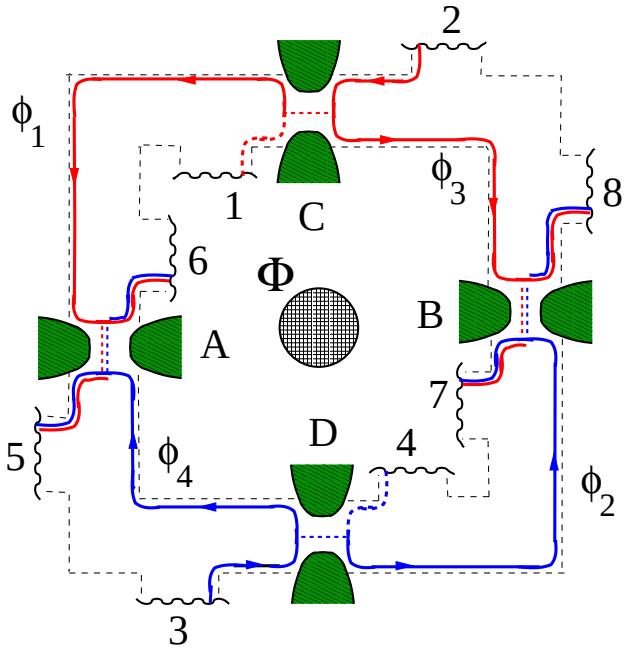
$$G_{52} = -\frac{e^2}{h} T_A T_C$$

All elements of the conductance matrix are independent of AB-flux

Two-particle Aharonov-Bohm Effect

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)





Fourth-order interference: Current-current correlation

$$S_{58} = -2\frac{e^2}{h}\int dE|s^*_{52}s_{82} + s^*_{53}s_{83}|^2(f - f_0)^2$$

For $T_A = T_B = T_C = T_D = 1/2$;

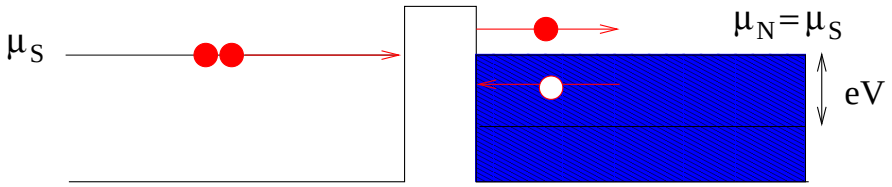
$$S_{58} = -\frac{e^2}{4h}|eV|\left[1 + \cos\left(\phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\frac{\Phi}{\Phi_0}\right)\right]$$

Orbital Entanglement

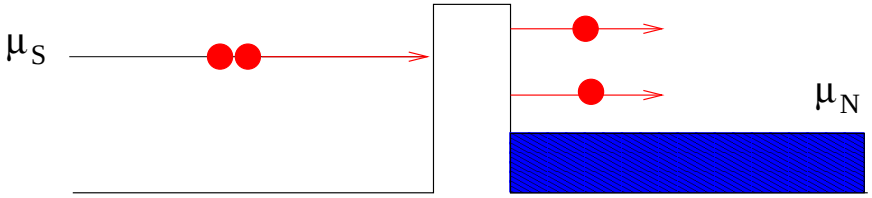
Two-particle sources

Superconducting-normal hybrid structures

Bogoliubov-de Gennes picture



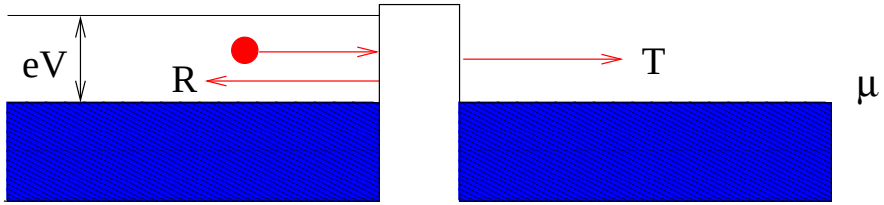
Pair-tunneling picture



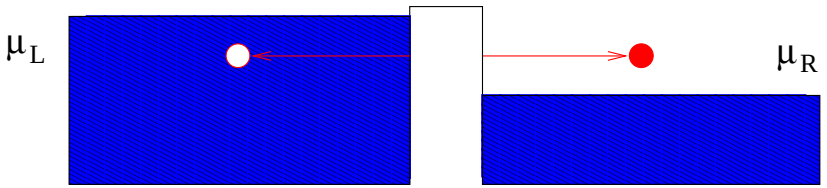
Samuelsson, Sukhorukov and Buttiker,
PRL 91, 157002 (2003)

