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Regeneration of small-scales by large-scale data assimilation in three-dimensional turbulence

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0 Introduction

0.1 Sensitivity of turbulence

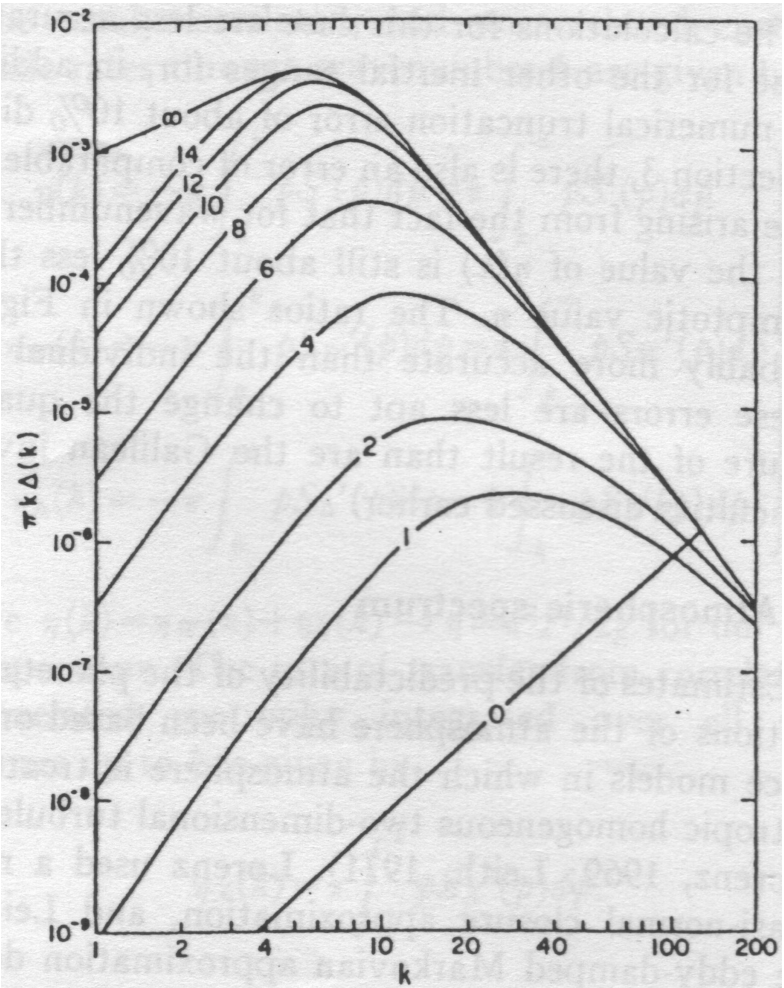
Turbulence

- Nonlinear, forced-dissipative dynamical systems with a huge number of degrees of freedom.
- sensitive to small differences in the flow conditions.

$\Longrightarrow$ Unpredictability

  - Chaos (e.g, Lorenz model).
  - Predictability of atmospheric turbulence is limited to 2 weeks or so [Leith and Kraichnan (1967)].

0.2 Predictability of turbulence



$\mathbf{u}^{(1)}$: correct field

$\mathbf{u}^{(2)}$: simulated field

(with error at initial time)

$\delta\mathbf{u} = \mathbf{u}^{(2)} - \mathbf{u}^{(1)}$

:error field

Error spectrum

$$\Delta(k) = \sum_{|\mathbf{k}'|=k} \delta\mathbf{u}(\mathbf{k}') \cdot \delta\mathbf{u}(-\mathbf{k}')$$

upper limit of predicition time ~ 14 days.

Leith and Kraichnan, J. Atmos. Sci. (1972)

0.3 Data assimilation

Recent studies suggest that

small scales are predictable from large-scale data,

that is, small-scale error is suppressed if correct large-scale data are continuously assimilated in simulations.

Previous studies

- Numerical studies
  - Reynolds numbers are too small.

    [Henshaw, Kreiss *et al.* (1998, 2003), Hayashi *et al.* (2002), Olson and Titi (2003)]
- Mathematical studies
  - Derived conditions for effective data assimilation are too strong.

