Flux Instability in Type-II Superconductors

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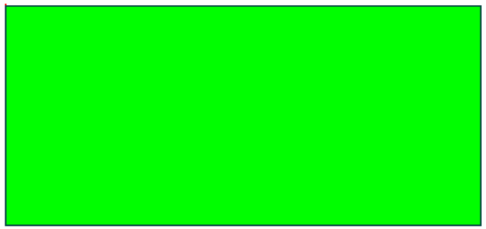
Introduction. Critical State. Instability

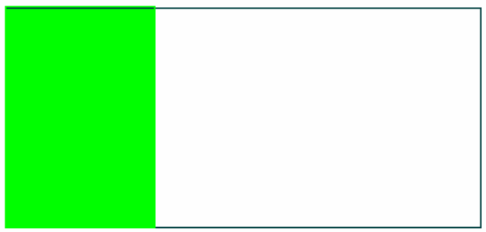
- **$B>B_{c1}$ No pinning**

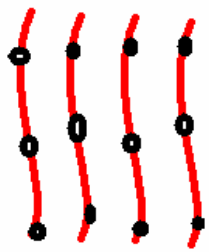


Slowly growing magnetic field results in abrupt creation of the large dendritic structure

MgB_2 Nb_3Sn

- **Been Critical state**





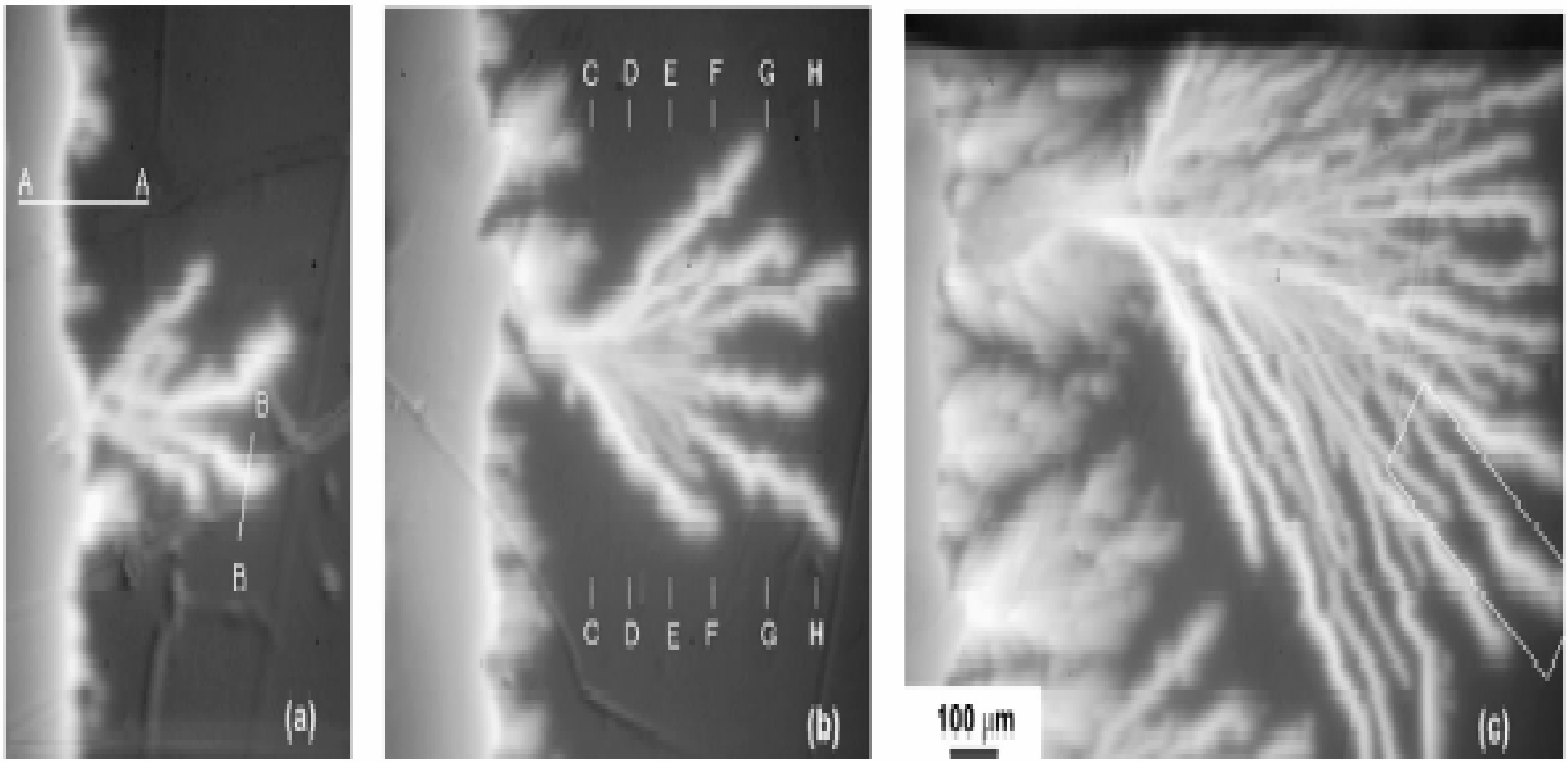
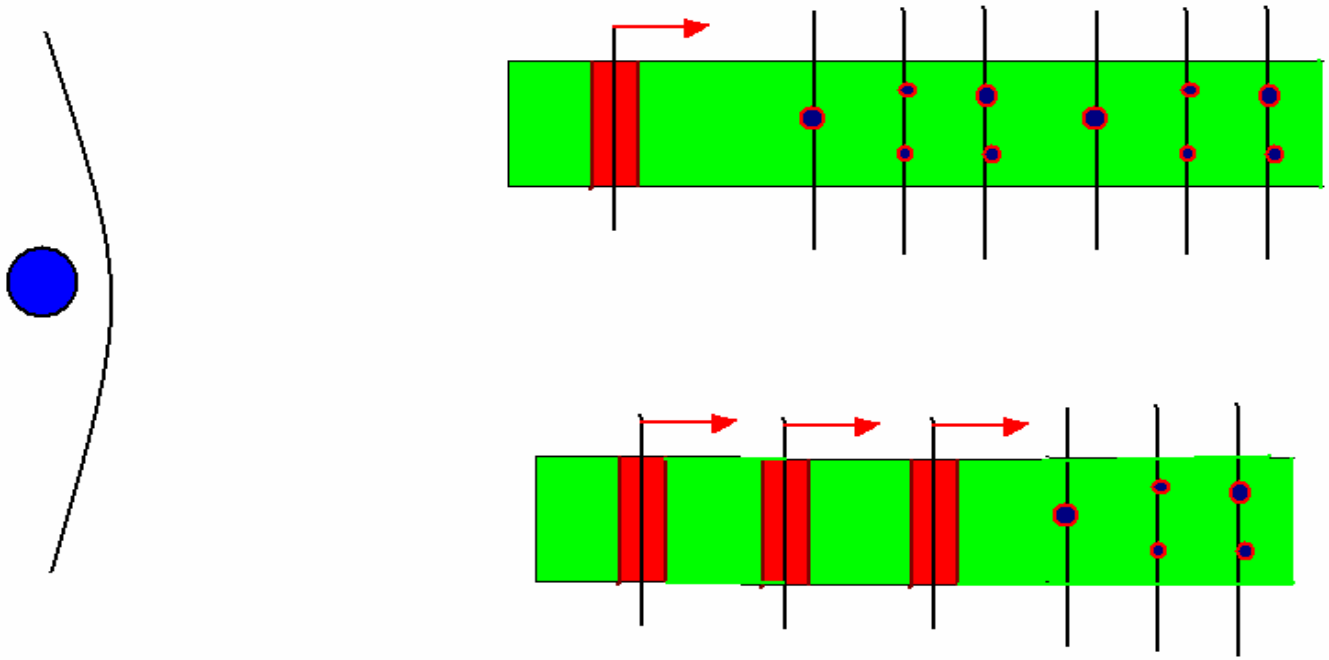


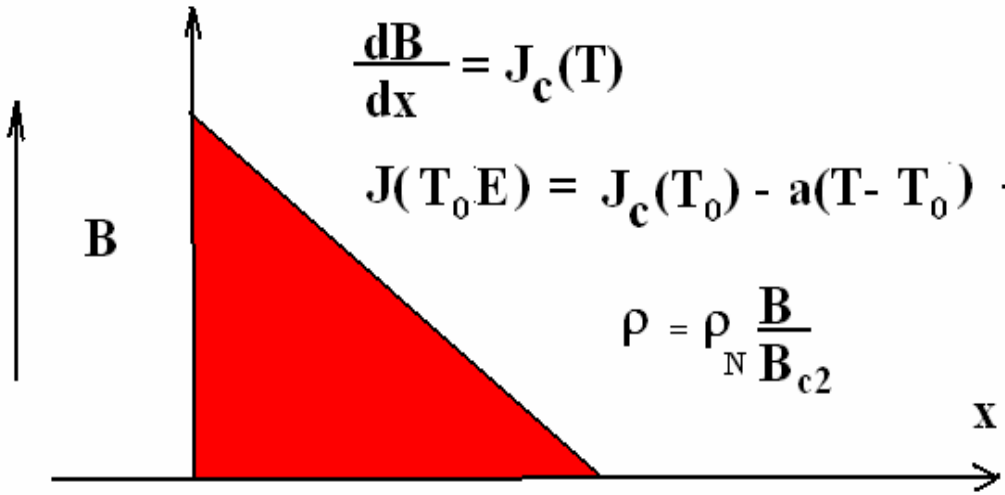
FIG. 2. MO images showing dendritic flux structures formed near the edge of the MgB_2 film at applied fields, which in (a)–(c) are B_a=2.3, 3.2, and 7.4 mT, respectively. The dendritic structures for different B_a differ in size, but not in flux density (image brightness) along the core of the individual branches.