Normal-conductors

Electron picture of tunneling



Electron-hole picture

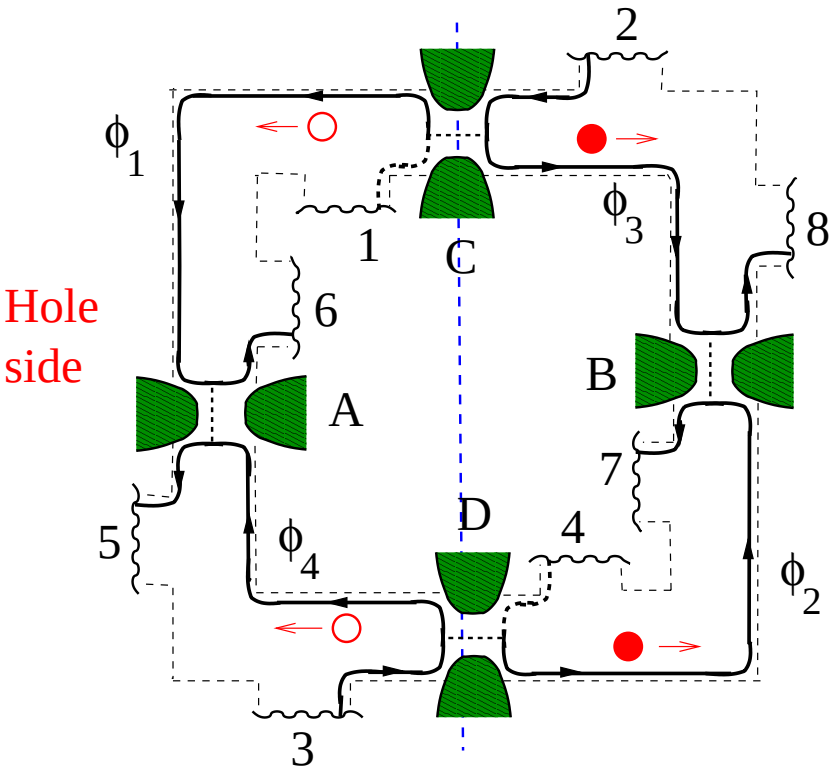


Beenakker et al ,
PRL 91, 147901 (2003)

P. Samuelsson, E.V. Sukhorukov, M. Buttiker **cond-mat/0503016**

Two-particle entanglement

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)



$$\left[\begin{array}{l} c_2^\dagger = t_C c_{2A}^\dagger + r_C c_{2B}^\dagger \\ c_3^\dagger = r_D c_{3A}^\dagger + t_D c_{3B}^\dagger \end{array}\right]$$

$R_C = T_D = R \ll 1 \;;\tau_C = \hbar/eV \;;\tau \sim \hbar/eVR\;,$ tunneling limit

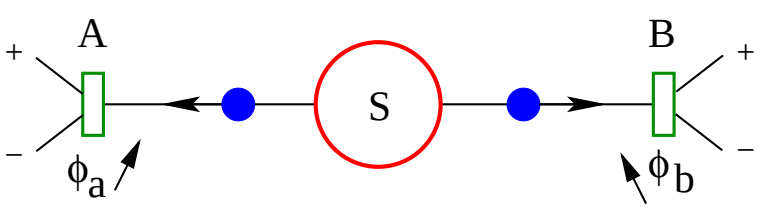
$$|\Psi_{in}\rangle = \prod_{0<E<eV} c_2^\dagger(E) c_3^\dagger(E)|0\rangle$$ incident state

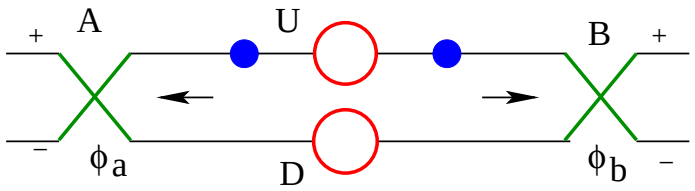
$$|\Psi\rangle = |\bar{0}\rangle + \sqrt{R}\int_0^{eV} dE\left[c_{3B}^\dagger c_{3A} + c_{2B}^\dagger c_{2A}\right]|\bar{0}\rangle + O(R)$$ orbitally entangled e-h-state

Bell Inequality

Comparison of a classical local theory with quantum mechanical predictions

Here: entanglement test





Bell Inequality: Clauser et al. , PRL 23 , 880 (1969)

$$S_B = |E(\theta_A,\theta_B) - E(\theta'_A,\theta_B) + E(\theta_A,\theta'_B) + E(\theta'_A,\theta'_B)| \leq 2$$

$$E(\theta_A,\theta_B) = P_{++} + P_{--} - P_{+-} - P_{-+} = \frac{\langle (I_{A+} - I_{A-})(I_{B+} - I_{B-})\rangle}{\langle (I_{A+} + I_{A-})(I_{B+} + I_{B-})\rangle}$$

$$P_{\alpha\beta}(\theta_A,\theta_B) = (1 + \alpha\beta\cos[2(\theta_A - \theta_B)])/4$$