0.4 Number of determining modes

- Smallest number N such that

$$\lim_{t\to\infty} |\delta\mathbf{u}(\mathbf{k}^{(m)}, t)| = 0 \ (m = 1, \cdots, N) \Longrightarrow \lim_{t\to\infty} |\delta\mathbf{u}(\mathbf{k}, t)| = 0 \ (\forall\ \mathbf{k})$$

Foias and Prodi (1967), Foias *et al.* (1983), Jones and Titi (1993)

Exisiting rigorous estimates for upper-bounds of N seem to be too large compared to DNS results.

1 Purpose of the research

Get some idea on the quantitative aspect of the effect of the data assimilation at large Reynolds numbers by means of DNS.

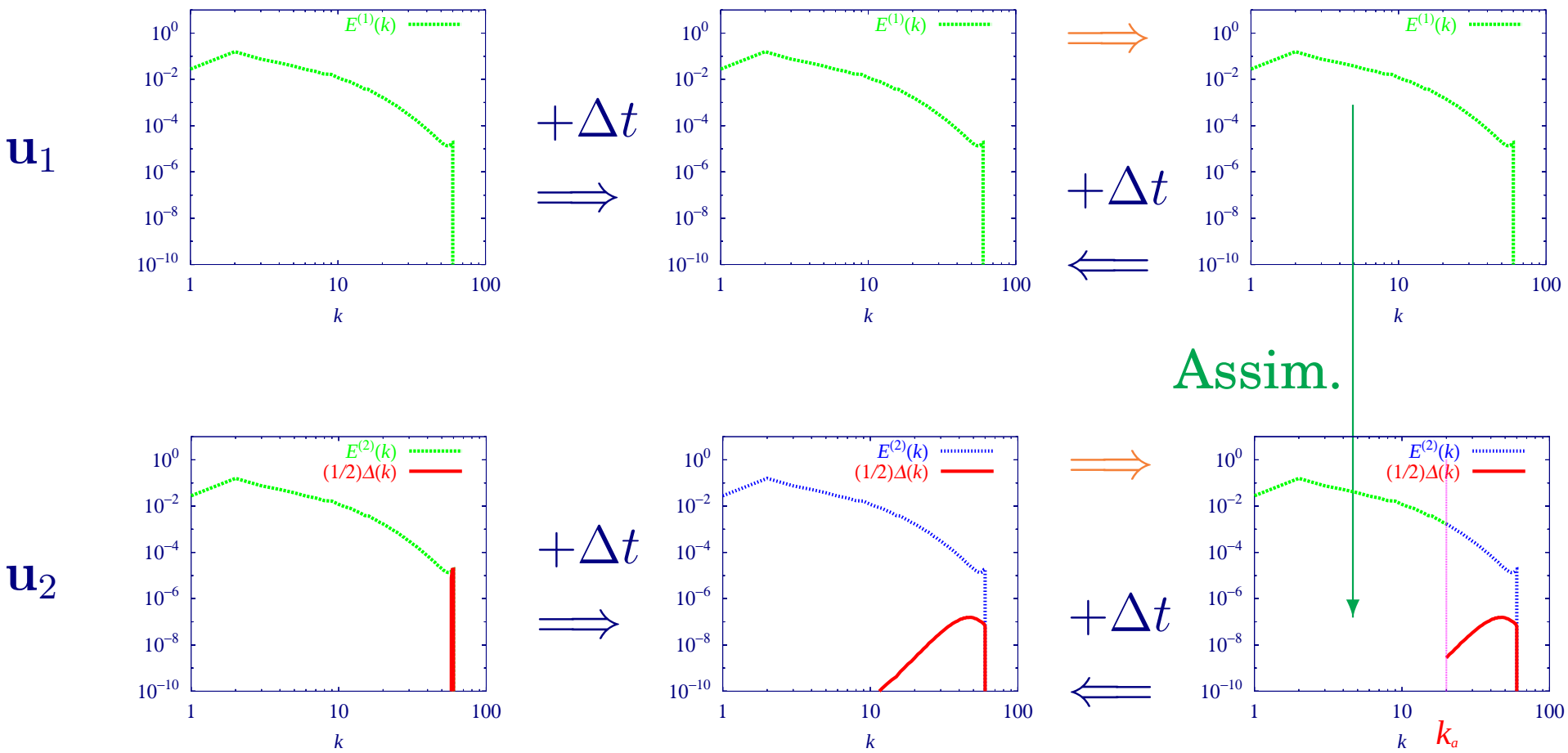
Quantitative study of the critical wavenumber k^*

such that the small-scale error is suppressed if data of Fourier modes with wavenumber up to k^* (or larger) are continuously assimilated to those of the correct field.