Thermo-magnetic feedback



Avalanche like penetration in the Critical State

I. Maksimov (1994)

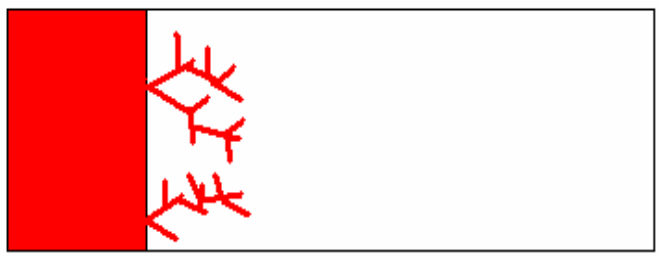


$$\frac{dB}{dx} = J_c(T)$$

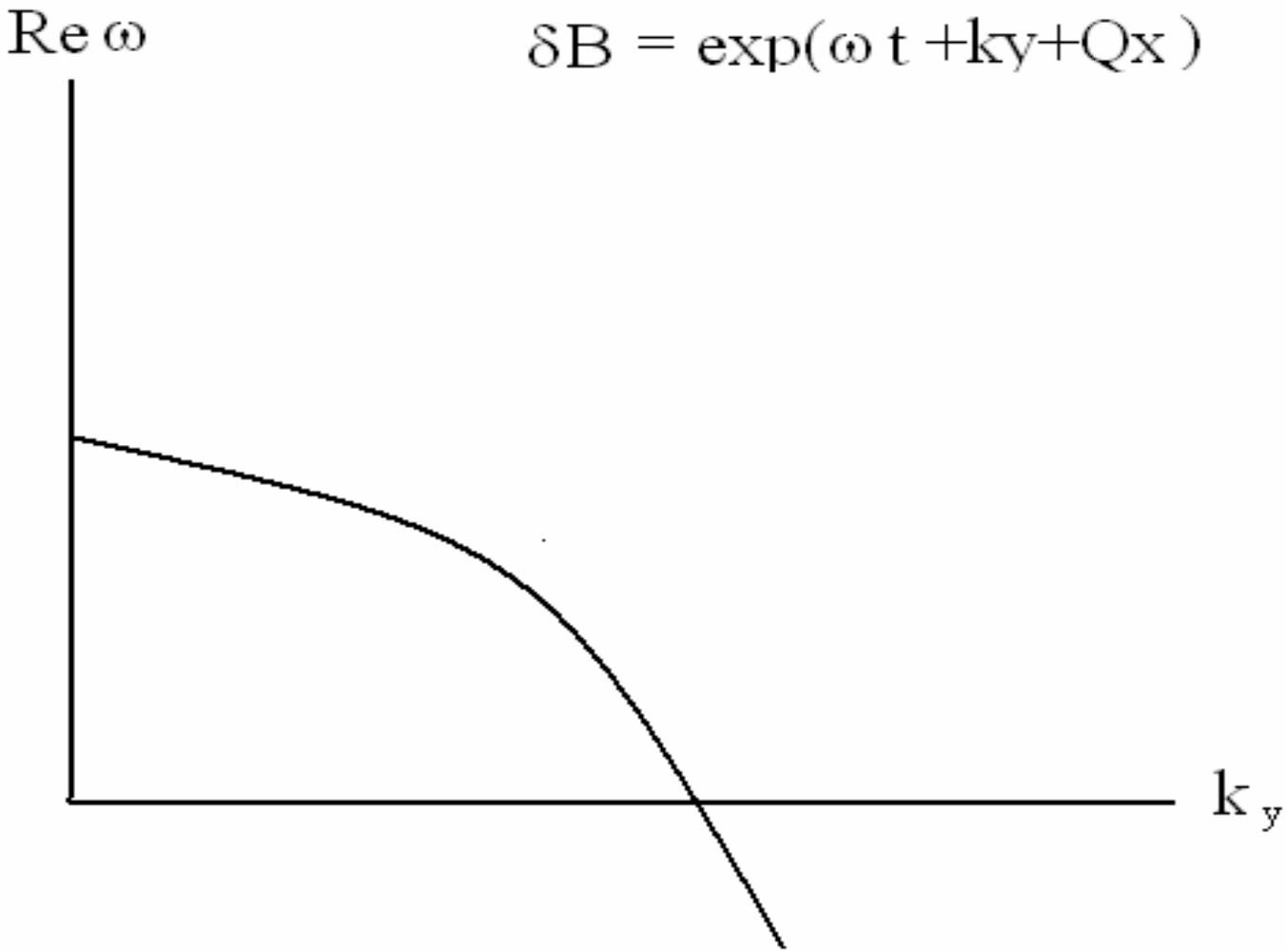
$$J(T_0 E) = J_c(T_0) - a(T - T_0) + \frac{E}{\rho}$$

$$\rho = \rho_N \frac{B}{B_{c2}}$$

Thermal softening of the pinning force leads to instability



The instability increment



Motivation: “Avalanches” without initial critical state.

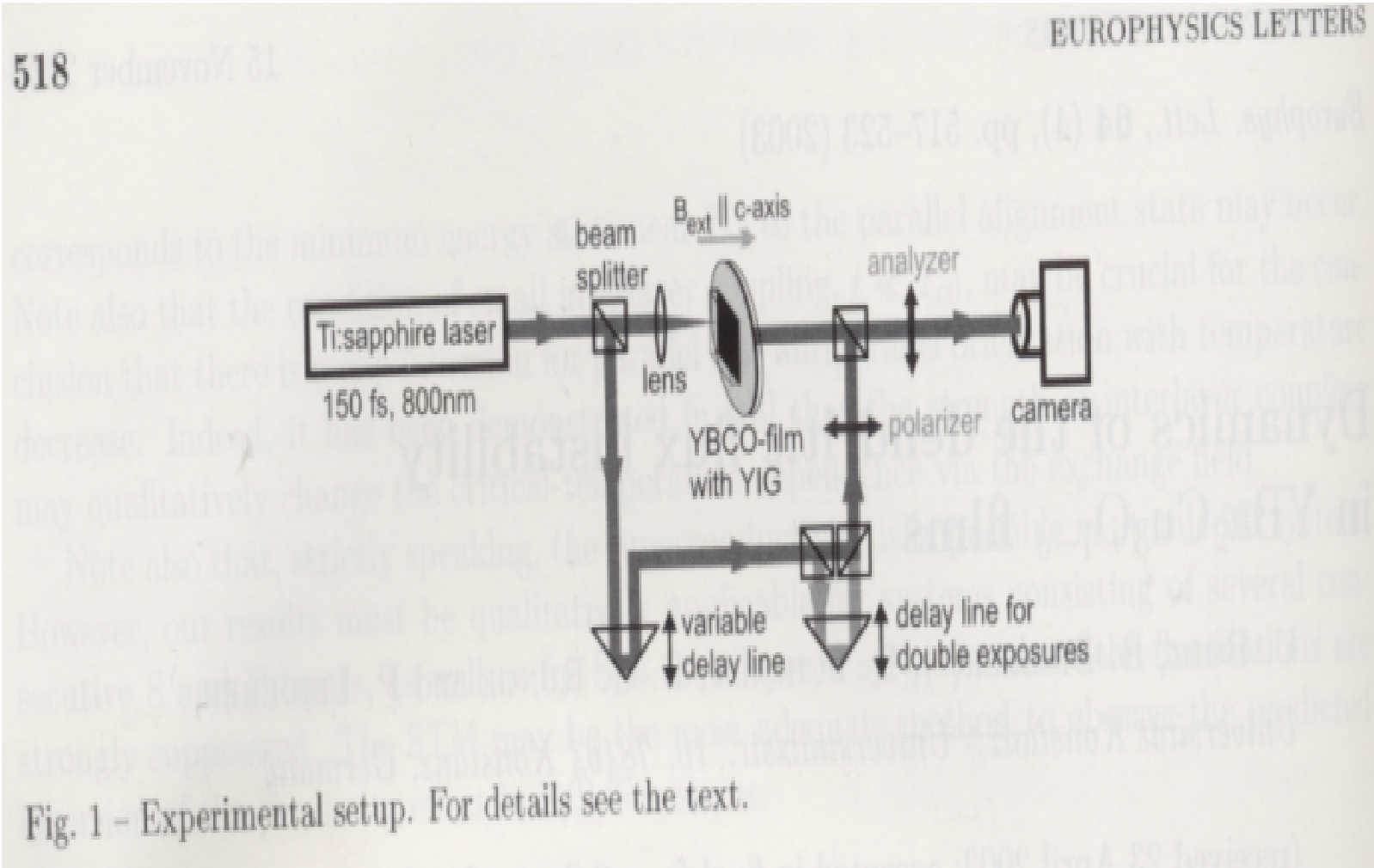
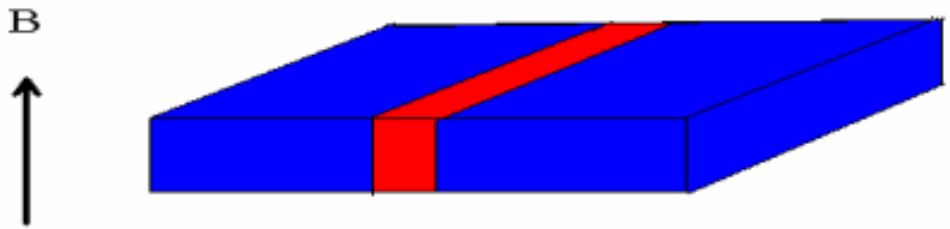


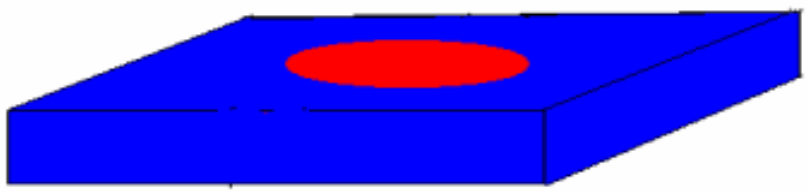
Fig. 1 – Experimental setup. For details see the text.