$$P_{\alpha\beta} \propto \langle b^\dagger_\beta(t) b^\dagger_\alpha(t+\tau) b_\alpha(t+\tau) b_\beta(t)\rangle \qquad (\tau\Delta\omega \leq 1)$$

spin

$\alpha = \uparrow, \downarrow,$ θ_A, θ_B angles of spin filters, polarizers

orbital

$\alpha = U, D,$ θ_A, θ_B rotation angles: splitter

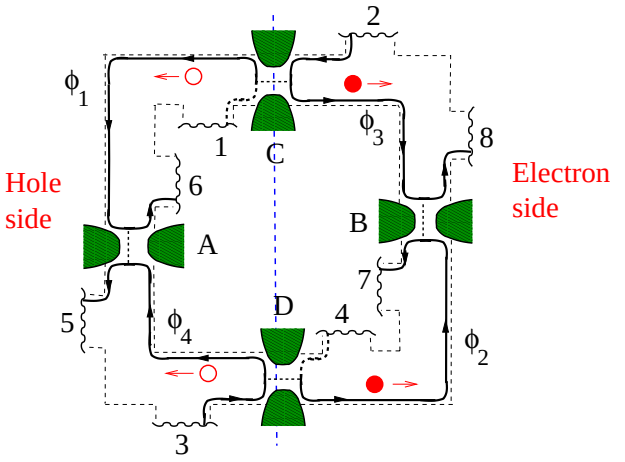
Entanglement test: Bell Inequality

With $\phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0 = 2\pi$

$$S_{A/B} = \begin{pmatrix} \cos\theta_{A/B} & -\sin\theta_{A/B} \\ \sin\theta_{A/B} & \cos\theta_{A/B} \end{pmatrix}$$

using e-h-state

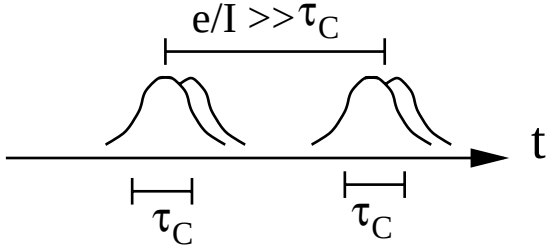
$$P_{\alpha\beta} = (1 + \alpha\beta\cos[2(\theta_A - \theta_B)])/4$$



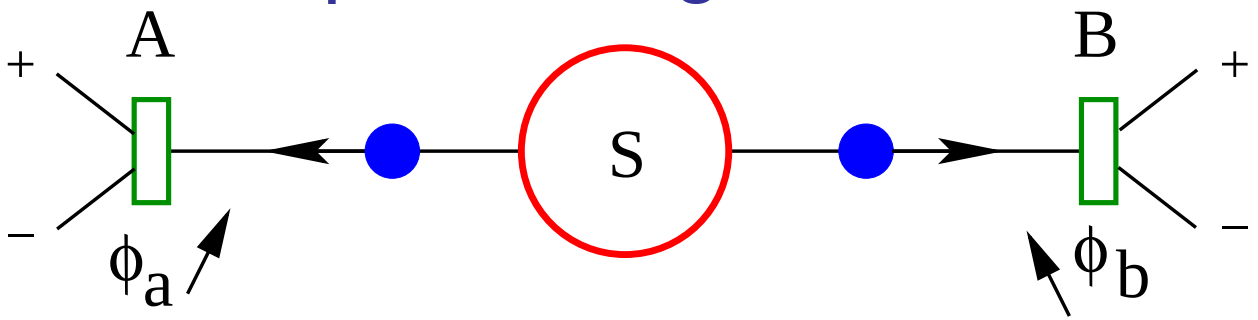
Noise correlators

$$S_{58} = S_{67} = -S_0 P_{++}, \quad S_{57} = S_{68} = -S_0 P_{+-}, \quad S_0 = -(4e^2/h)|eV|R$$

In the tunneling limit $R_C = T_D = R \ll 1$; $\tau_C = \hbar/eV$; $\tau = e/I = \hbar/eVR$, measuring the noise cross-correlation is equivalent to coincidence detection in a long time interval: Only two particles within a pair are correlated with each other.

