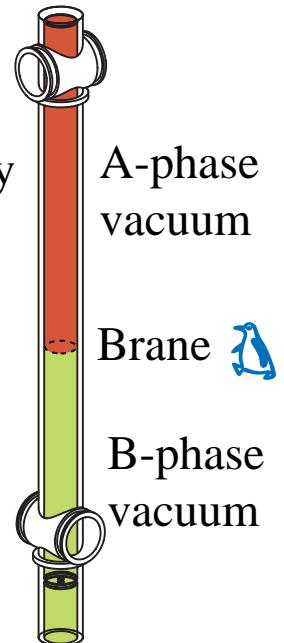
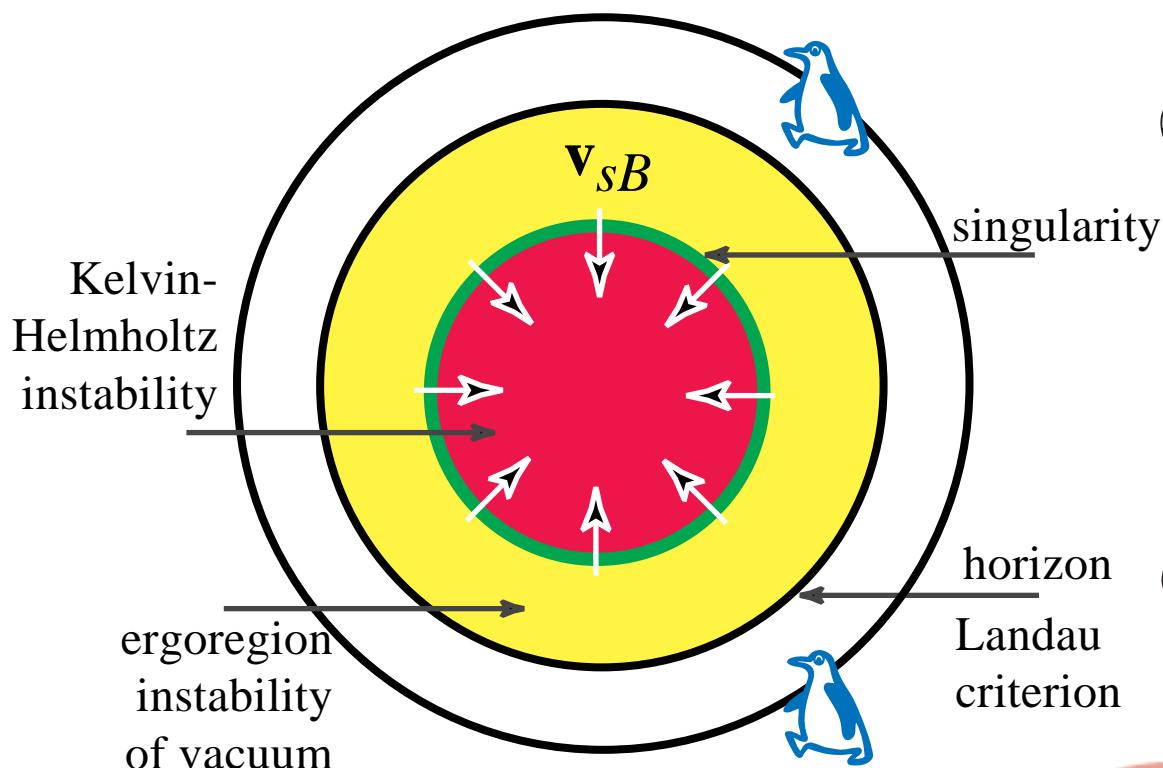
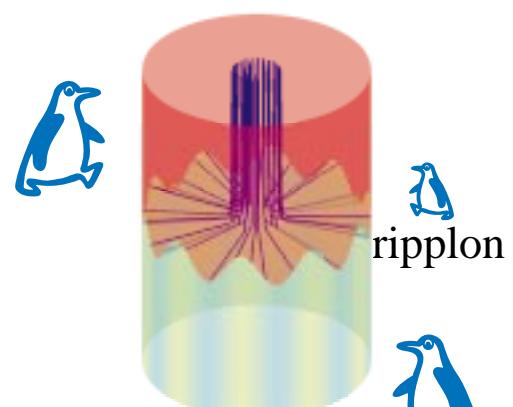


# Vacuum decay behind the horizon

G. Volovik



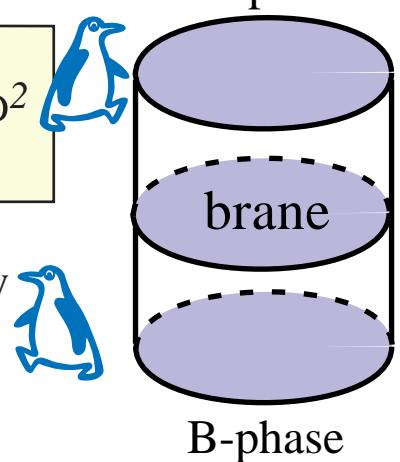
effective metric for ***relativistic ripplons***  
(surface waves in shallow water)