Hot spot and layer

After a Heat Impact



Leiderer et al (2002)



Leiderer et al (1993),Wyder et al (2002),Polturak et al (2003)

Hydrodynamics Instability of the Hot Spot

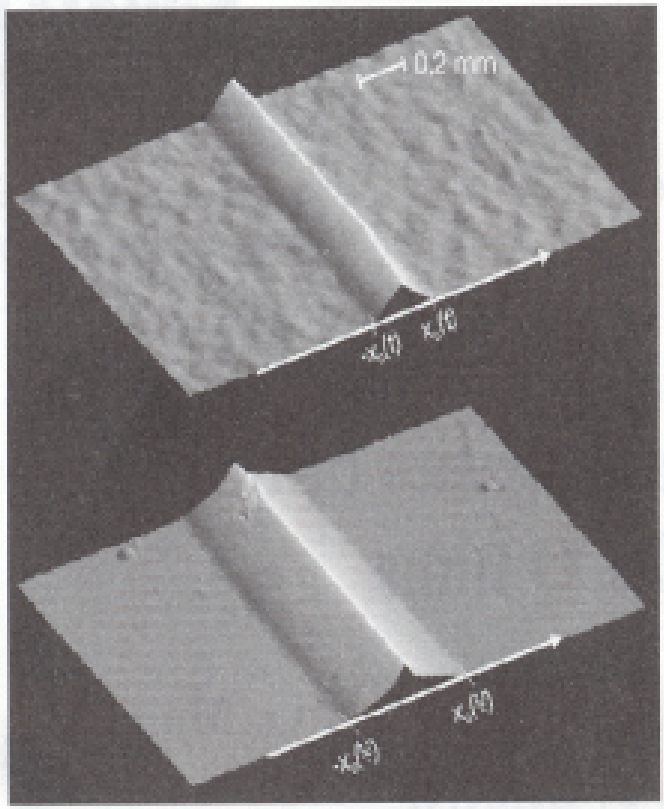


Fig. 1. Magnetic flux profiles after a time delay of 67.8 ns and of the final state ($T = 30$ K, $B_a = 15.2$ mT).

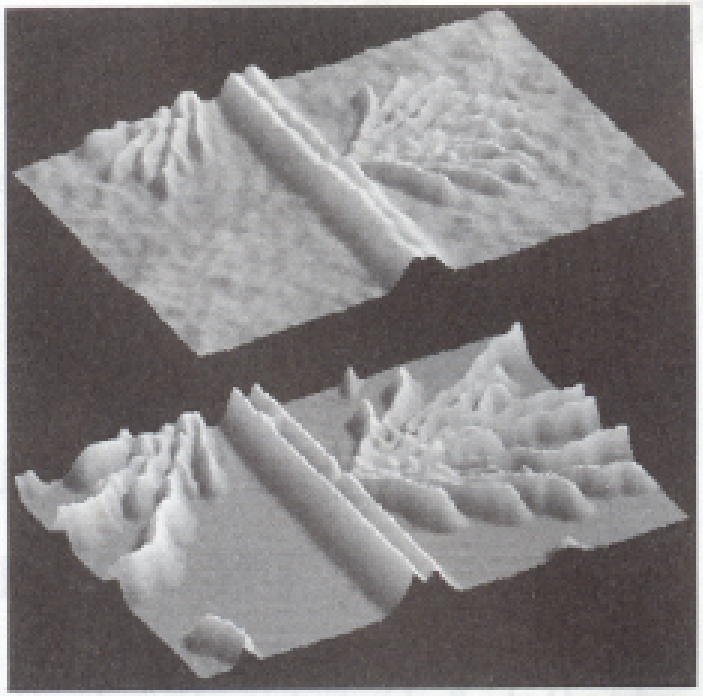
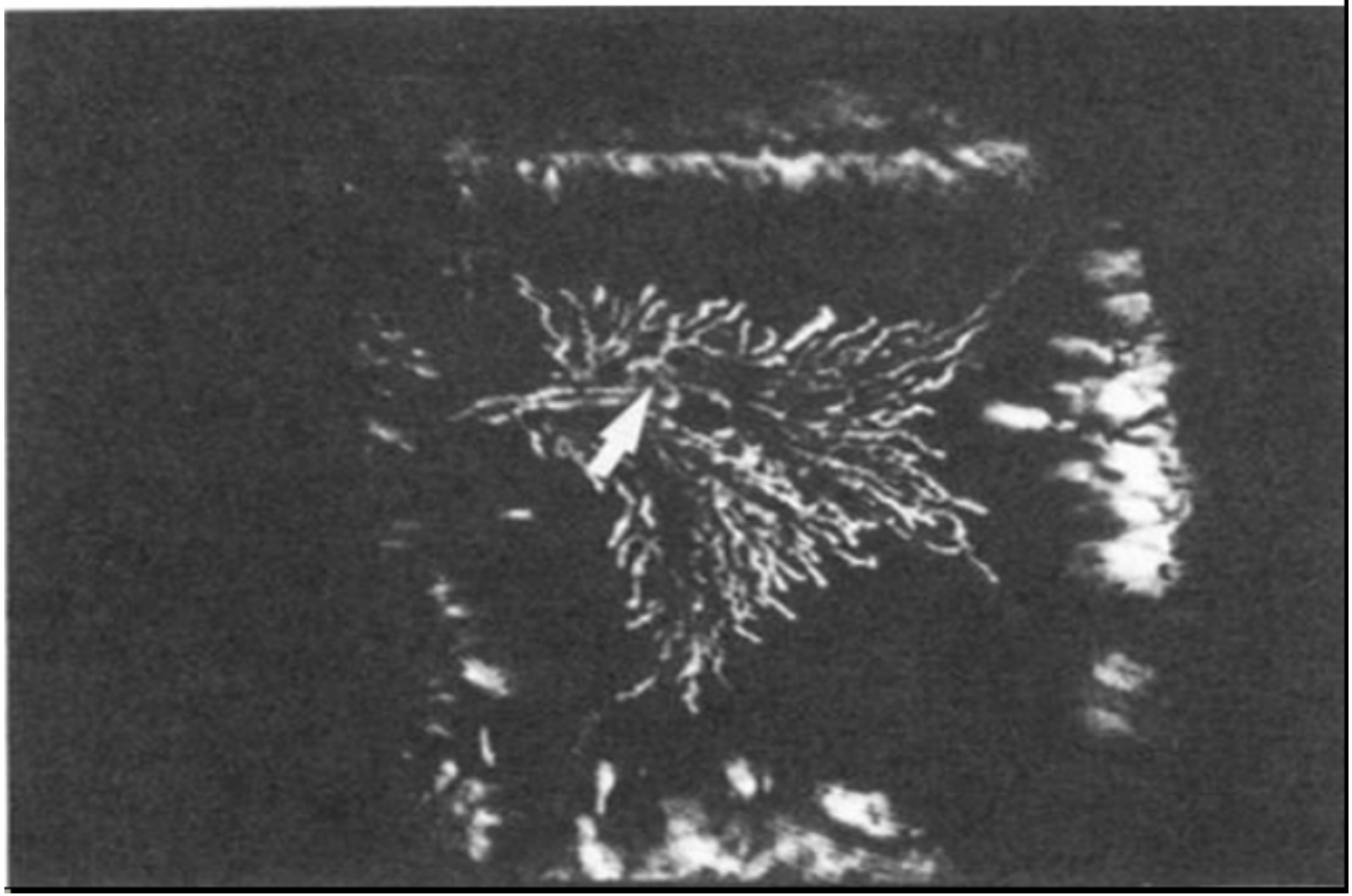


Fig. 3. 3D magneto-optical images of the flux density distribution after 50 ns and of the final state at 10 K with an applied field of 30 mT. The size of the area shown is 2.28×1.52 mm^2 and the maximal magnetic field density is 95 mT.

Instability of the Topologically Charged Hot Spot



Microscopic Basic Equations

TDGL Equations

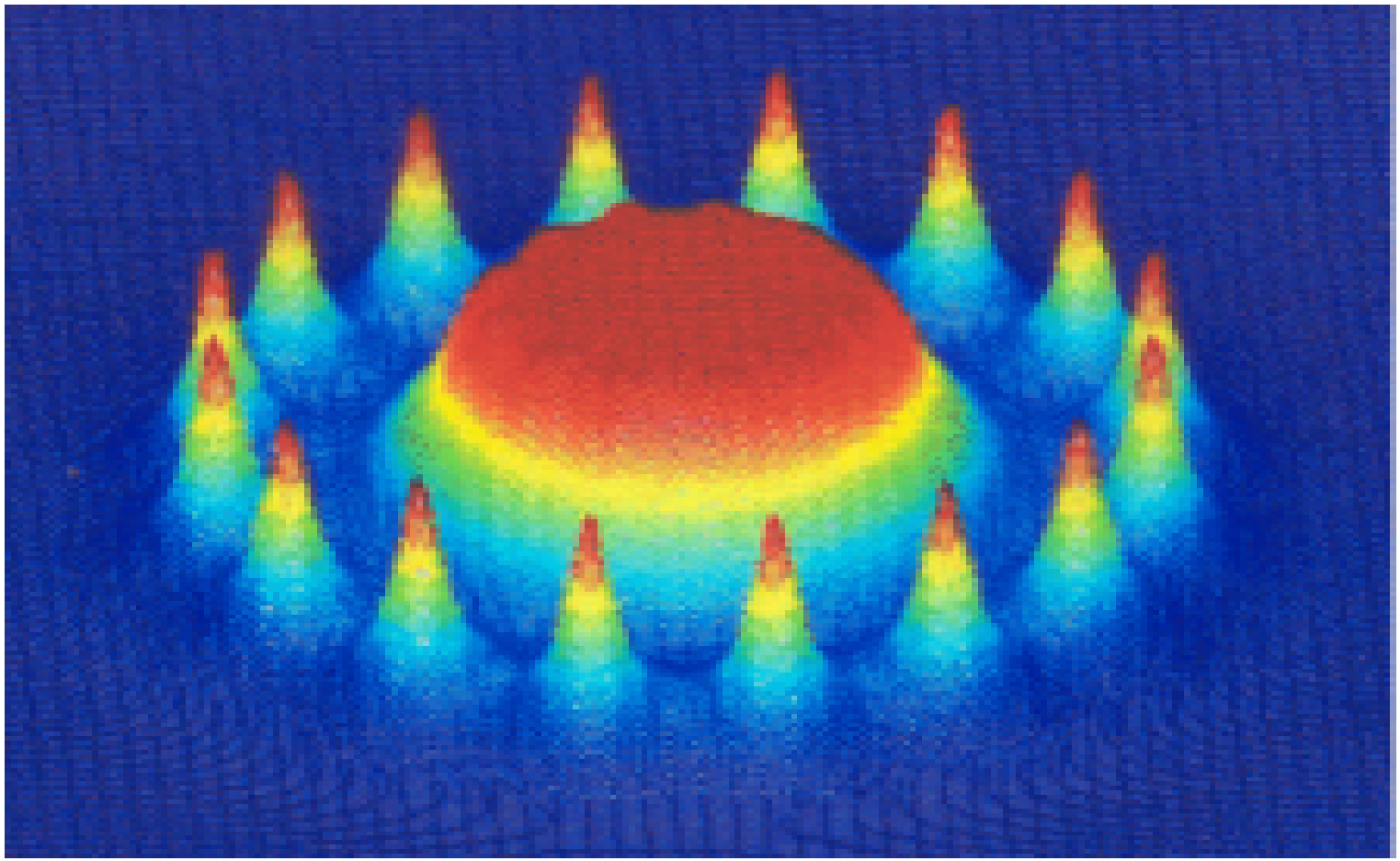
$$\Gamma \frac{\partial \psi}{\partial t} = [1 - \Theta(\vec{r}, t)]\psi - |\psi|^2 \psi - (i\vec{\nabla} + \vec{A})^2 \psi$$

