August 21, 2004, Lammi, Finland

Turbulence in Cosmology

Igor Tkachev

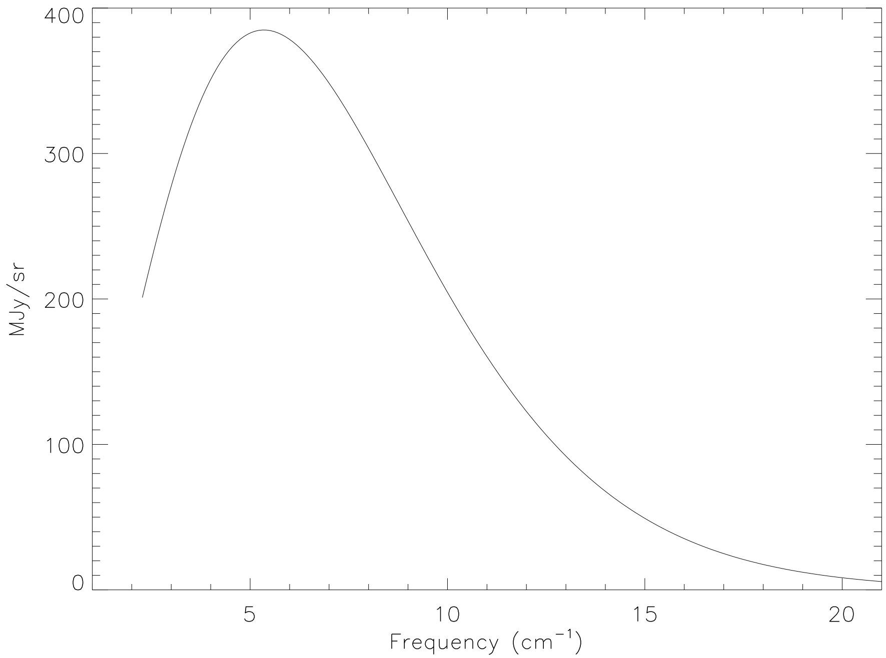
CERN, Geneva

Some articles on astro-ph this year

1. TURBULENCE AND GALACTIC STRUCTURE
2. PROBING TURBULENCE IN THE COMA GALAXY CLUSTER
3. INTERSTELLAR TURBULENCE: IMPLICATIONS AND EFFECTS
4. GRAVITY, TURBULENCE, AND STAR FORMATION
5. HOT DISK CORONA AND MAGNETIC TURBULENCE IN RADIO-QUIET ACTIVE GALACTIC NUCLEI: OBSERVATIONAL CONSTRAINTS
6. COSMIC RAY SCATTERING AND STREAMING IN COMPRESSIBLE MAGNETOHYDRODYNAMIC TURBULENCE
7. STRONG TURBULENCE IN THE COOL CORES OF GALAXY CLUSTERS: CAN TSUNAMIS SOLVE THE COOLING FLOW PROBLEM?
8. GENERATION OF MAGNETIC FIELDS IN THE MULTI-PHASE ISM WITH SUPERNOVA-DRIVEN TURBULENCE

Turbulence in Cosmology

- Turbulent Universe today
- Perfect thermal equilibrium at $T > 1$ eV



- What was at the very beginning ?


COSMOLOGY MARCHES ON
Where the hell did it all come from?
Where the hell did it all come from?
S. Harris

Turbulence in the Early Universe Cosmology

Outline:

- Inflationary cosmology: an introduction
- Creation of matter
- Relaxation towards equilibrium
- Thermalization

BASICS OF INFLATION

Puzzles of classical cosmology which Inflation solves:

WHY THE UNIVERSE

- is so old, big and flat ?
  $t > 10^{10}$ years
- homogeneous and isotropic?
  $\delta T/T \sim 10^{-5}$
- contains so much entropy?
  $S > 10^{90}$
- does not contain unwanted relics?
  (e.g. magnetic monopoles)

Recipe

Inflationary Universe: accelerated expansion or $\ddot{a} > 0$

Friedmann equations

$$\ddot{a} = -\frac{4\pi}{3} G a (\rho + 3p)$$

We have inflation when $p < -\rho/3$

Getting something for nothing

$$T_\mu^{\ \nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

Energy-momentum conservation $T^{\mu\nu}{}_{;\nu} = 0$ can be written as

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0$$

Consider $T_{\mu\nu}$ for a vacuum. Vacuum has to be Lorentz invariant, hence $T_\mu^{\ \nu} = V\,\delta_\mu^{\ \nu}$ and we find $p = -\rho \quad \Rightarrow \quad \dot{\rho} = 0$

Energy of the vacuum stays constant despite the expansion !

The Inflaton field

Consider $T_{\mu\nu}$ for a scalar field φ

$$T_{\mu\nu} = \partial_\mu \varphi \, \partial_\nu \varphi - g_{\mu\nu} \, \mathcal{L}$$

with the Lagrangian :

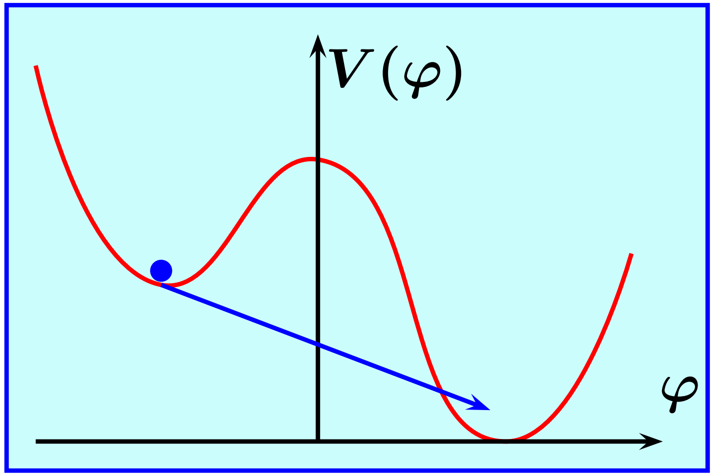
$$\mathcal{L} = \partial_\mu \varphi \, \partial^\mu \varphi - V(\varphi)$$