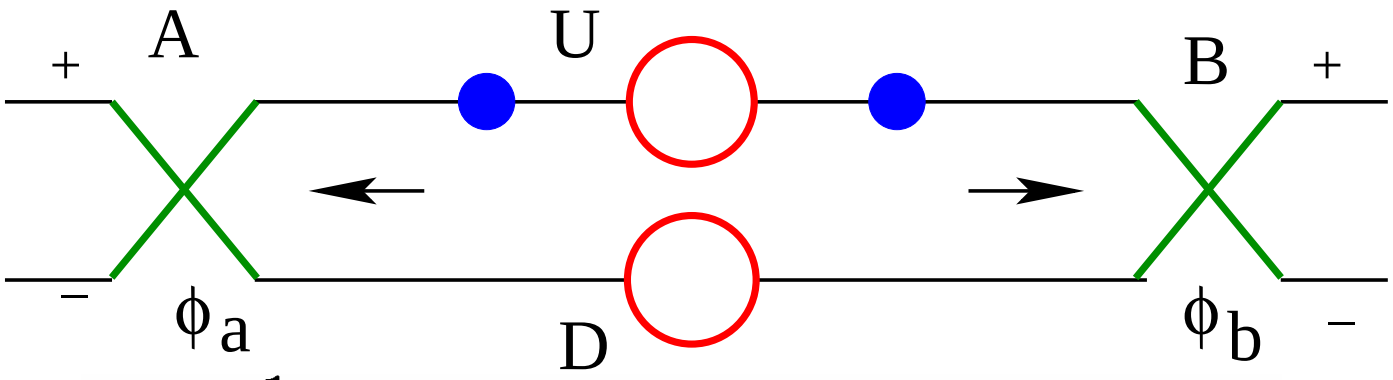


Spin entanglement



$$\Psi = \frac{1}{\sqrt{2}}\left[\Psi_{\uparrow}(A)\Psi_{\downarrow}(B) - \Psi_{\downarrow}(A)\Psi_{\uparrow}(B)\right]$$

Orbital entanglement



$$\Psi = \frac{1}{\sqrt{2}}\left[\Psi_U(A)\Psi_U(B) + \Psi_D(A)\Psi_D(B)\right]$$

Electron-electron entanglement through postselection

Symmetric interferometer $T, R \approx 1/2$

Electron-hole picture not appropriate

Incident electron state is a product state: no intrinsic entanglement

Two-particle effects nevertheless persists

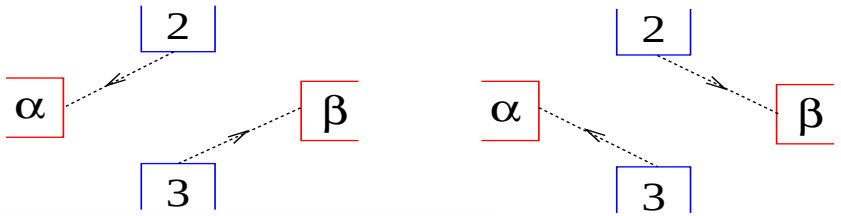
A Bell Inequality can be violated

Explanation: Entanglement through ``postselection" (measurement)

Joint detection probability

$$P_{\alpha\beta} \propto \langle b^\dagger_\beta(t) b^\dagger_\alpha(t) b_\alpha(t) b_\beta(t)\rangle = (h^2/e^2)[(1/2\tau_c)S_{\alpha\beta} + I_\alpha I_\beta]$$
$$= |s_{\alpha 3}s_{\beta 2} - s_{\alpha 2}s_{\beta 3}|^2 \qquad \tau_c = \hbar/eV$$

$$\langle I_\alpha\rangle = \frac{e^2}{h} V(|s_{\alpha 2}|^2 + |s_{\alpha 3}|^2)\,,$$

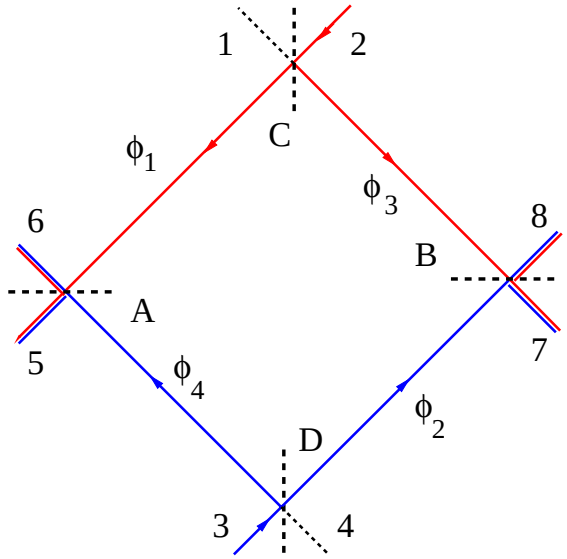


Bell parameter (Bell Inequality):

$$S_B^{max} = 2\sqrt{1+\cos^2\phi_0},\ \ \phi_0 = \phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0$$

Short time statistics: Pauli principle leads to injection of at most one electron in a short time interval: only two-particle transmission probability enters

Thermal photon sources



Sources: black body

Energy window: narrow band filters $\Delta\omega = 2\pi/\tau_C$

$$P_{\alpha\beta} \propto \langle b^\dagger_\beta(t) b^\dagger_\alpha(t) b_\alpha(t) b_\beta(t)\rangle \propto [(1/2\tau_c) S_{\alpha\beta} + I_\alpha I_\beta] \ ,$$

$$S_B^{max} = (2/3)\sqrt{1+\cos^2\phi_0} \ ,$$

No violation: In contrast to electron injection through a single quantum channel where in each time-slot only one particle is injected, in the bosonic case, many particles can be injected.