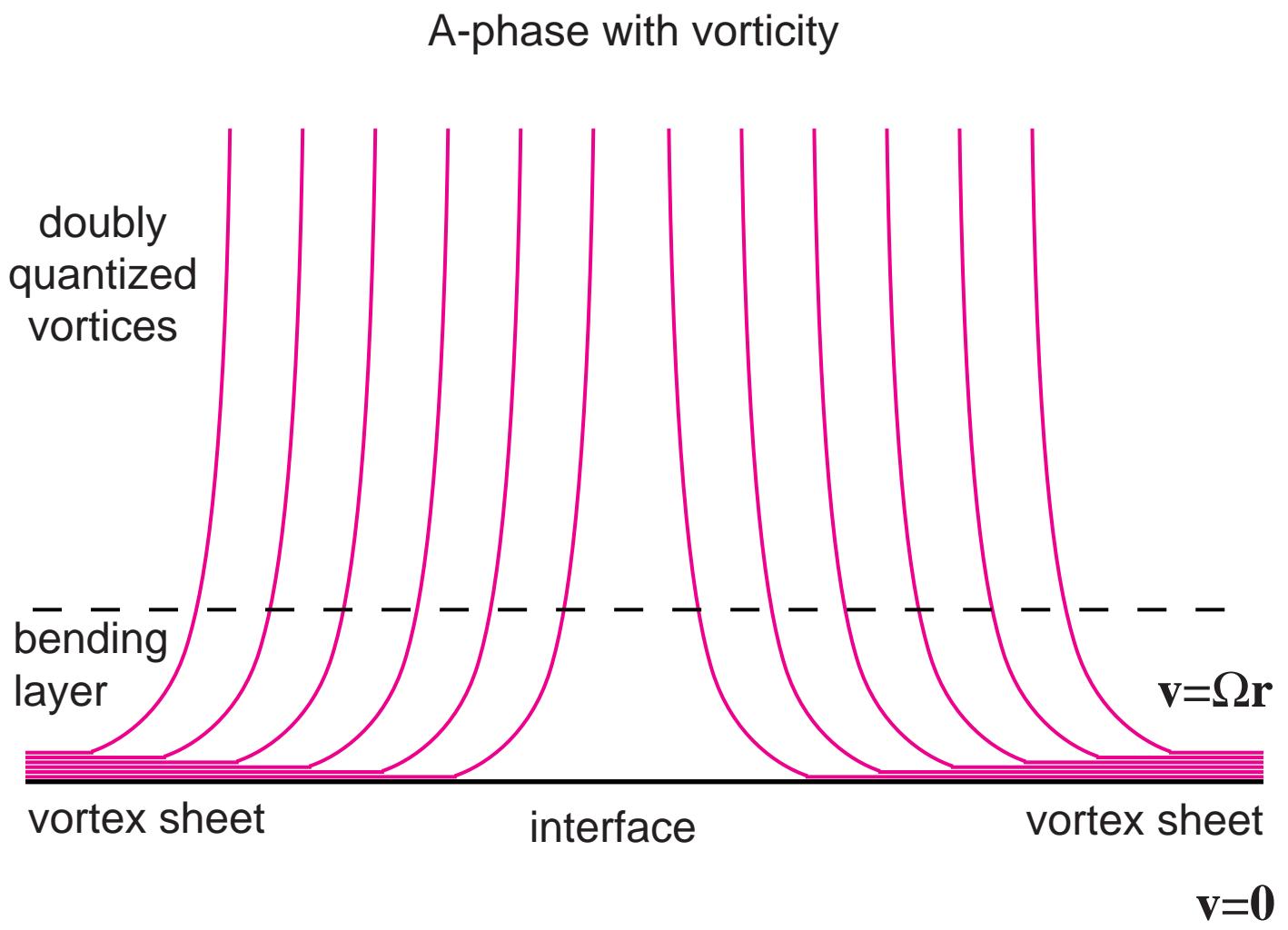
horizon = ergoregion instability

$$ds^2 = -dt^2 \frac{c^2 - W^2 - U^2}{c^2 - U^2} + dr^2 \frac{1}{c^2 - W^2 - U^2} + r^2 d\phi^2$$