In a state when all derivatives of φ are zero, the stress-energy tensor of a scalar field is that of a vacuum

$$T_{\mu\nu} = V(\varphi) \, g_{\mu\nu}$$

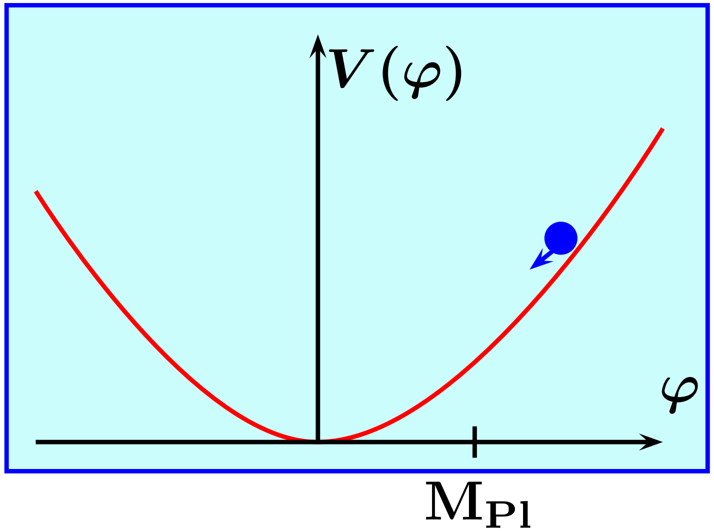
There are two basic ways to arrange $\varphi \approx$ const and hence to imitate the vacuum-like state.

1. A. Guth: consider potential with two minima



2. A. Linde: consider the simplest potential

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2$$

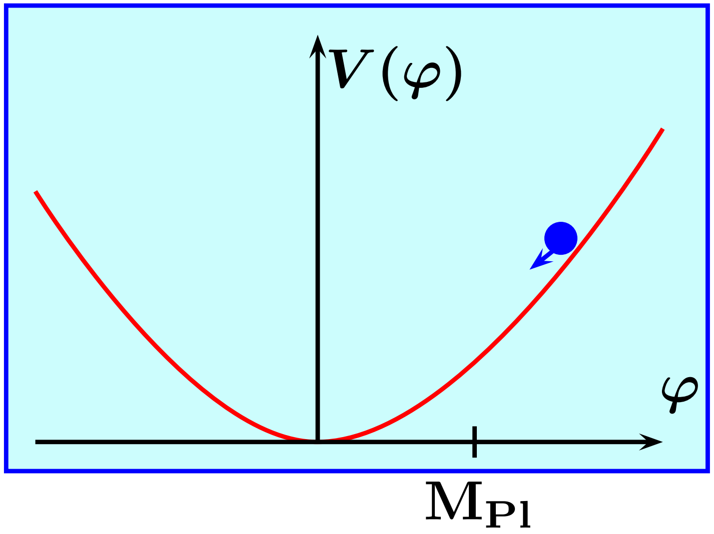


Chaotic Inflation

Equation of motion

$$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0$$

If $H \gg m$ the field (almost) does not move.



$$H \sim \sqrt{\rho/M_{Pl}^2} \sim m\,\varphi/M_{Pl}:$$

$\varphi > M_{Pl}$ Inflation

$\varphi < M_{Pl}$ Field oscillates.

During and after Inflation the Universe is empty, in a vacuum state.

How vacuum was turned into radiation ?

Where all matter and seeds for structure formation came from ?

Unified Theory of Creation

During Inflation the Universe is “empty”. But small fluctuations obey

$$\ddot{u}_k \; + \; [k^2 + m^2_{\mathrm{eff}}(\tau)] \, u_k = 0$$

and it is not possible to keep fluctuations in vacuum

if m_{eff} is time dependent

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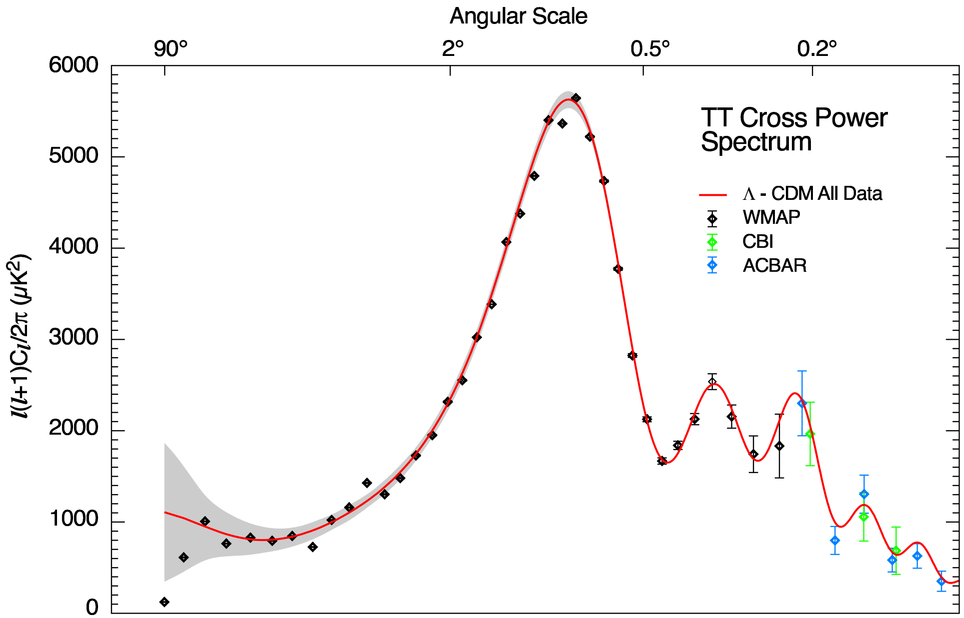
Sources of matter (and structure) creation

- Expansion of space-time itself, $a(\tau)$
- Evolution of the inflaton field, $\phi(\tau)$

Technical remarks:

- This is true for all species
- Equations look that simple in conformal reference frame $ds^2 = a(\eta)^2\,(d\eta^2 - dx^2)$
- For conformally coupled, but massive scalar $m_{\rm eff} = m_0\, a(\eta)$
- $m_{\rm eff}$ may be non-zero even for massless fields.
  - graviton is the simplest example $m^2_{\rm eff} = -\ddot{a}/a$
- Of particular interest are ripples of space-time itself
  - curvature fluctuations (scalar)
  - gravitons (tensor)