Summary of recent advances

Orbital entanglement

P. Samuelsson, E. V. Sukhorukov, M. Buttiker, PRL 91, 157002 (2003)

Zero-frequency measurement (Bell Inequality)

X. Maitre, W. D. Oliver, Y. Yamamoto, Physica E6, 301 (2000)

N.M. Chtchelkatchev et al., Phys. Rev. B 66, 161320 (2002)

P. Samuelsson, E.V. Sukhorukov and M. Buttiker, PRL 91, 157002 (2003)

Normal components: e-h-pairs

C.W.J. Beenakker et al, PRL 91, 147901 (2003);

P. Samuelsson, E.V. Sukhorukov and M. Buttiker, PRL 92, 026805 (2004).

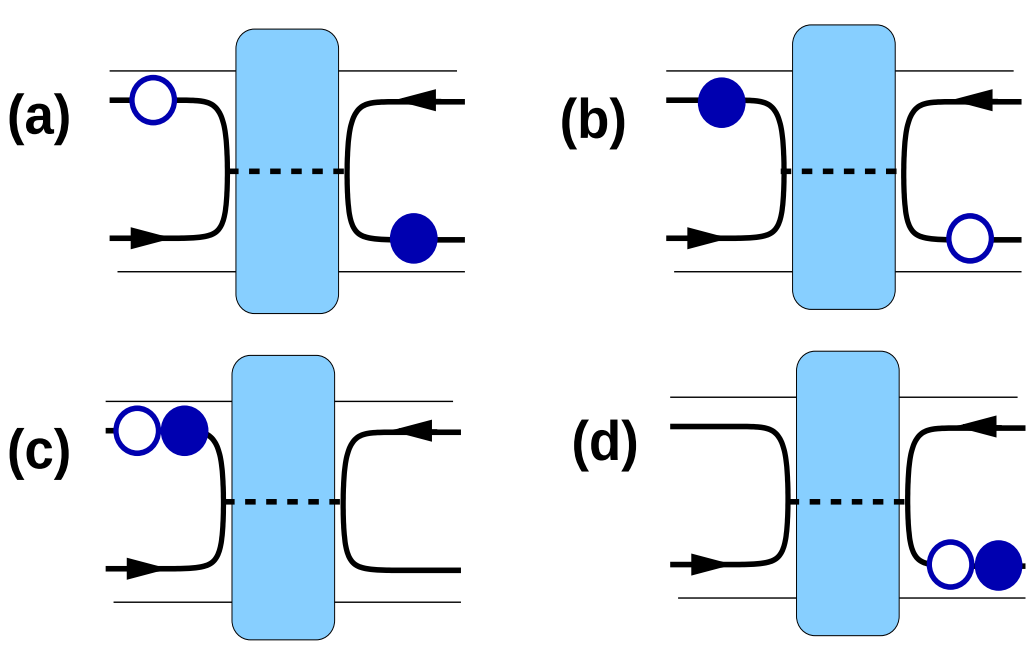
Controllable geometries

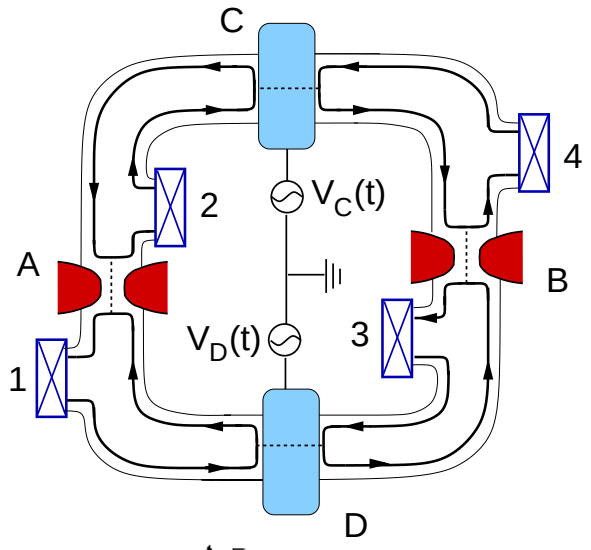
P. Samuelsson, E.V. Sukhorukov and M. Buttiker, PRL 92, 026805 (2004)