DNS with wide range of Reynolds numbers,
$R_\lambda = 31 - 284$ (the highest resolution: 512^3)

2 Numerical experiments

2.1 Method of Data Assimilation



Fourier modes with wavenumber $k < k_a$ are continuously assimilated.

2.2 DNS Parameters

N	$k_{\max}$	$\nu(\times 10^3)$	E	ϵ	η	R_λ
64	30	10.0	0.500	0.171	0.0491	**31**
128	60	2.70	0.500	0.138	0.0194	**67**
256	120	1.10	0.500	0.132	0.0100	**107**
512	241	0.289	0.500	0.0712	0.0043	**284**

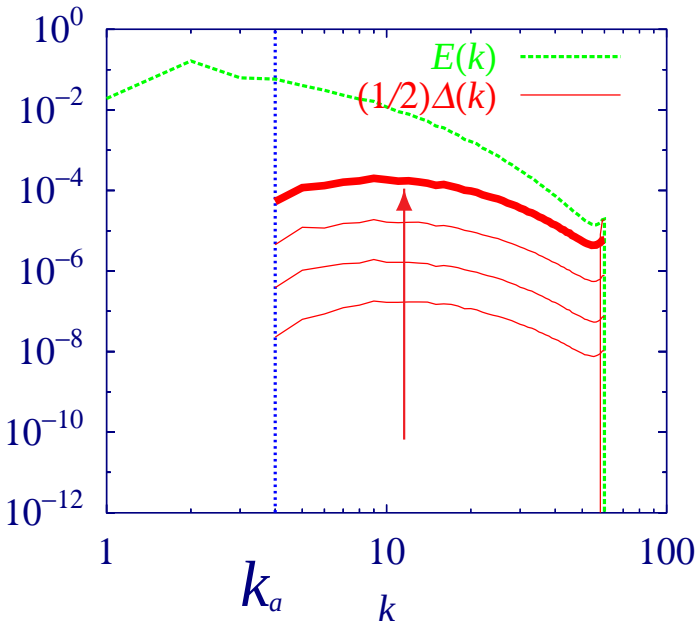
External force: $\mathbf{f}(\mathbf{k}, t) = -\gamma \mathbf{u}(\mathbf{k}, t) \qquad 2 < k < 3$

Viscosity ν: $k_{\max}\eta \sim 1, \qquad \eta = (\nu^3/\epsilon)^{1/4}$

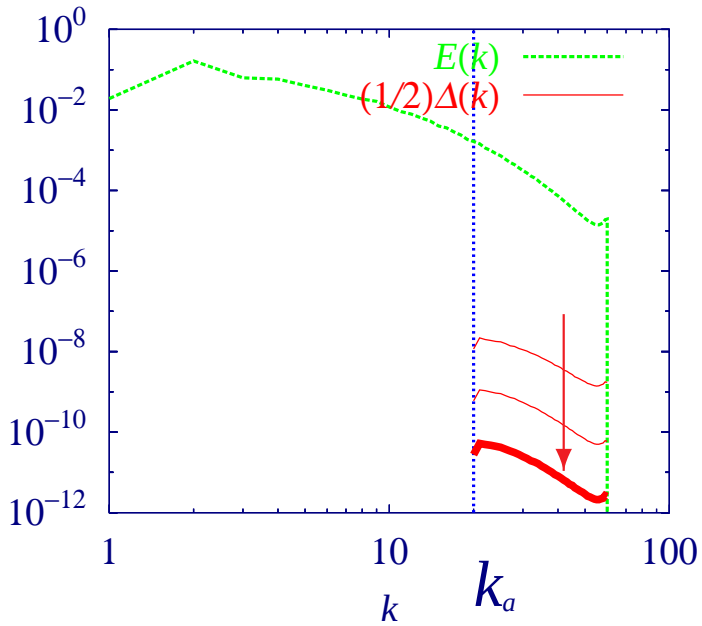
Numerical scheme: Alias-free spectral method,
4-th order Runge-Kutta method