Creation of metric perturbations



Data are in agreement with Inflationary predictions

$\mathbf{\Omega_0 = 1.0 \pm 0.03}$,

$\mathbf{n_s = 0.99 \pm 0.04}$

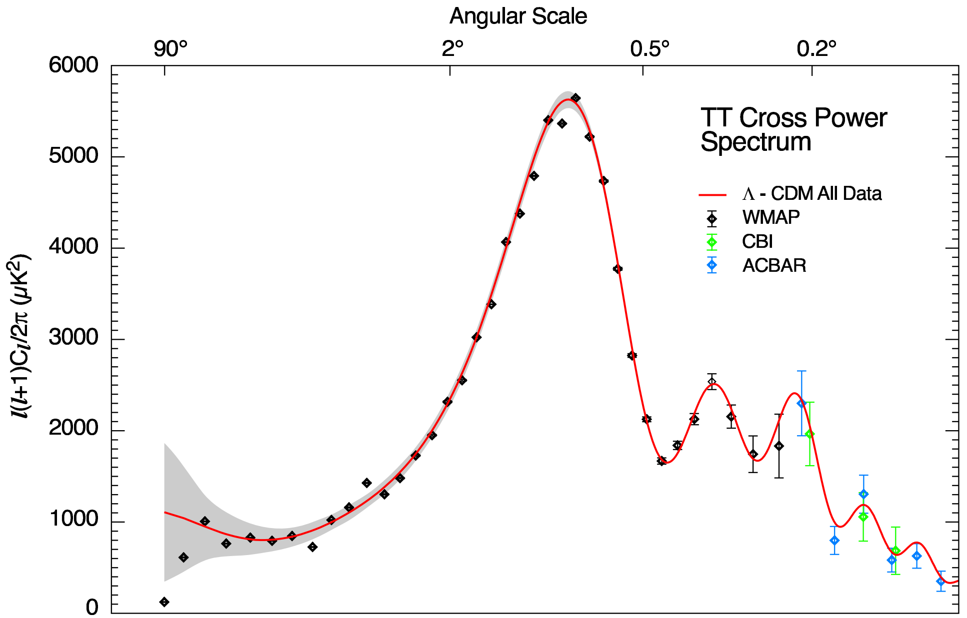
□

Derived parameters of the inflaton potential

$$m \sim \frac{\delta T}{T} M_{\rm Pl} \sim 10^{13}\ \mathrm{GeV}$$

$$\lambda \sim \left(\frac{\delta T}{T}\right)^2 \sim 10^{-12}$$

Creation of metric perturbations



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Gravitational Creation of matter

Source of creation: $m_{\mathrm{sh}}\, a(\tau)$

It is effective at: $H \sim m_{\mathrm{sh}}$

Number density of created particles: $n \sim m_{\mathrm{sh}}^3$

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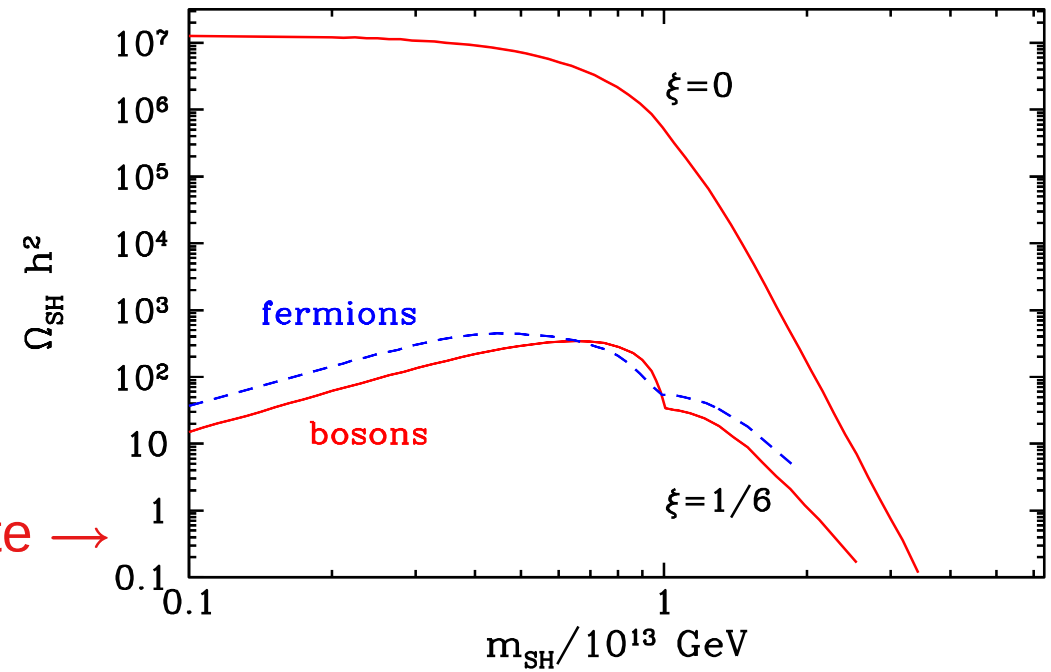

New dark matter candidate $\longrightarrow$

Coupling to the inflaton as a source of creation

Scalar X

$m_{\text{eff}}^2 = m_X^2 + g^2\phi^2(t)$

Fermion ψ

$m_{\text{eff}} = m_\psi + g\phi(t)$

Coupling to the inflaton as a source of creation

Scalar X

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Fermion ψ

$$m_{\rm eff} = m_\psi + g\phi(t)$$

Relevant parameter:

$$g^2 \;\longrightarrow\; q \equiv \frac{g^2\phi^2}{4m_\phi^2}$$

Note: q can be very large since

$$\frac{\phi^2}{m_\phi^2} \approx 10^{12}$$

Bose versus Fermi :

Scalar X

$m^2_{\text{eff}} = m^2_X + g^2\phi^2(t)$

Fermion ψ

$m_{\text{eff}} = m_\psi + g\phi(t)$

Bose stimulation.

Occupation numbers grow,

$n = e^{\mu t}$

Pauli blocking.

Occupation numbers

$n < 1$

Bose versus Fermi :

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Explosive decay of the inflaton

Bose versus Fermi :

Scalar X	Fermion ψ
$m_{\text{eff}}^2 = m_X^2 + g^2\phi^2(t)$	$m_{\text{eff}} = m_\psi + g\phi(t)$

Explosive decay of the inflaton

$\downarrow$

Quantum to classical transition

Pauli blocking.