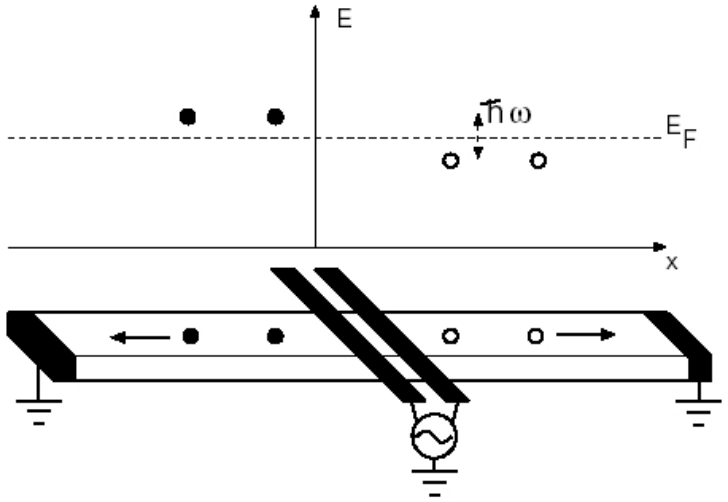
Entanglement on demand

Electron-hole generation: oscillating barrier potentials

P. Samuelsson and M. Buttiker, Phys. Rev. B (unpubl). cond-mat/0410581





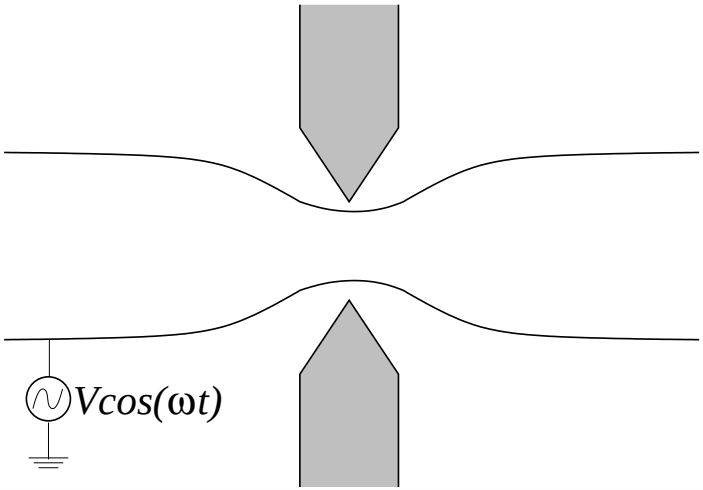


C. W. J. Beenakker, M. Titov and B. Trauzettel, cond-mat/0502055

Electron-hole generation: oscillating contact potentials

Reydellet, Roche, Glattli, Etienne, Jin, PRL 90, 176803 (2004)

QPC: left contact V_{ac} with frequency ν , right contact grounded



Ac-voltage excites electron-hole pairs:

electrons with energy $-\epsilon$ below E_F excited to $h\nu - \epsilon > 0$

leaves behind a hole at $-\epsilon$

$$I_0 = a\, e\, \omega \;\; , a = (eV_{ac}/h\omega)^2$$

independent electron and hole partition noise ?