$$\frac{\partial \vec{A}}{\partial t} = -\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) - \frac{i}{2\kappa^2}\left(\psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^*\right) - \frac{1}{\kappa^2}|\psi|^2 \vec{A}$$

$$\frac{\partial \Theta}{\partial t} = \chi \nabla^2 \Theta + \varepsilon \left(\frac{\partial \vec{A}}{\partial t} \right)^2 - \gamma \left(\Theta - \Theta_0 \right)$$

$$\psi(\vec{r}, t) = |\psi(\vec{r}, t)| \exp\left[i \chi(\vec{\rho}, t) \right]$$

Hot spot relaxation



Hydrodynamics of Vortex Matter

$$n(\vec{r},t) = \sum \delta(\vec{r} - \vec{r}_a(t)); \qquad \mathrm{I}_f = \sum v_a \delta(\vec{r} - \vec{r}_a(t))$$

$$\partial_t n + \partial_\alpha \mathrm{I}_f = 0; \; B = \varphi_0 n; \quad B = (0, 0, B_z)$$

$$\partial \times B = J; \quad J = \left(\frac{\partial B}{\partial y}, -\frac{\partial B}{\partial x}, 0\right)$$

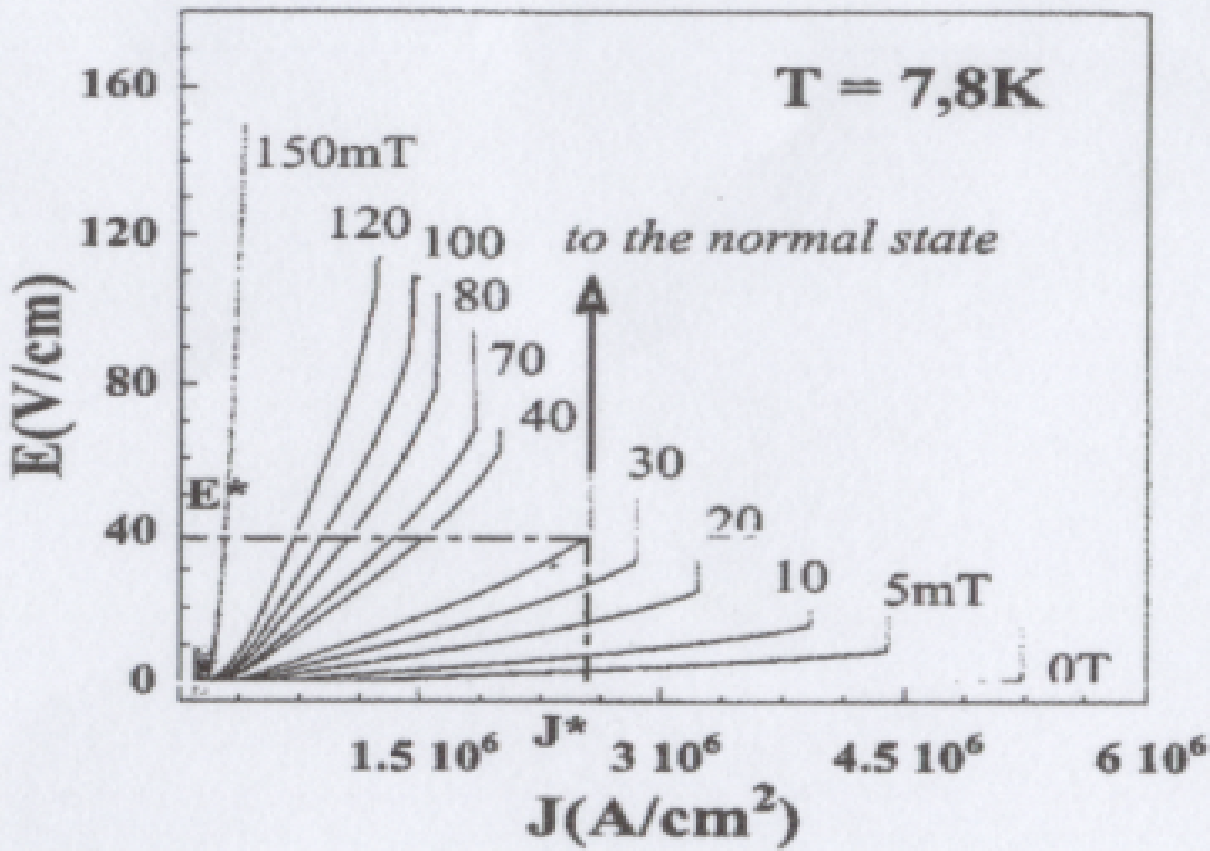
$$E = v \times B = (v_y B, -v_x B, 0)$$

$$\partial_t B + \frac{\partial v_x B}{\partial x} + \frac{\partial v_y B}{\partial y} = 0; \quad \partial_t B - \frac{\partial E_y}{\partial x} + \frac{\partial E_x}{\partial y} = 0$$

$$\rho_f J = E; \qquad \partial_t B - \frac{\partial \rho_f J_y}{\partial x} + \frac{\partial \rho_f J_x}{\partial y} = 0$$

$$\frac{\partial B}{\partial t} = \frac{\partial}{\partial x}\left(\rho_f \frac{\partial B}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho_f \frac{\partial B}{\partial y}\right)$$

J-V characteristics in a resistive state of the vortex matter



. Typical set of (J,E) curves at $T = 7.8$ K for the cool Nb fil

Basic Equations

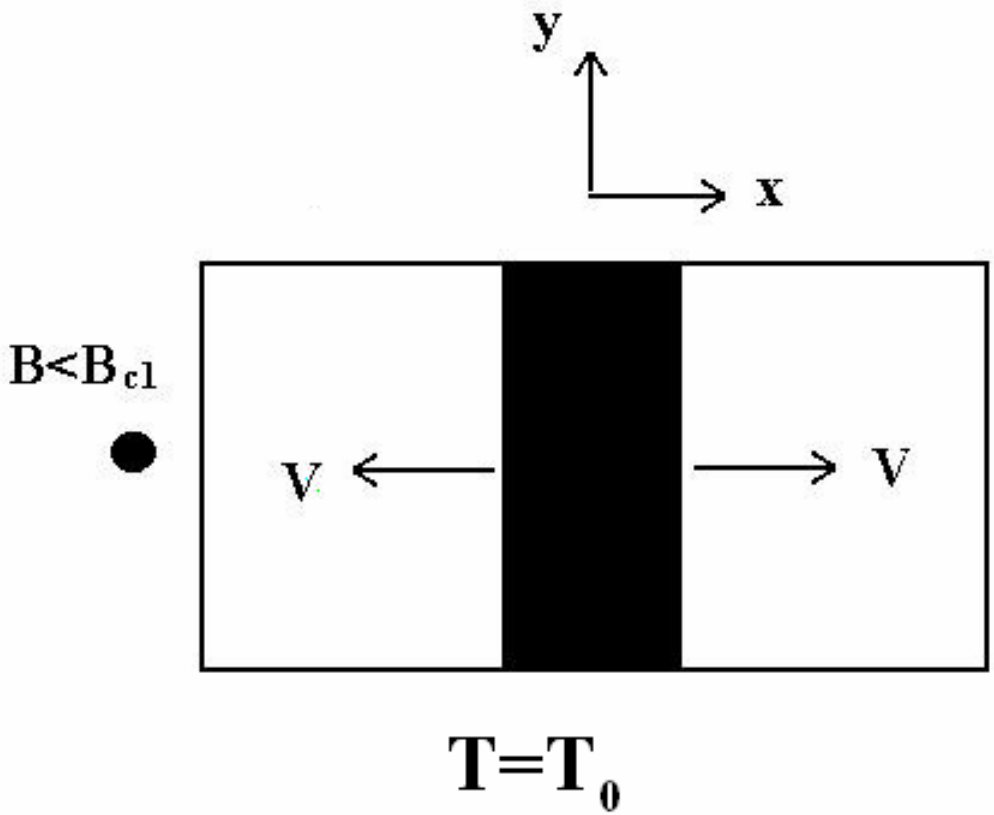
$$-\frac{1}{c}\frac{\partial B}{\partial t} = \nabla \times E; \qquad \nabla \times B = \frac{4\pi}{c} J$$

$$\frac{\partial T}{\partial t} = D\nabla^2 T + \frac{J^2 R}{C} - \gamma (T - T_0)$$

$$RJ = E$$

$$R = R_N \left(\frac{B}{B_{c2}(T)} \right)^{\nu} \left(\frac{J}{J_c(T)} \right)^{\mu}$$

Geometry of the problem



Dimensionless equations

$$\frac{\partial b}{\partial t} = \frac{\partial}{\partial x}\left(\rho \frac{\partial b}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho \frac{\partial b}{\partial y}\right)$$

$$\frac{\partial \Theta}{\partial t} = \nabla^2 \Theta + P_J - \Gamma(\Theta - \Theta_0)$$

$$P_J = j^2 \rho$$

$$j^2 = \left(\frac{\partial b}{\partial x}\right)^2 + \left(\frac{\partial b}{\partial y}\right)^2$$

$$\rho = \left(\frac{b}{1-\Theta}\right)^{\nu} \left(\frac{j}{1-\Theta}\right)^{\mu}$$