3 Numerical Results

3.1 Time evolution of the error spectrum



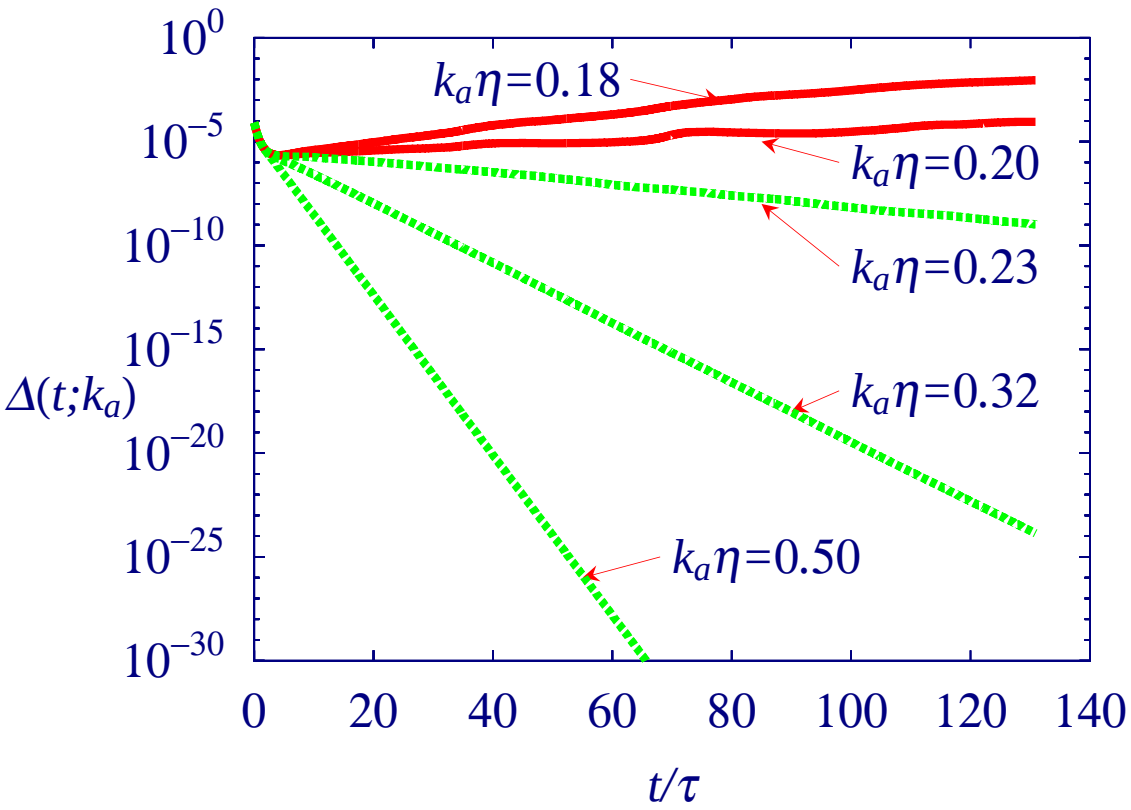
$k_a < k^*$



$k_a > k^*$

3.2 Time evolution of the error

$R_\lambda = 107$



$$\Delta(t;k_a) = \frac{1}{2}\langle \delta\mathbf{u}(t)\cdot\delta\mathbf{u}(t)\rangle$$