physical singularity = Kelvin-Helmholtz instability



## Interface between two superfluids in rotating vessel: shear flow without viscous dissipation

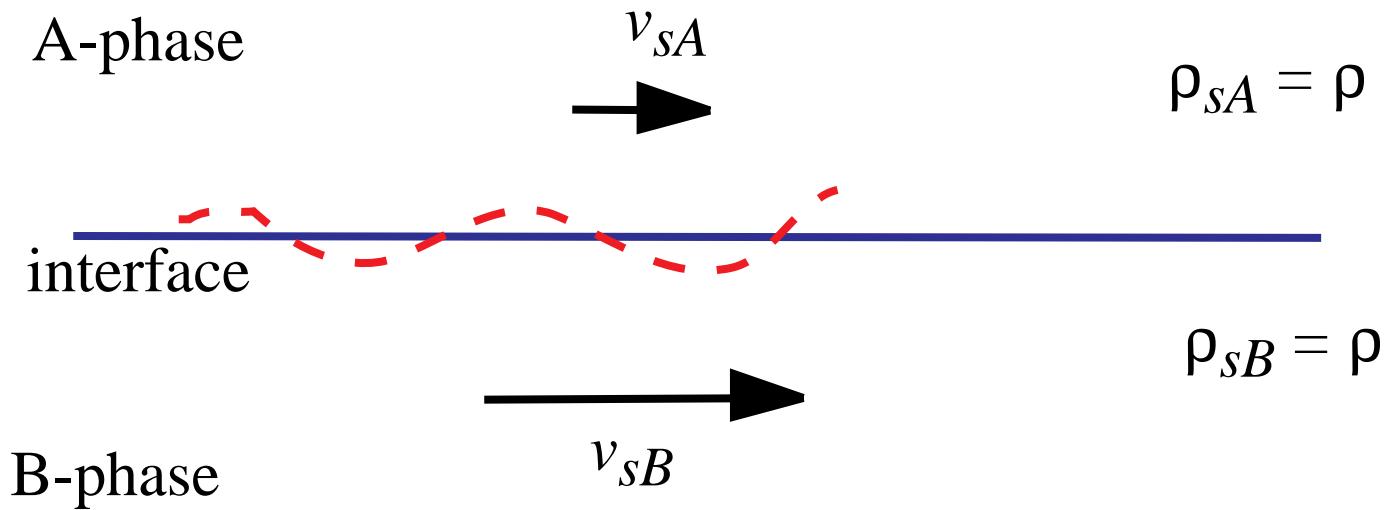


vortex free B-phase

instability of discontinuity in tangential flow at  $v > v_c$

$$v_c = ?$$

# Interface instability at $T=0$



\* Kelvin-Helmholtz criterion

$$k_c = \sqrt{F/\sigma}$$

(dynamic instability of interface under shear flow)

$$\tilde{\rho}(v_{sB} - v_{sA})^2 = 4\sqrt{F\sigma}$$

magnetic forces in  ${}^3He$

$$F = \nabla(\chi_B H^2 - \chi_A H^2)$$

gravity in liquids

$$F = g(\rho_2 - \rho_1)$$

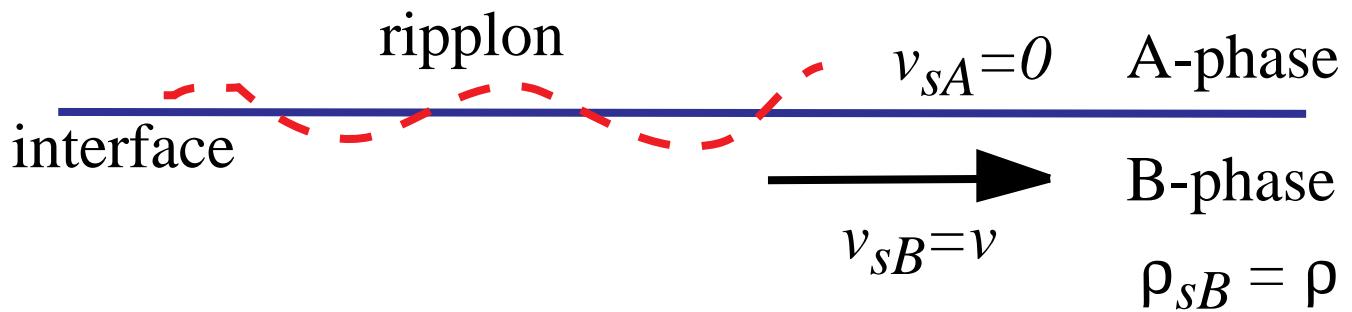
$$\tilde{\rho} = \rho$$

$$\tilde{\rho} = 2\rho_2\rho_1/(\rho_2 + \rho_1)$$

$$\rho(v_{sB} - v_{sA})^2 = 4\sqrt{F\sigma}$$

### 3 criteria for interface instability at $T=0$

$$\rho_{sA} = \rho$$



#### \* Kelvin-Helmholtz criterion

(dynamic instability of interface under shear flow)