Normal domain at the front

$$b = b(x - Vt); \quad \Theta = \Theta(x - Vt)$$

$$b = A(V)\left|X\right|^{(\mu+1)/(\mu+\nu)}\theta(X); \quad \Theta = \Theta_f;$$

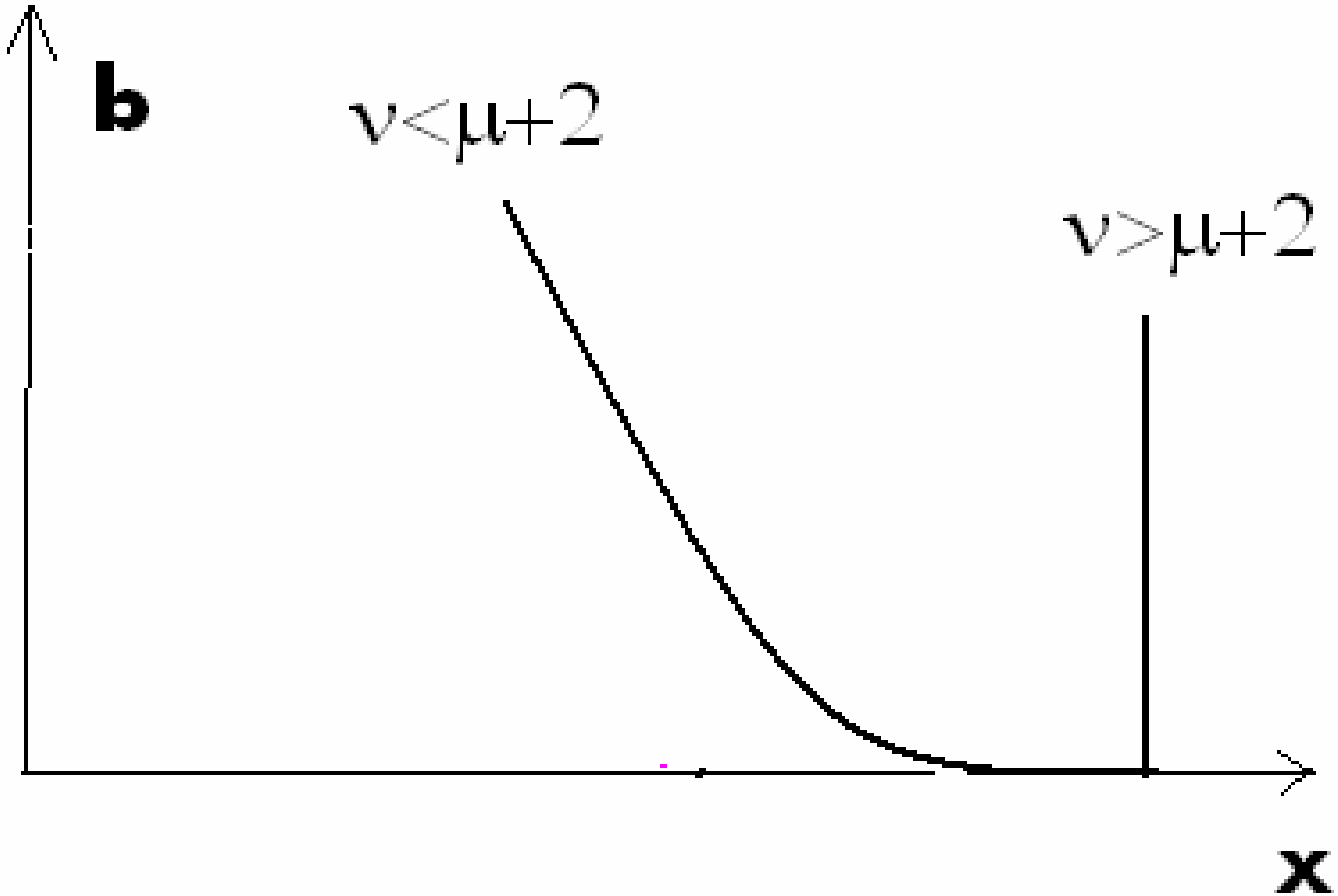
$$X = x - Vt$$

$$A(V) = (1 - \Theta_f)V^{1/(\nu+\mu)}\left(\frac{\mu+1}{\mu+\nu}\right)^{-\left(\frac{\mu+1}{\mu+\nu}\right)}$$

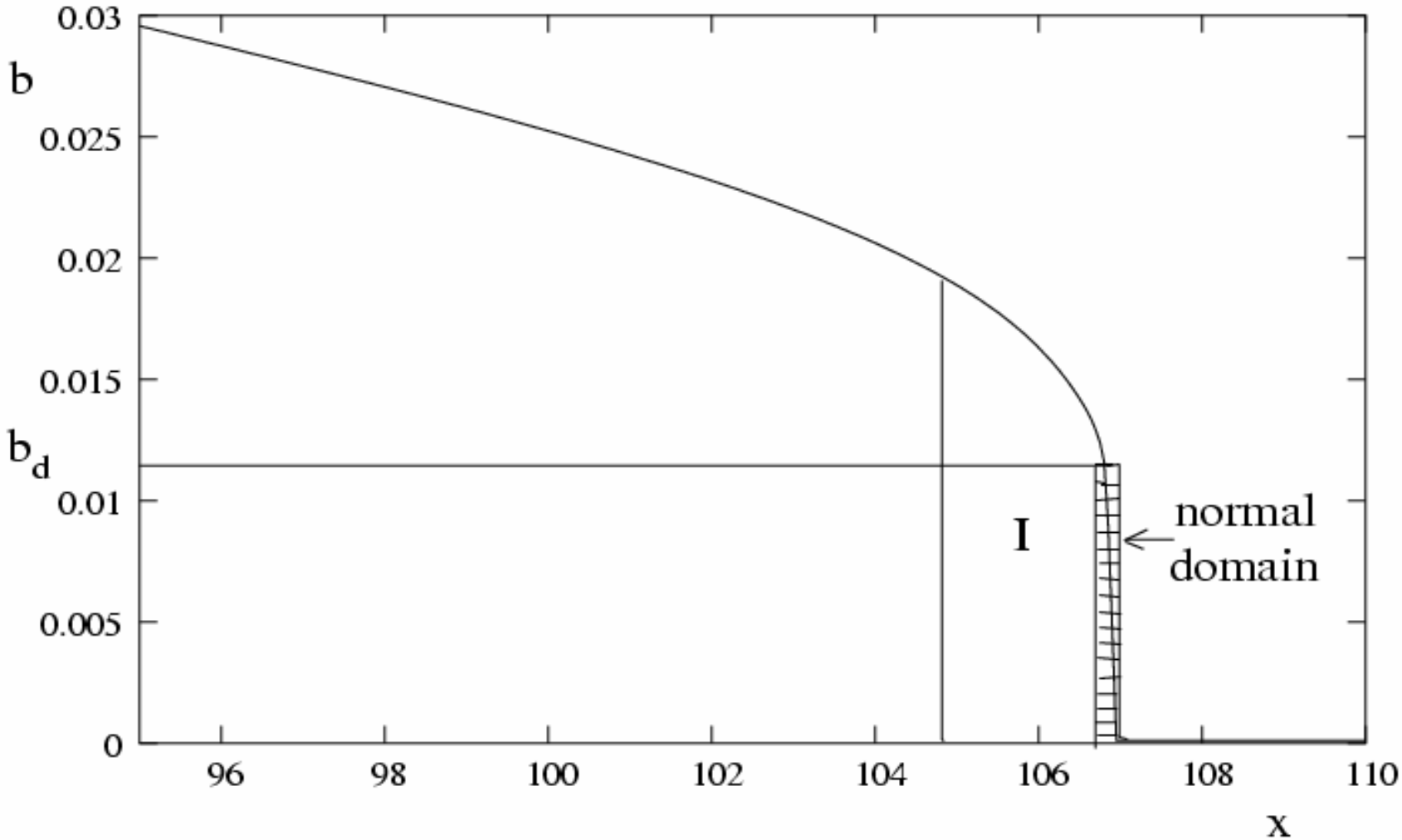
$$P_J = j^2\rho \propto \left|X\right|^{\left(\frac{\mu+2-\nu}{\mu+\nu}\right)} \to \infty \text{ at } \mu + 2 < \nu$$

$$\frac{\partial \mathrm{b}}{\partial \mathrm{x}} = j \to \infty \quad \text{at } X \to 0$$