Occupation numbers

$n < 1$

Bose versus Fermi :

Scalar X

$$m_{\rm eff}^2 = m_X^2 + g^2\phi^2(t)$$

Fermion ψ

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Quantum to classical transition

$\downarrow$

Non-thermal phase transitions

Pauli blocking.

Occupation numbers

$n < 1$

Bose versus Fermi :

Scalar X	Fermion ψ
$m_{\mathrm{eff}}^2 = m_X^2 + g^2\phi^2(t)$	$m_{\mathrm{eff}} = m_\psi + g\phi(t)$

Heavy particles are always heavy

Heavy particles are massless at $\phi(t) = -m_\psi/g$

Bose versus **Fermi** :

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Heavy particles are always heavy

Heavy particles are massless at $\phi(t) = -m_\psi/g$

$\downarrow$

Superheavy Fermion creation

Thermalization after Inflation

With R. Micha

Questions:

- How system approaches equilibrium ?
- When ? What is thermalization temperature ?

Are of general interest and important for parctical applications.
It influences:

- Inflationary predictions
- Baryogenesis
- Abundance of gravitino and dark matter relics
- Primordial fluctuations

Approach:

- Lattice simulations (as a guidance)
- Kinetic theory

Approach:

Consider simplest $\lambda\varphi^4$ model

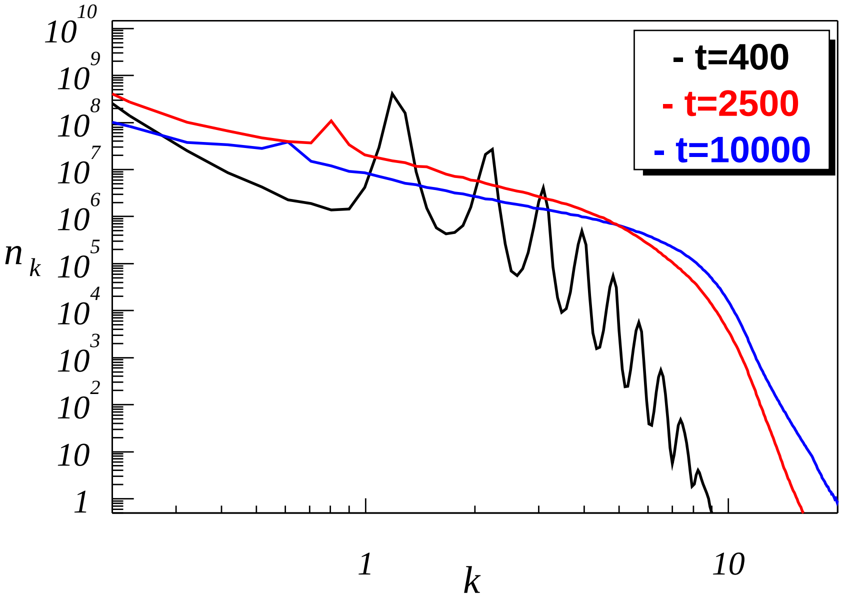
In conformal frame, $\phi = \varphi/a$, and rescaled coordinates, $x^\mu \to \sqrt{\lambda}\varphi(0)\, x^\mu$, the equation of motion

$$\Box\phi + \phi^3 = 0$$

can be solved on a lattice and various quantities be measured

- Zero mode, $\phi_0 = \langle\phi\rangle$
- Variance, $\langle\phi^2\rangle$ - ϕ_0^2
- Particle number, $n_k = \langle a^\dagger(k)a(k)\rangle$
- Correlators, $\langle aa\rangle$, $\langle a^\dagger a^\dagger aa\rangle$, $\langle\pi^2\rangle$, ...

Particle spectra on a lattice



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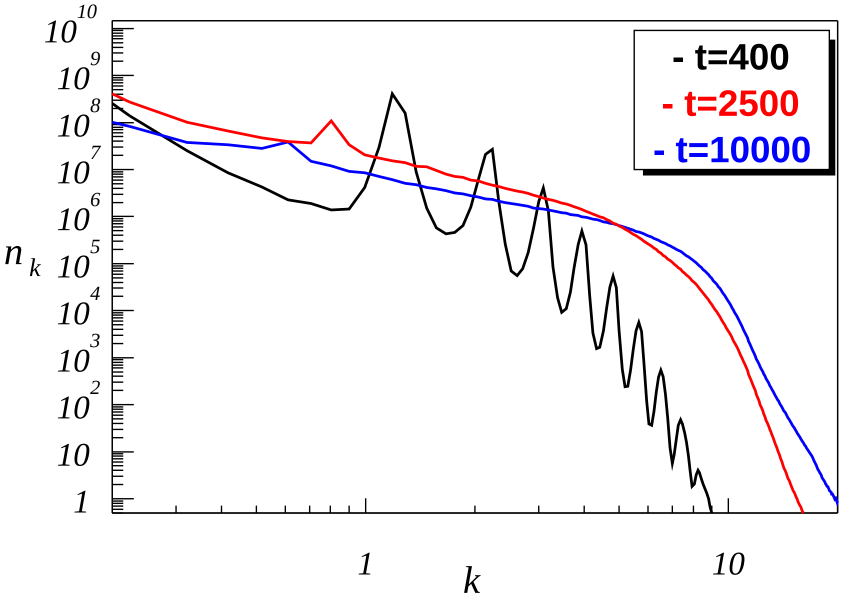


Complications:

- Insufficient dynamical range in k
- Hopelessly long integration time

Simple kinetic description ?