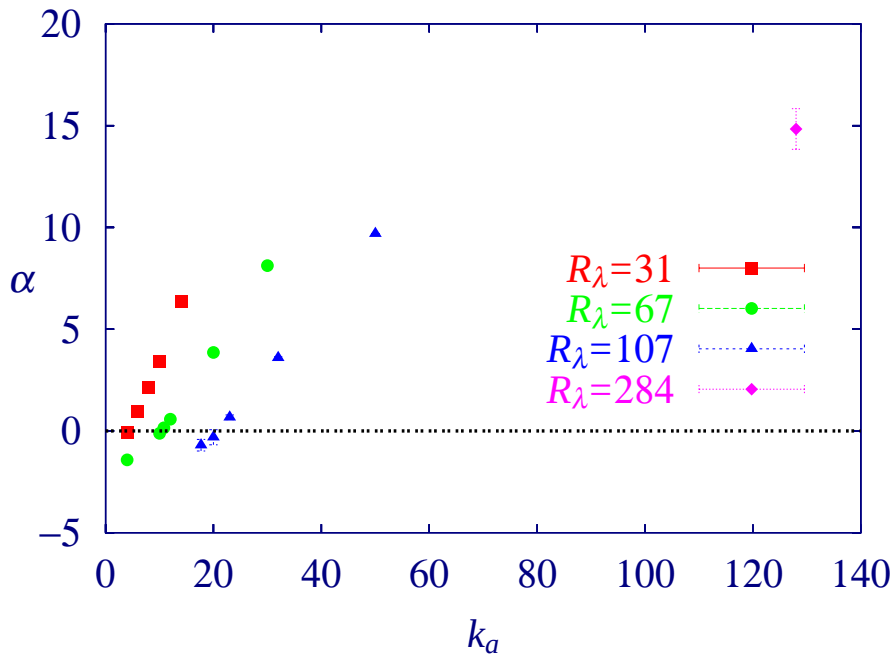
$\alpha(k_a)$: **decay coefficient.**

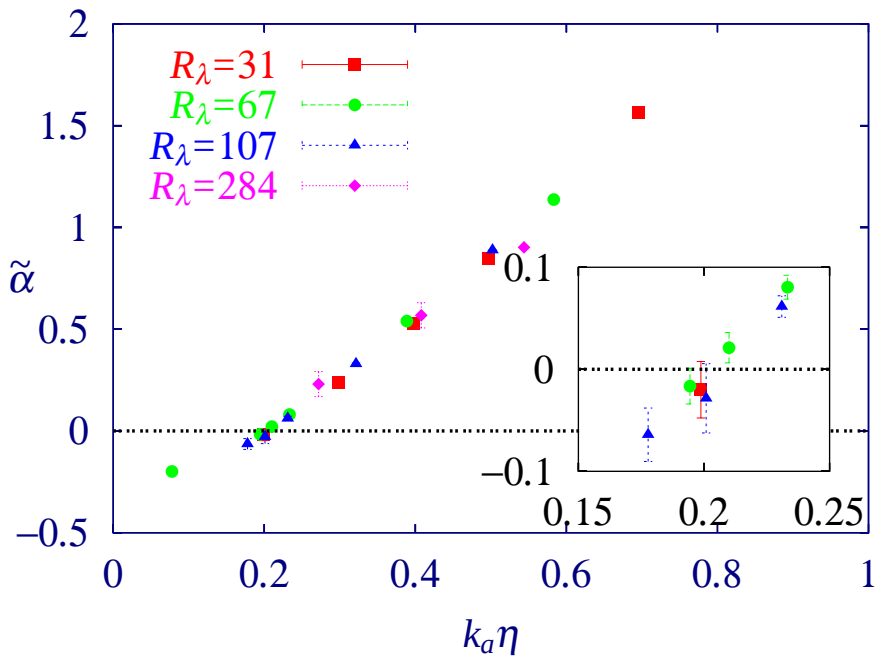
$$\Delta(t;k_a) \sim C\exp[-\alpha(k_a)t].$$