Sharp and smooth interface



Normal domain



Front velocity

$$P_J = \Xi \delta(x - x_f)$$

$$\Xi = j_c^{\,2} \rho_N d$$

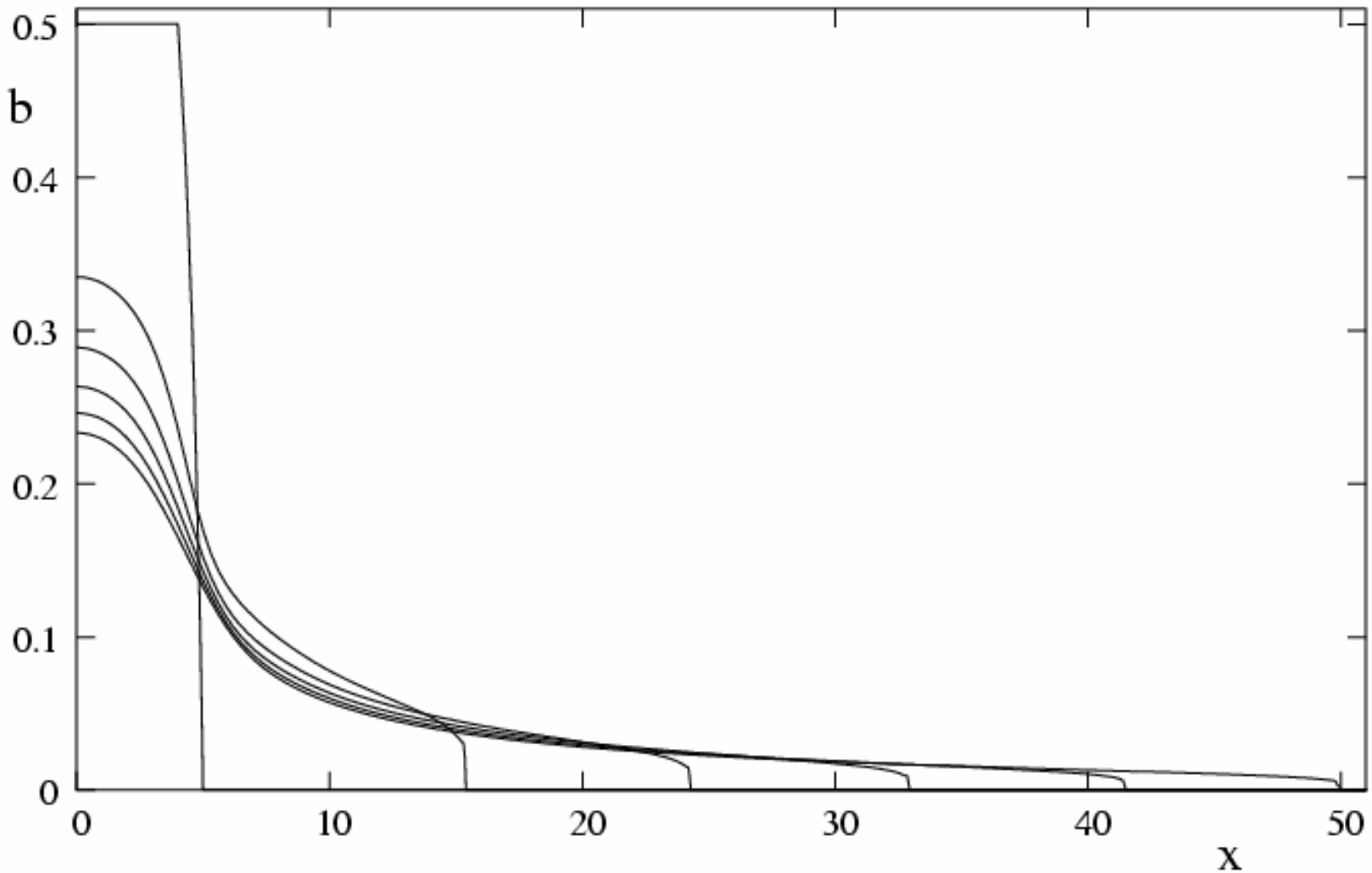
$$j_c d = b_d$$

$$V = \frac{\Xi}{\Theta_d} \text{ dimensionless velocity}$$

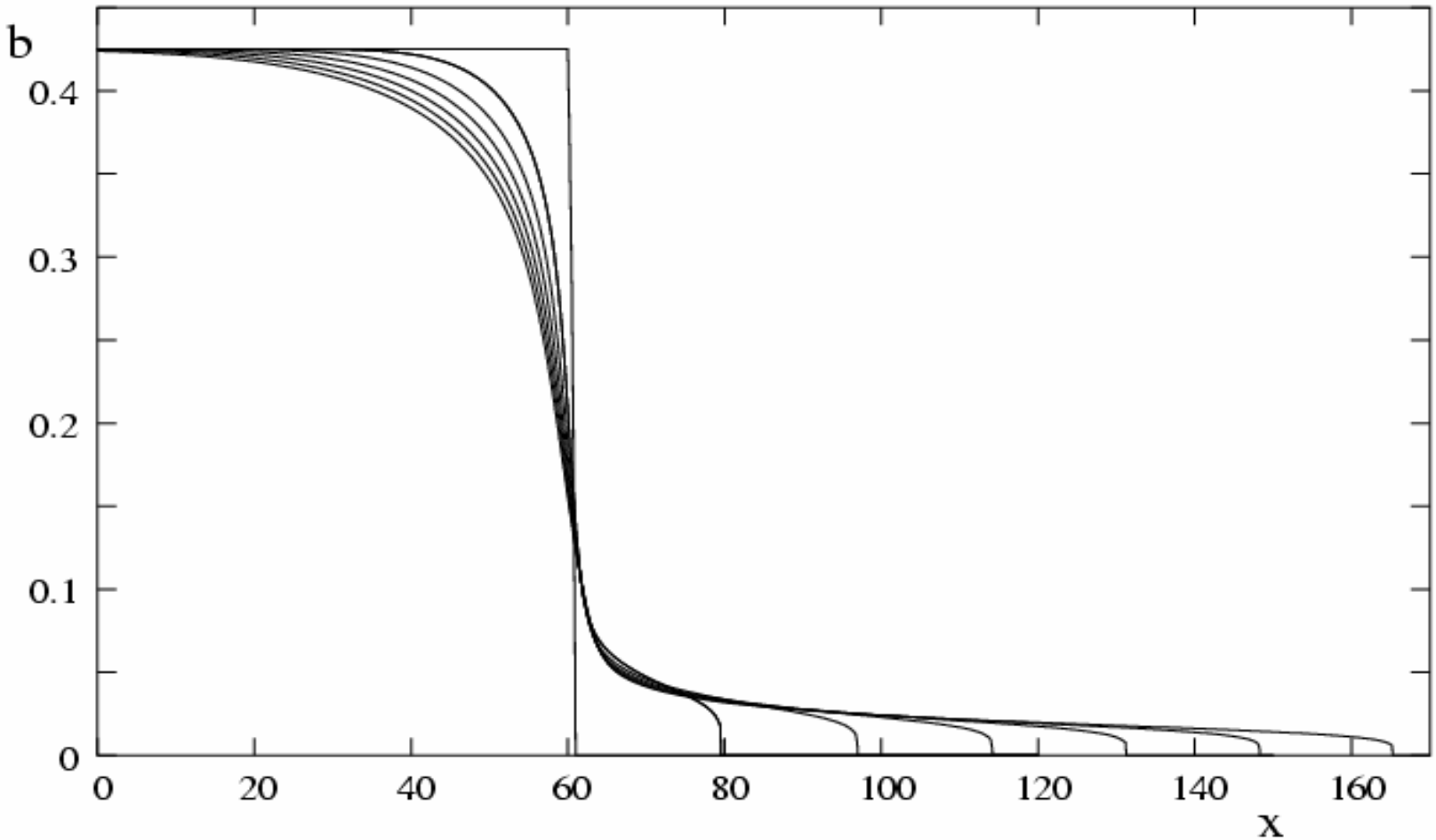
$$\Theta_d = \Theta_f - \Theta_0$$

$$U = \frac{\Xi}{\Theta_d} \frac{D}{cR_N} = c \frac{J_c^2 R_N^2}{CT_c} \text{ Front velocity}$$