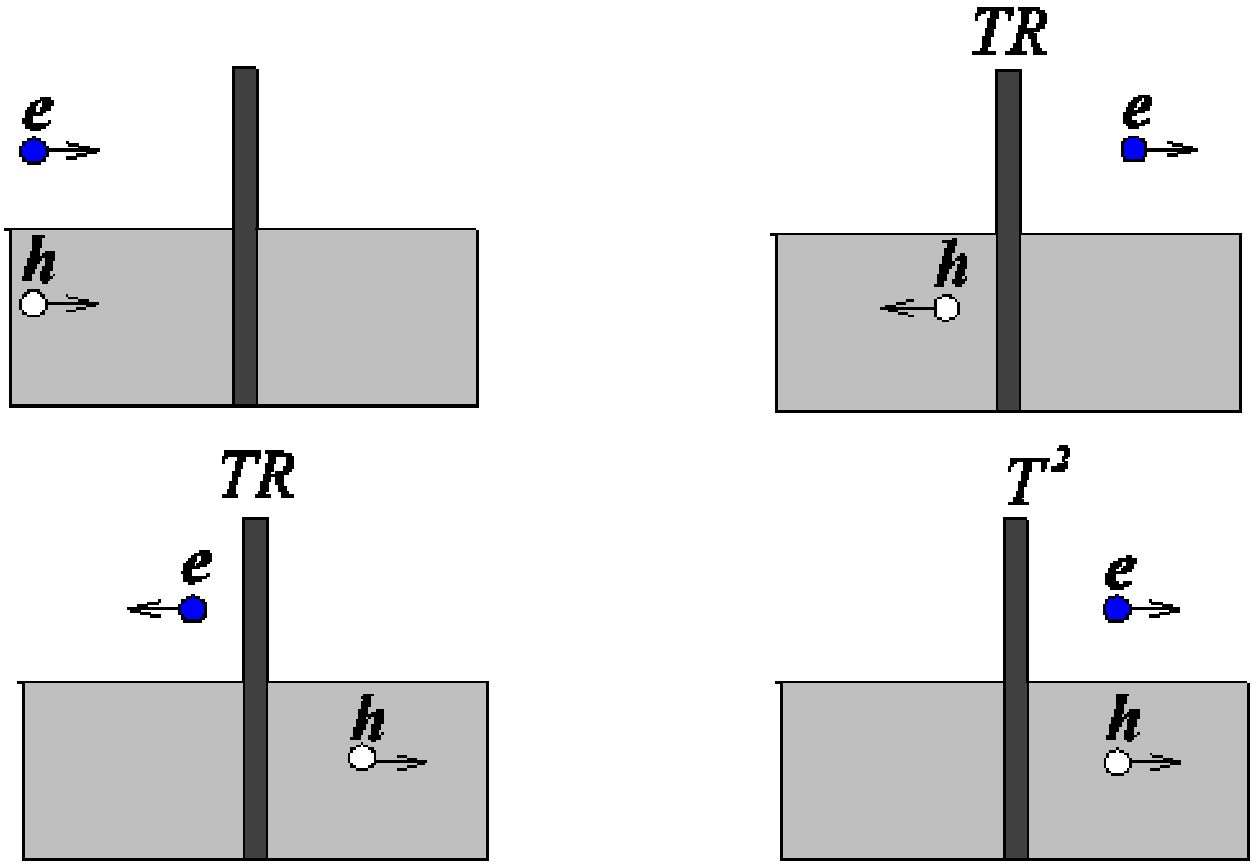
$$S^{ee} = eI_0T(1-T)$$

$$S^{hh} = eI_0T(1-T)$$

$$S = 2eI_0T(1-T)$$

Electron-hole autocorrelations

Rytchkov, Polianski, Buttiker (unpubl).