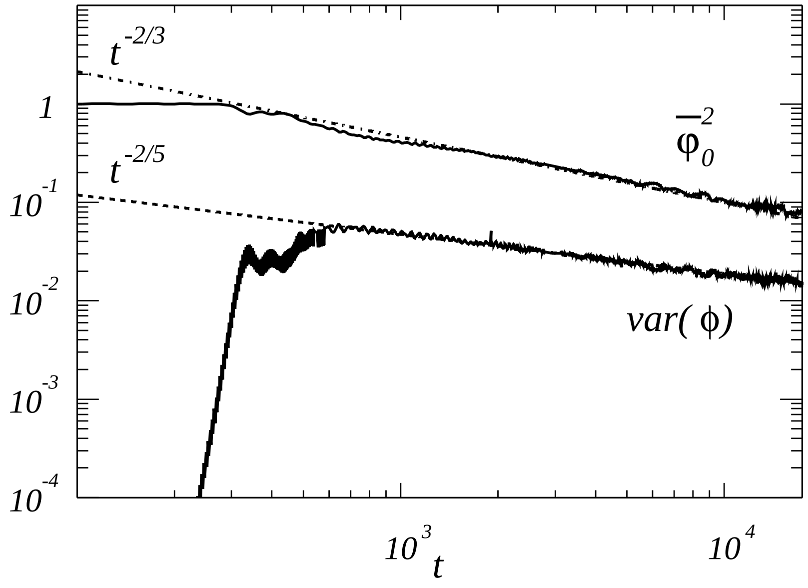
Complications:

- Occupation numbers are too big
- Are anomolous correlators important ?

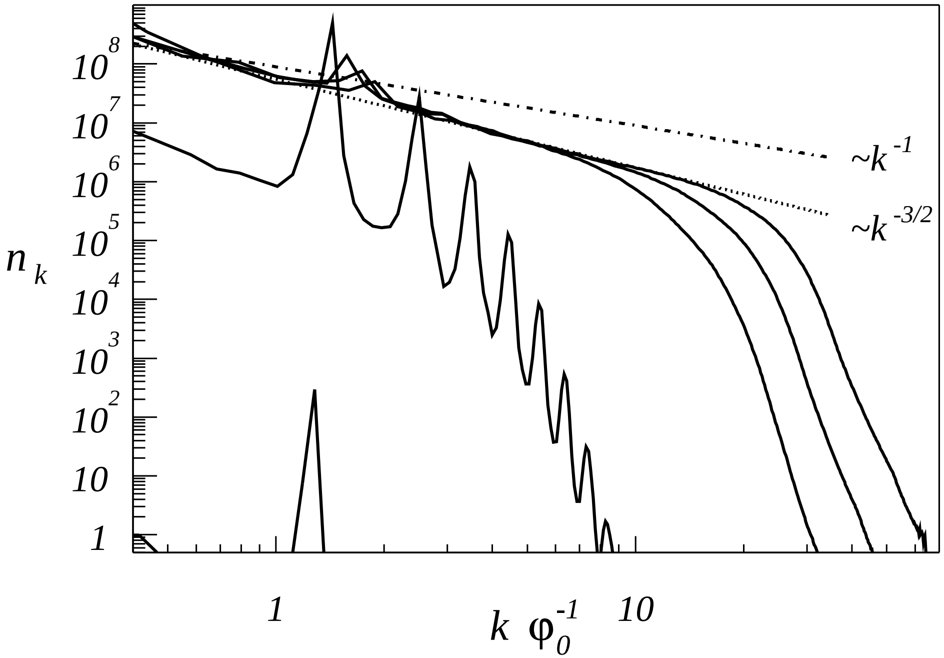
Simple kinetic description ?

Complications:

- Occupation numbers are too big
- Are anomolous correlators important ?
- Zero mode never dies



Turbulent spectra

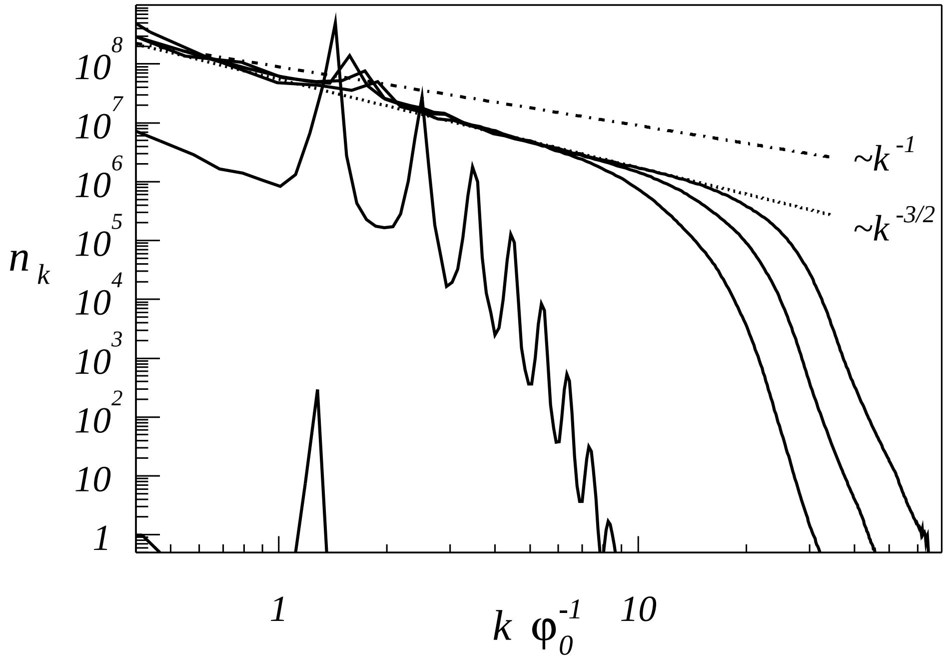


Re-scale the field and coordinates by the current amplitude of the zero mode

$$\Box\phi + \phi^3 = 0$$

Here $x^\mu \;\to\; x^\mu\,\phi_0$ and therefore $k \to k\,/\phi_0$

Turbulent spectra



Re-scale the field and coordinates by the current amplitude of the zero mode

$$\Box\phi + \phi^3 = 0$$

Here $x^\mu \longrightarrow x^\mu\,\phi_0$ and therefore $k \longrightarrow k\,/\phi_0$

Let $n \sim k^{-\alpha}$. Theory of a stationary wave turbulence (Zakharov, L'vov, Falkovich) predicts