Kolmogorov length scale

$$\eta = (\nu^3/\epsilon)^{1/4}$$

3.3 Decay coefficient vs k_a





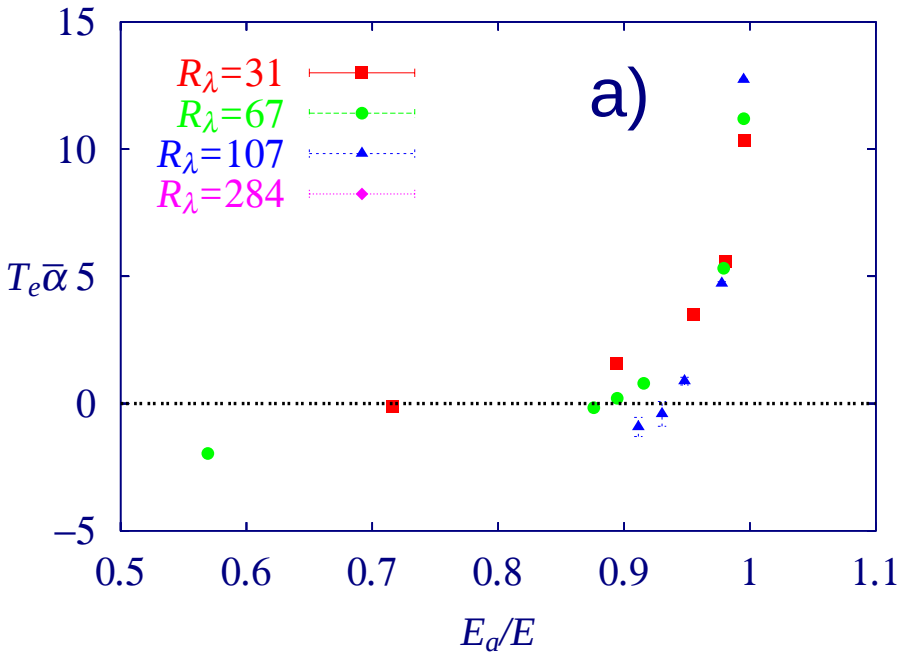
Nondimensionalized decay coefficient

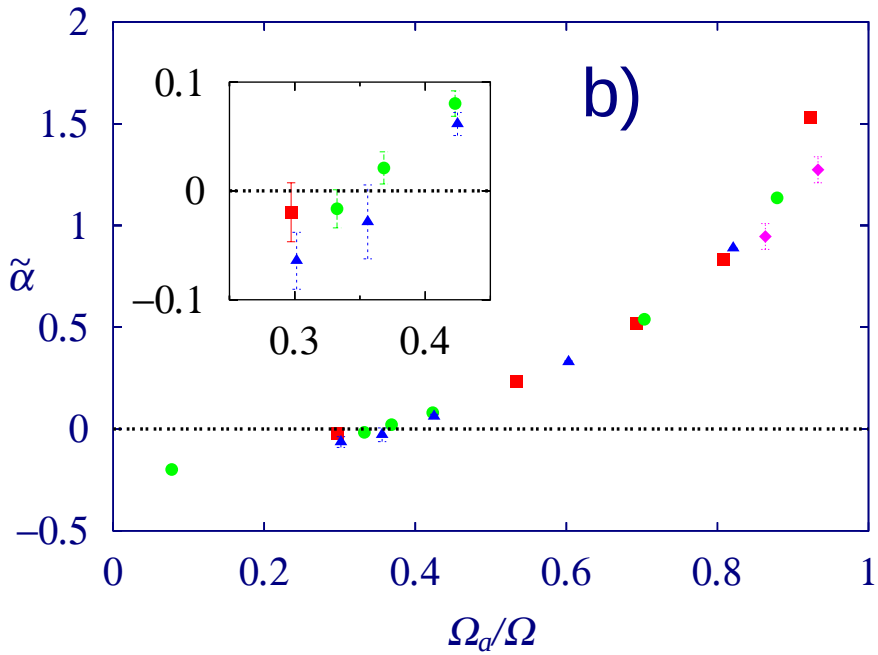
$$\boxed{\tilde{\alpha}(k_a\eta) = \tau\alpha(k_a)} \qquad \eta = (\nu^3/\epsilon)^{1/4}, \tau = (\nu/\epsilon)^{1/2}.$$

Critical wavenumber

$$\boxed{k^*\eta \sim 0.2}.$$

3.4 Decay coefficient vs assim. energy/enstrophy ratio





Energy of assimilated modes

$$E_a = \int_0^{k_a} dk E(k)$$

Eddy-turnover time

$$T_e = \frac{L_0}{u'}$$

Enstrophy of assimilated modes

$$\Omega_a = \int_0^{k_a} dk k^2 E(k)$$

Kolmogorov time scale

$$\tau = (\nu/\epsilon)^{1/2}$$

4 Conclusion

- In turbulence, small-scale error is suppressed, if a sufficient amount of correct data of large scales are continuously assimilated in simulation
  (small scales are functions of large-scale data).

- The condition for the amount is given by $\boxed{k^*\eta \sim 0.2}$,
  k^*: the maximum wavenumber of the assimilated modes,
  $\eta = (\nu^3/\epsilon)^{1/4}$: Kolmogorov length scale.
  - The enstrophy ratio: $\boxed{\Omega^*/\Omega \sim 0.30 - 0.35}$
  - The ratio of number of degrees of freedom : $\boxed{N^*/N \sim 0.01}$

- The condition is expected to be universal for turbulence at very large Reynolds numbers.

Small eddies are subordinate to large eddies.

(Small eddies are a lizard's tail!)