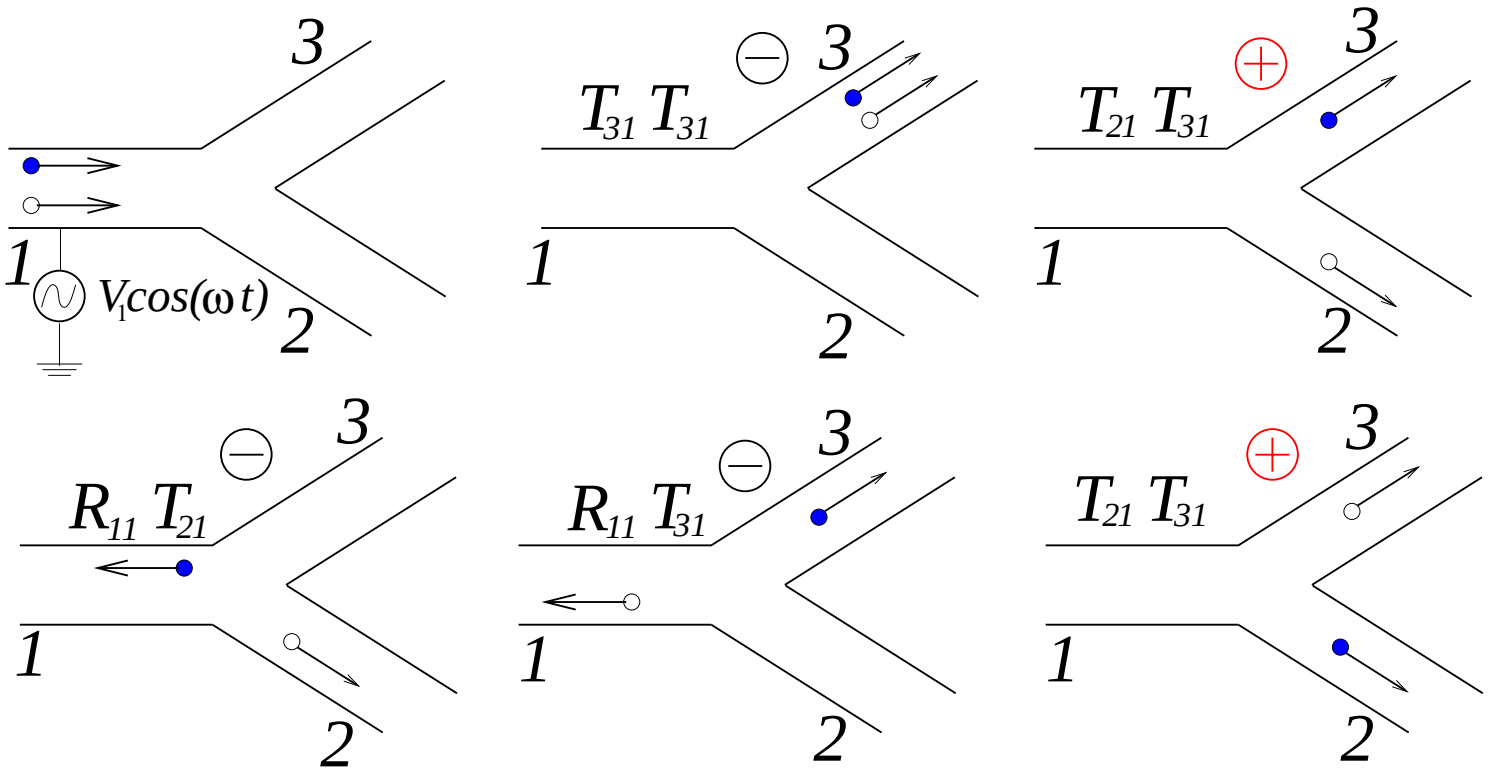


$$S^{ee} = eI_0 T \ , S^{hh} = eI_0 T \ , S^{he} = S^{eh} = -eI_0 T^2$$

$$S = 2eI_0 T(1-T)$$

Electron-hole cross-correlations

Rytchkov, Polianski, Buttiker (unpubl).

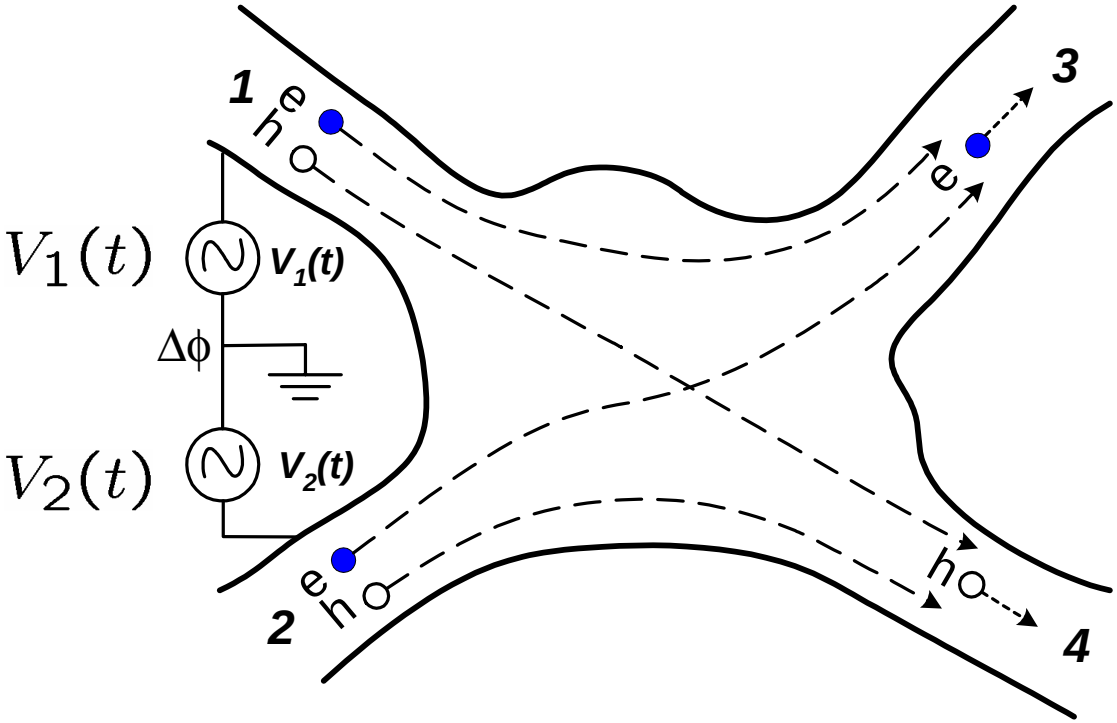


$$S_{23}^{ee} = S_{23}^{hh} = 0\ ,\ S_{23}^{he} = S_{23}^{eh} = -eI_0\,T^2/2$$

$$S_{23} = -eI_0\,T^2$$

Photon-exited HBT-exchange interference

Rytchkov,Polianski, Buttiker (unpubl)



$$V_1(t) = V\,cos(\omega t)$$
$$V_2(t) = V\,cos(\omega t + \phi)$$

Scattering paths are in reality chaotic

$$S_{34}(\phi) = -\frac{e^2\omega}{2\pi}\mathcal{P}[s_{31}^\dagger\, s_{31}\, s_{41}^\dagger\, s_{41} + s_{32}^\dagger\, s_{32}\, s_{42}^\dagger\, s_{42} + s_{32}^\dagger\, s_{31}\, s_{41}^\dagger\, s_{42} e^{i\phi} + s_{31}^\dagger\, s_{32}\, s_{42}^\dagger\, s_{41} e^{-i\phi}]\,.$$

$\Rightarrow$ Full exchange interference correlation

Summary

Scattering theory conductance and noise

Two-particle Aharonov-Bohm effect

Orbital quasi-particle entanglement

Entanglement test: Bell Inequality

Dynamic generation of entangled states