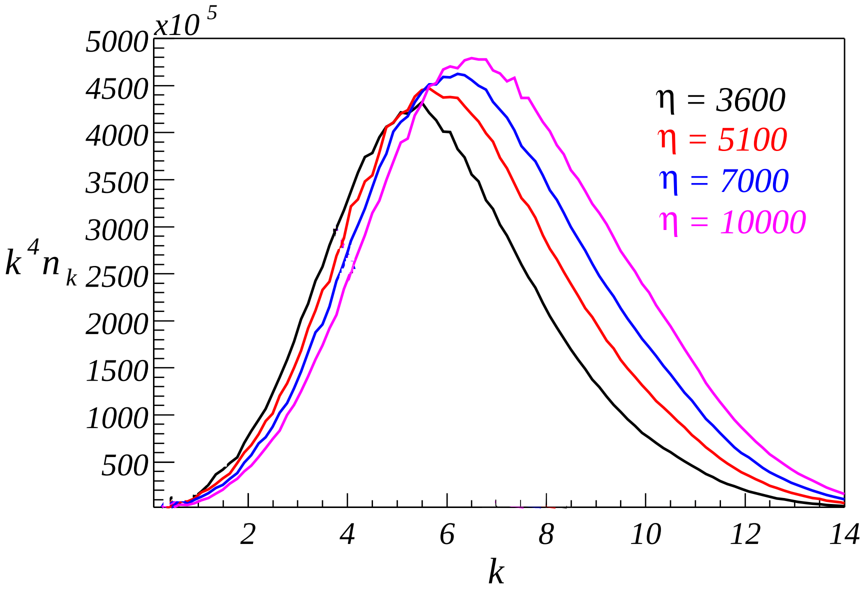
- $\alpha = \frac{5}{3}$ for 4-particle interaction
- $\alpha = \frac{3}{2}$ for 3-particle interaction

Self-similar evolution

At late times we expect self-similarity with conserved energy

$$n(k,t) = t^{-q}\, n_0(kt^{-p})$$

Excellent fit to numerical data with $q = 3.5p$ and $p = \frac{1}{5}$



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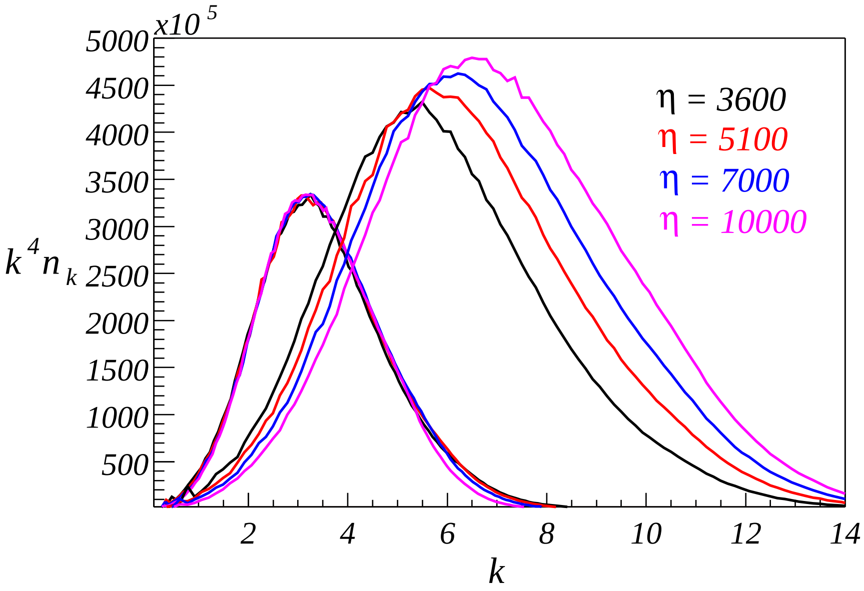

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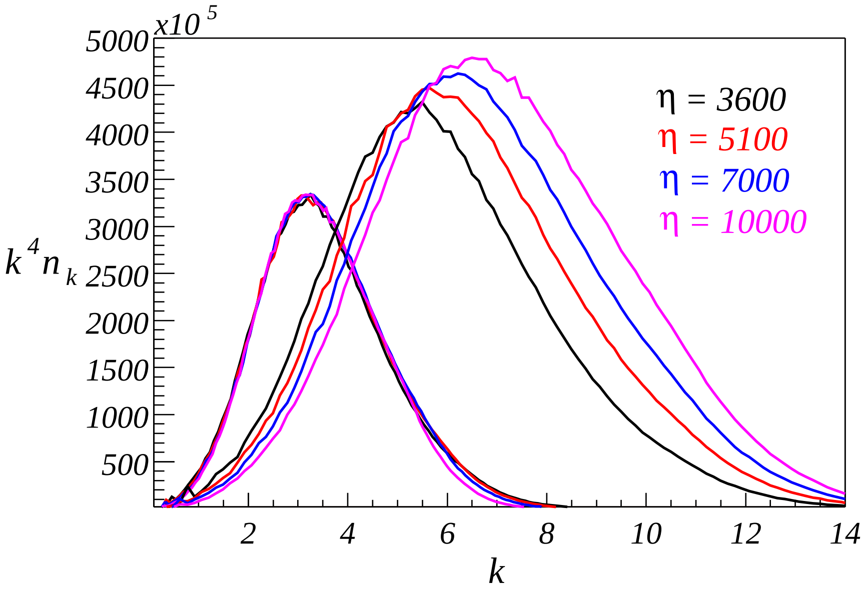
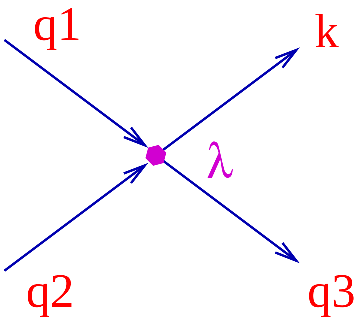


Theory of wave turbulence (Zakharov, L'vov, Falkovich) predicts

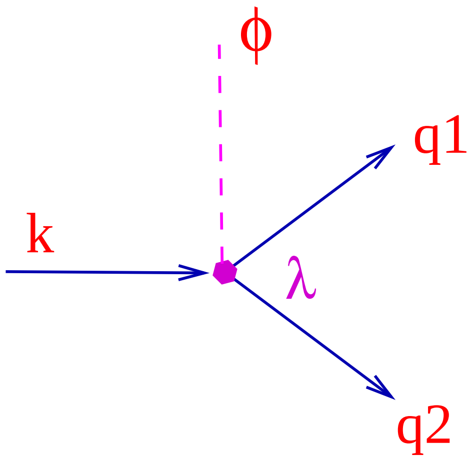
- $p = \frac{1}{7}$ for 4-particle interaction
- $p = \frac{1}{5}$ for 3-particle interaction