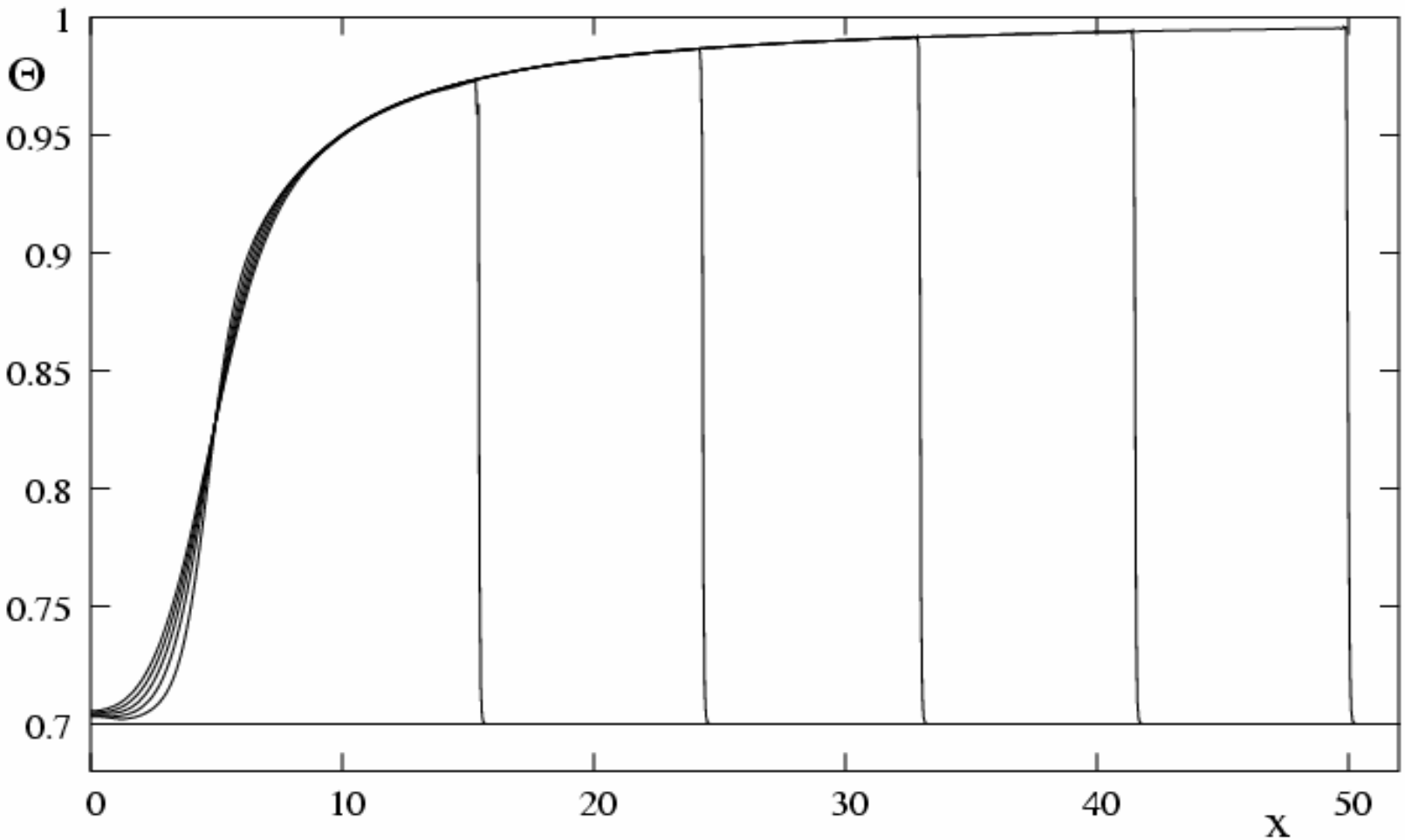
Evolution of magnetic flux



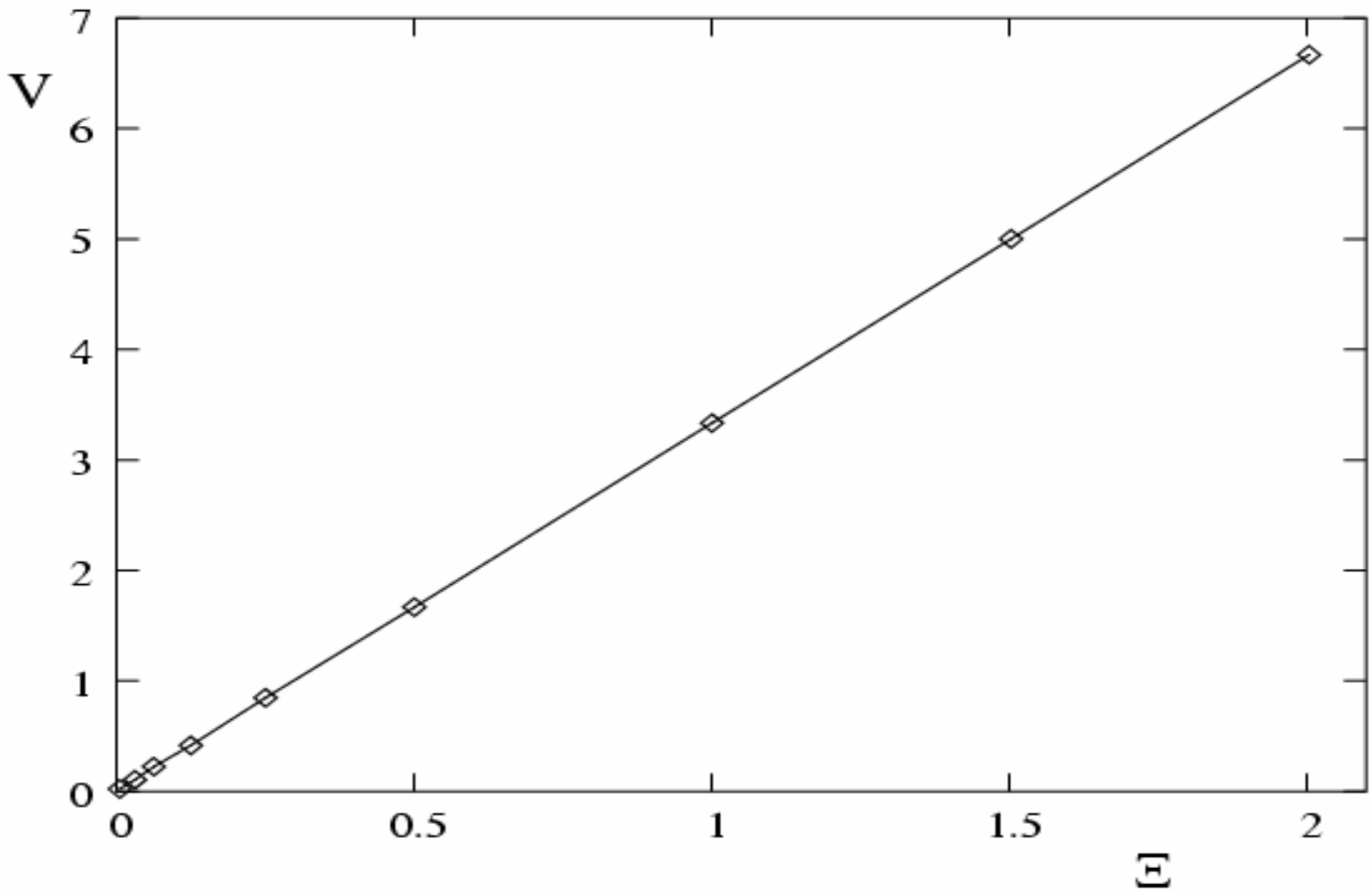
Evolution of magnetic induction



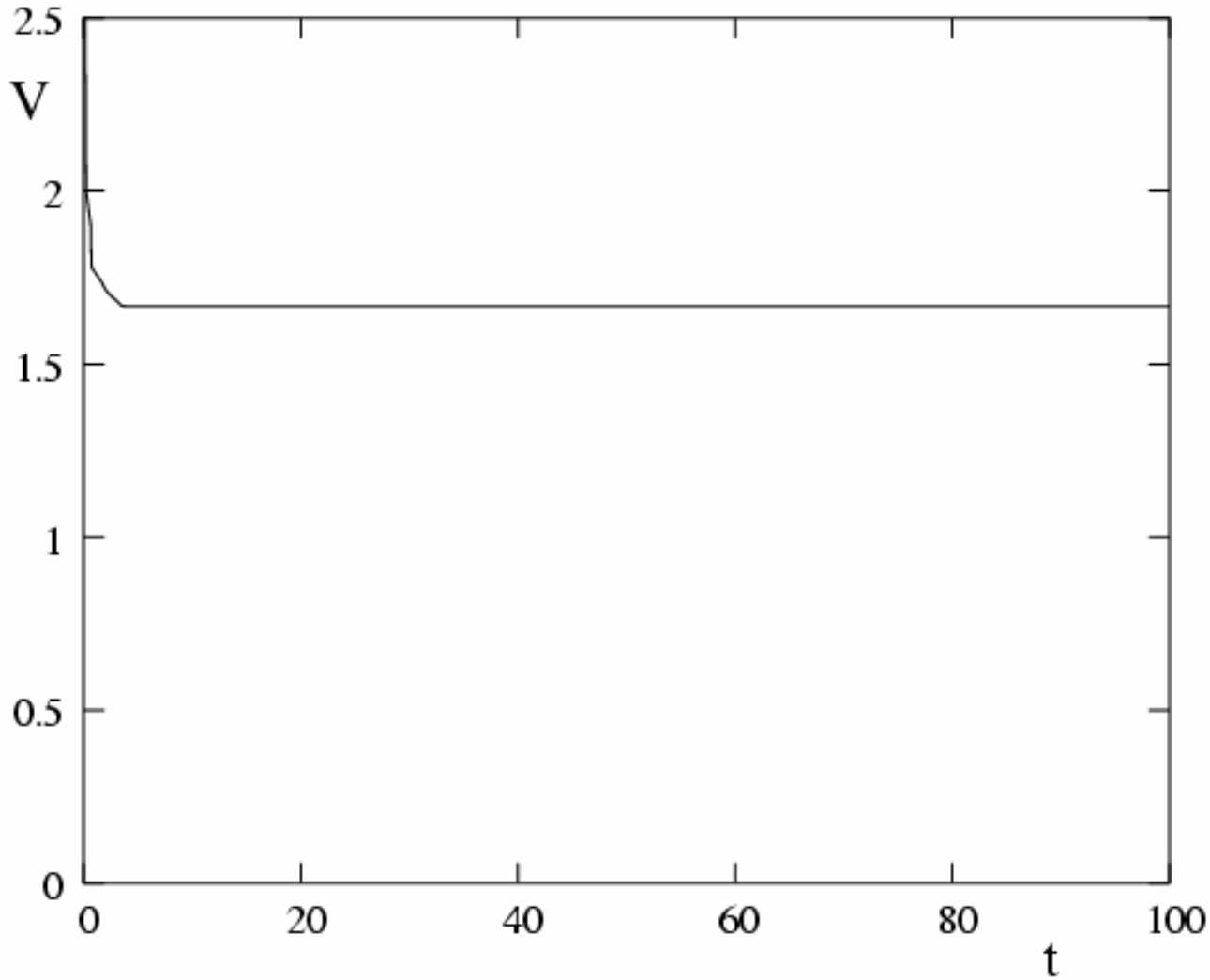
Temperature shock wave



Flux front velocity versus Joule power at the front



Velocity of the front



Linear analyses of instability

$$b = b_0(x - Vt) + \eta(x, y, t)$$

$$\Theta = \Theta_r(x - Vt) + \zeta(x, y, t)$$

$$-V \frac{\partial b_0}{\partial X} = \rho_0 \Theta_r \frac{\partial^2 b_0}{\partial X^2} + \rho_0 \frac{\partial \Theta_r}{\partial X} \frac{\partial b_0}{\partial X}$$

$$-V \frac{\partial \Theta_r}{\partial X} = \kappa \nabla^2 \Theta_r + \rho_0 \Theta_r \left(\frac{\partial b_0}{\partial X} \right)^2$$

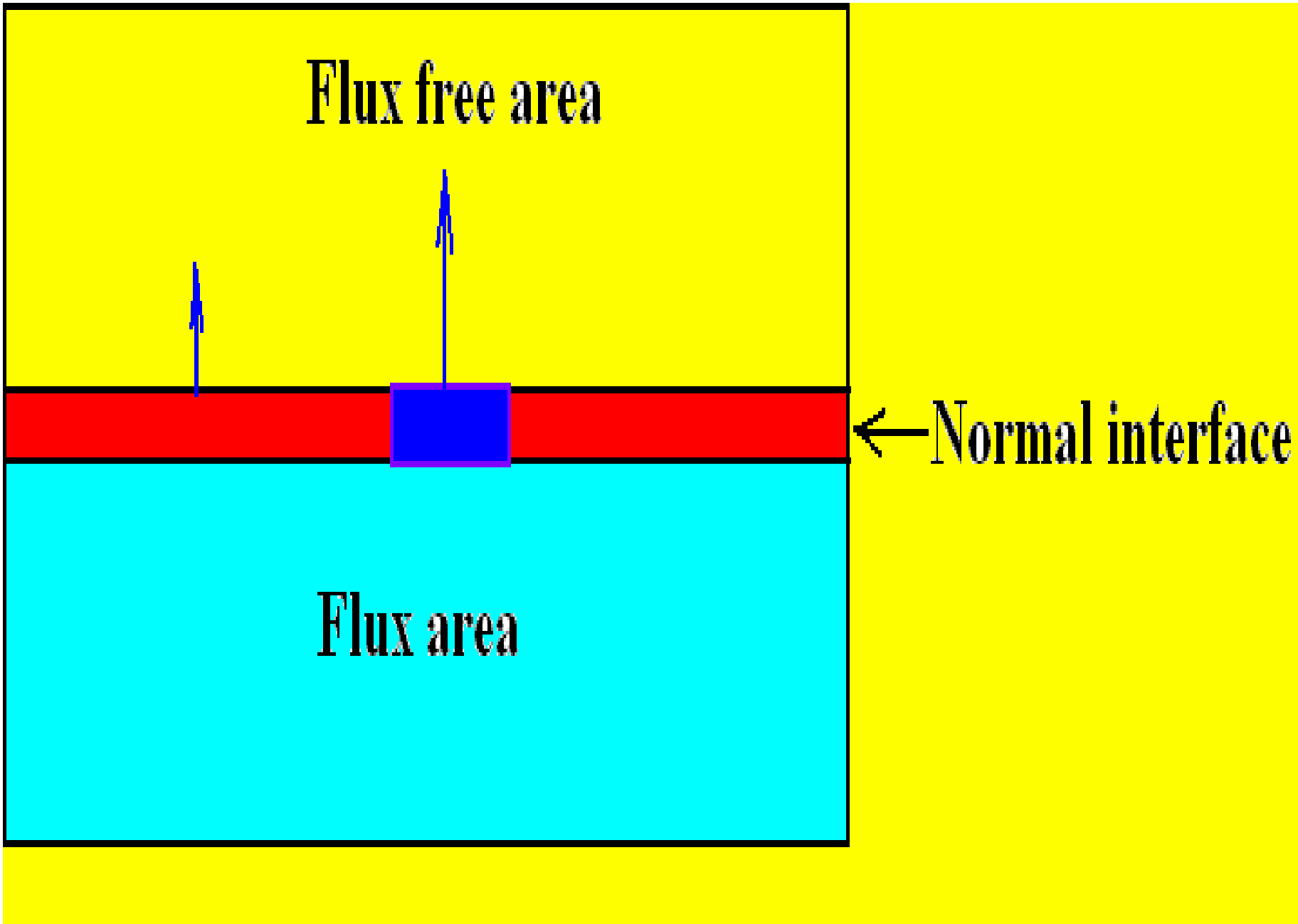
$$j_c = \left| \frac{\partial b_0}{\partial X} \right|; \quad \Theta_r = \Theta_0 - \frac{VX}{\rho_0}; \quad d = \sqrt{\frac{\Theta_0}{j_c}}$$

$$\frac{\partial \eta}{\partial t} = \rho_0 \Theta_0 \nabla^2 \eta - V \frac{\partial \eta}{\partial x} - \rho_0 j_c \frac{\partial \zeta}{\partial x}$$

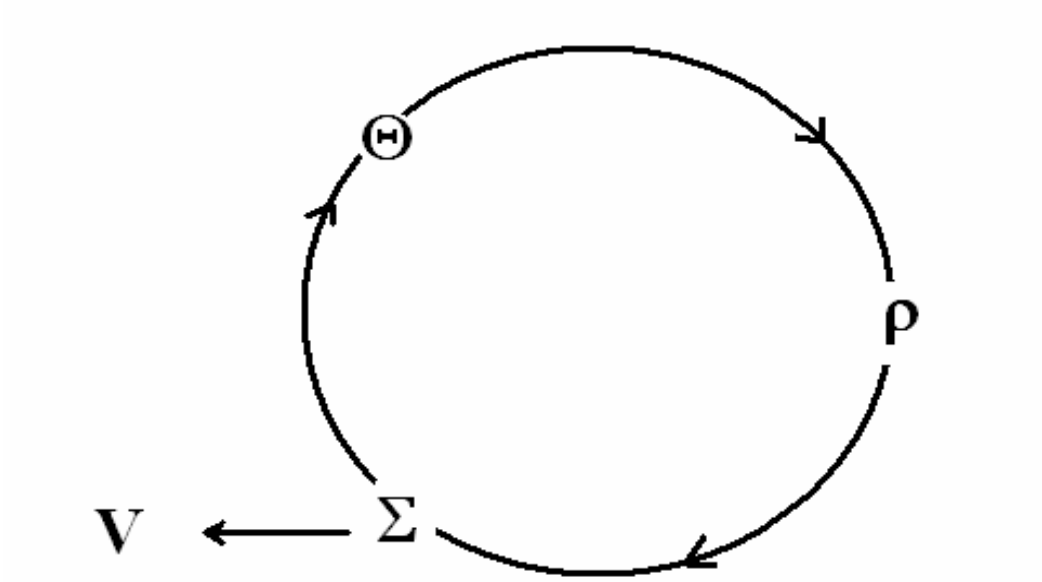
$$\frac{\partial \zeta}{\partial t} = \kappa \nabla^2 \zeta - 2 j_c \rho_0 \Theta_0 \frac{\partial \eta}{\partial x} + \rho_0 j_c^2 \zeta$$

$$\zeta, \eta \propto \exp(\Omega t + i k_x x + i k_y x)$$

$$\mathrm{Re}\{\Omega\} = \rho_0 j_c^2 \quad (k_x, k_y \to 0)$$


Flux free area
Normal interface
Flux area

Positive temperature feedback is responsible for instability



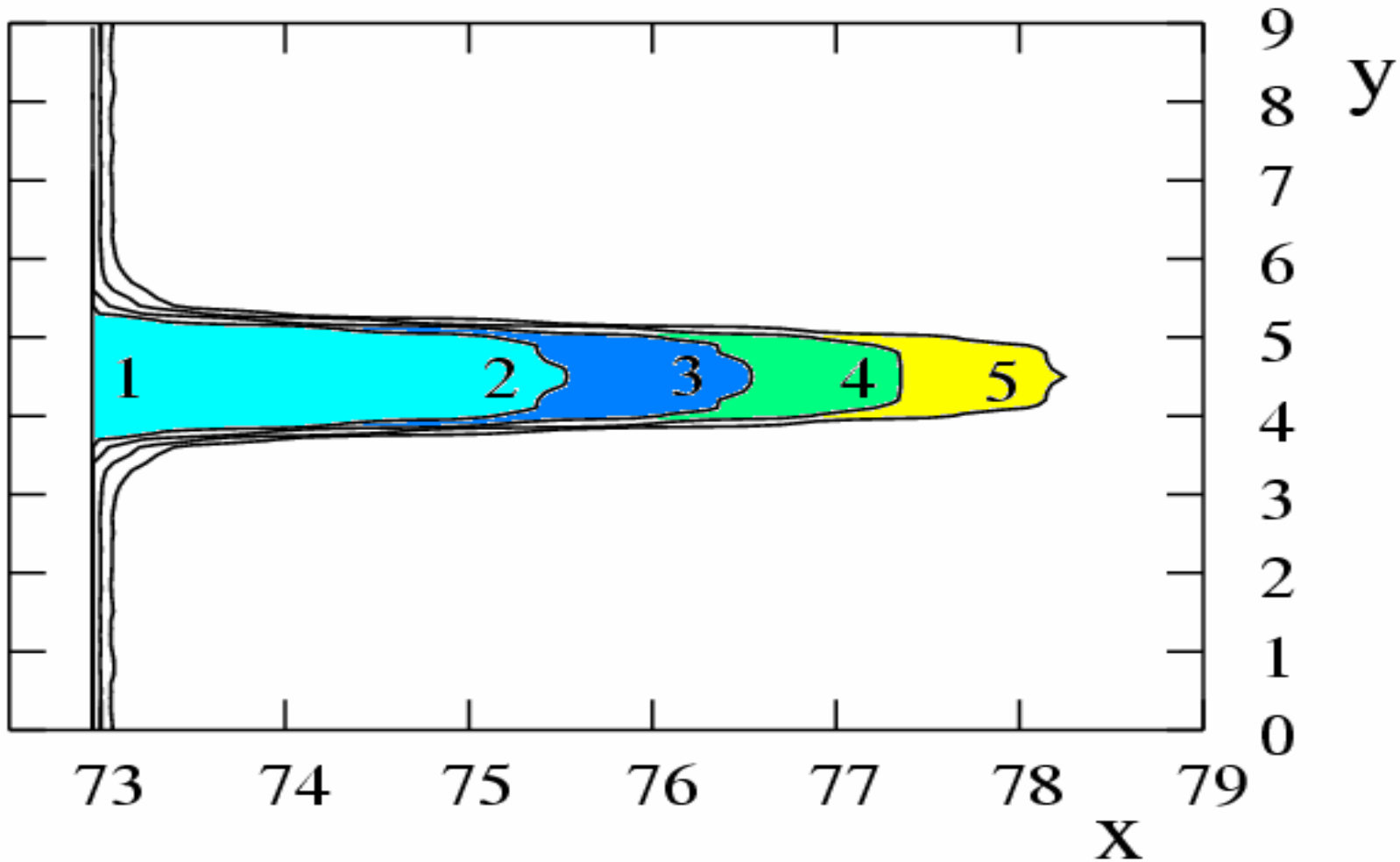
$$\frac{\rho_N(\Theta)}{\rho_0} = \left[1 + \alpha\left(\Theta(x,y,t) - \Theta(y,t=0)\right)\right]$$

$$\Theta_0(y,t=0) = 0.88 \text{ for } 4 < y < 5$$

$$\Theta_0(y,t=0) = 0.7 \quad \text{for } y \notin [4,5]$$

$$\alpha = 14.5$$

Development of Instability



Critical velocity

$$t_0 << t_a \qquad\qquad t_a = \Gamma^{-1}$$

$$V_d t_0 >> \sqrt{\kappa t_0} \qquad\qquad V_d = V_f - V$$

$$U_c = \min\left\{ \frac{\Gamma d D}{\Theta_b c R_N}, \frac{\Gamma j_c D}{c R_N}\sqrt{\kappa \rho_0} \right\}$$

$$U_c = 10^5 \, m/s$$

Conclusion

1. The avalanche is developed when the voltage-current characteristics of the uniform superconductor in its resistive state provides significant screening currents at the moving flux front interface. In this case, the small area of the interface becomes normal.

2. The voltage-current characteristics in the resistive state of the Type-II superconductor is the decisive factor for the instability onset while its development depends also on heat absorption properties of the sample.

3. The interface moves with constant velocity determined by the Joule heat released in the normal domain at the front. For $\nu<\mu+2$ the heat at the flux front vanishes. No instability of the flux front develops in this case.

4. The positive feedback between excessive local temperature at the front and Joule heat released there leads to instability. The hydrodynamics tangential instability of the flux front destroys the flat front. The instability develops for the flux velocities exceeding the critical value.