3-particle interactions

Replace



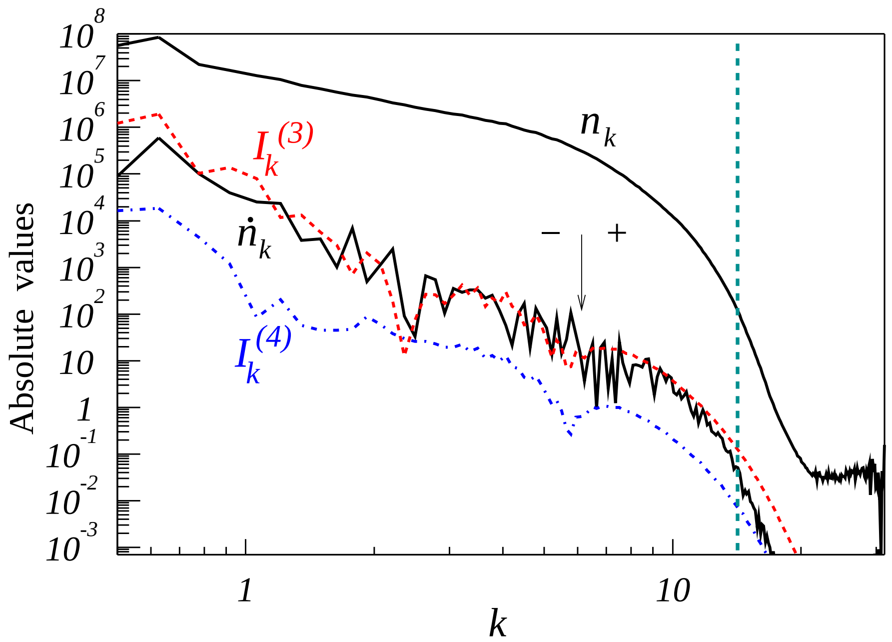
by



3-paricle collision integral is proportional to the amplitude of the zero-mode $\boldsymbol{I_k^{(3)} \sim \varphi_0^2}$

Since zero-mode decays , one expects $\boldsymbol{p = 1/15}$ ...(?)...

Test of kinetic description



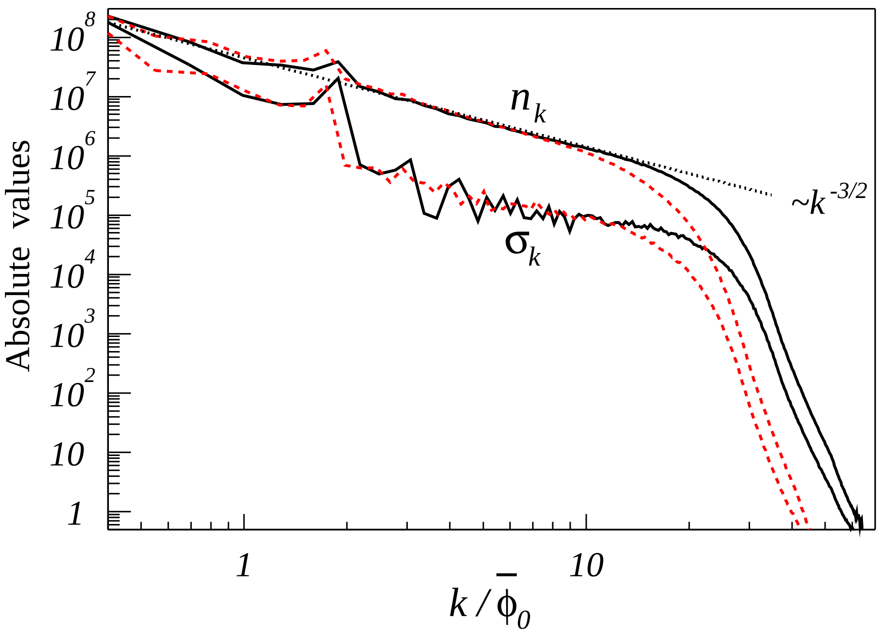
Collision integrals and $\dot{n}(k)$ at $\eta = 5000$.

$I_k^{(3)}$ agrees with $\dot{n}(k)$ to the left of the vertical dashed line

Red line: 3-particle collision integral, $I_k^{(3)}$

Blue line: 4-particle collision integral, $I_k^{(4)}$

Test of kinetic description

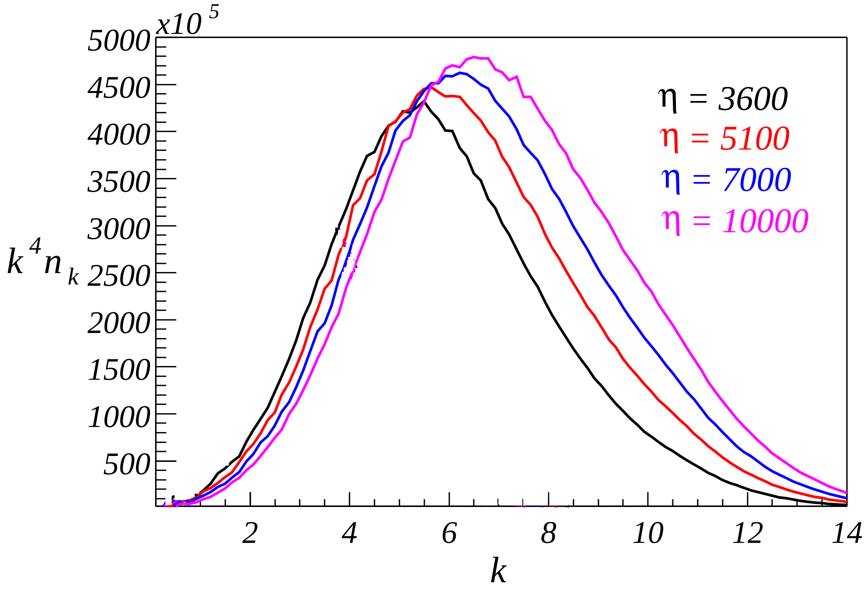


Dashed lines: $\eta = 5000$

Solid lines: $\eta = 10000$

Occupation numbers, $n_k = \langle a_k^\dagger a_k \rangle$, and absolute values of "anomalous" correlator $\sigma_k = \langle a_k a_k \rangle$

Thermalization



$$n(k,t) = t^{-q}\, n_0(kt^{-p})$$

The exponent p determines the rate with which a system approaches equilibrium $k_{\max}(\tau) = k_0\, \tau^p$, where $k_0 = \lambda^{1/2}\varphi_0$. Thermalization will occur when $k_{\max}^4 \sim T^4 \sim \lambda\varphi_0^4$.

Thermalization

At late times influence of the zero mode should become negligible and

$$p = \frac{1}{7}$$

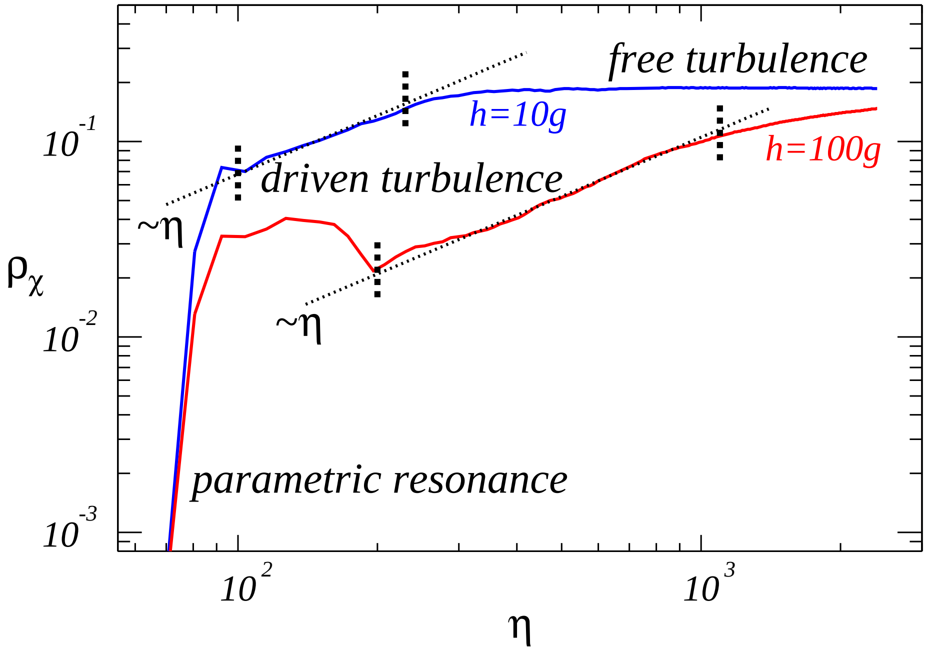
Time to thermalization $\tau \sim \lambda^{-7/4} \sim 10^{21}$.
Scale factor in comoving coordinates $a(\tau) = \tau$ and we find for thermalization temperature

$$T \sim \frac{k_{\max}}{a(\tau)} = \lambda^2 \varphi_0 = 10^{-26} M_{\rm Pl} = 100 \text{ eV} .$$

One can use “naive” perturbation theory to estimate thermalization tempreature.

Three major epochs of reheating

$$V(\chi, X) = \frac{\lambda_\phi}{4}\phi^4 + \frac{g}{2}\phi^2\chi^2 + \frac{\lambda_\chi}{4}\chi^4$$

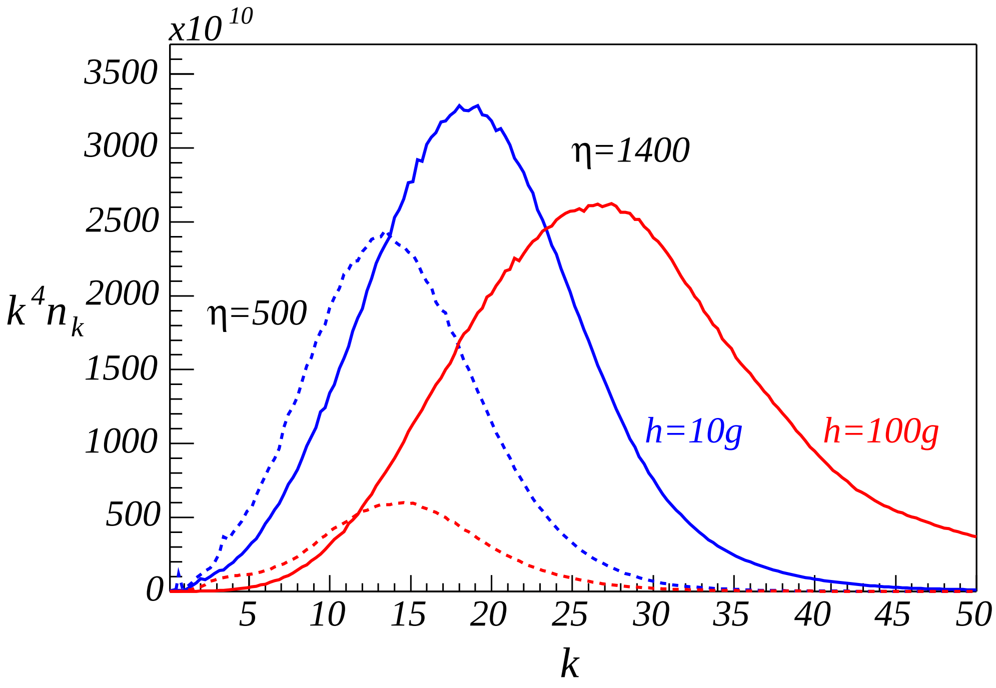


At large h and/or g the parametric resonance stops when n_χ are relatively low

$$h = \lambda_X/\lambda_\phi, \quad h = \lambda_{X\phi}/\lambda_\phi$$

Three major epochs of reheating

$$V(\chi, X) = \frac{\lambda_\phi}{4}\phi^4 + \frac{g}{2}\phi^2\chi^2 + \frac{\lambda_\chi}{4}\chi^4$$



But particle distributons for larger h and/or g move faster

$$h = \lambda_X/\lambda_\phi, \quad h = \lambda_{X\phi}/\lambda_\phi$$

Conclusions

- We identify three different stages of the Universe reheating
  - “Parametric resonance.” Fast exponential growth of energy in fluctuations, but only a small fraction is transferred.
  - Driven turbulence. Linear growth. Major mechanism of energy transfer.
  - Free turbulence. Long stage of thermalization.
- Turbulent evolution is self-similar and in agreement with kinetic theory developed by Falkovich and Shafarenko.
- Estiamtes for reheating time and temperature are found.