$$\rho v^2 = 4\sqrt{F\sigma}$$

$$k_c = \sqrt{F/\sigma}$$

#### \* Landau criterion

(excitation of quasiparticles --  
riplons = capillary-gravity waves)

$$v = \min_k \frac{\omega(k)}{k}$$

$$\omega(k) = \frac{\sqrt{kF + k^3\sigma}}{\sqrt{2\rho}}$$

$$\rho v^2 = \sqrt{F\sigma}$$

#### riplon spectrum in deep water

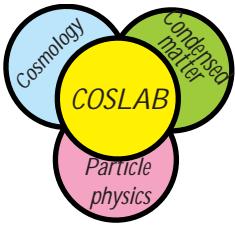
$$k_c = \sqrt{F/\sigma}$$

#### \* Thermodynamic (ergoregion) instability criterion

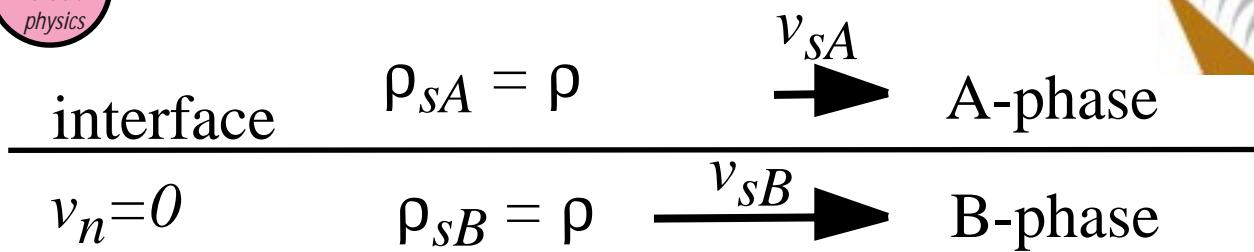
(negative free energy = ergoregion at T=0)

$$\rho v^2 = 2\sqrt{F\sigma}$$

$$k_c = \sqrt{F/\sigma}$$



## General Landau criterion



\* **Conventional Landau criterion** (one superfluid velocity)

$$\omega(\mathbf{k}, \mathbf{v}_s) = \omega(k) + \mathbf{k} \cdot \mathbf{v}_s < 0$$

$$v = \min \frac{\omega(k)}{k}$$

\* **General Landau criterion** (two superfluid velocities)

$$\omega(\mathbf{k}, \mathbf{v}_{sA}, \mathbf{v}_{sB}) < 0$$

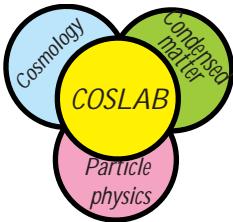
**ripplon spectrum**

$$\omega(\mathbf{k}, \mathbf{v}_{sA}, \mathbf{v}_{sB}) = \mathbf{k} \cdot (\mathbf{v}_{sA} + \mathbf{v}_{sB})/2 + \frac{\sqrt{kF + k^3\sigma - k^2\rho(\mathbf{v}_{sA} - \mathbf{v}_{sB})^2/2}}{\sqrt{2\rho}}$$

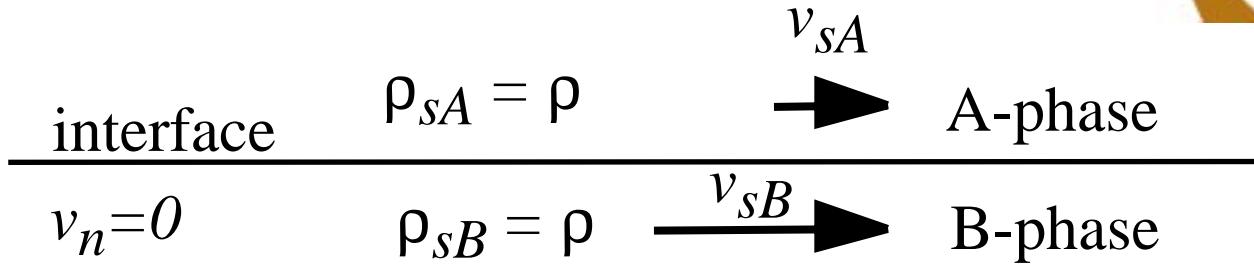


$$\rho(v_{sB} - v_n)^2 + \rho(v_{sA} - v_n)^2 = 2\sqrt{F\sigma}$$

**Coincides with thermodynamic instability criterion**



## Kelvin-Helmholtz analysis



\* Friction force between interface & normal component

$$F_{friction} = -\Gamma(v_{AB} - v_n)$$

(thermal environment)  
establishes connection with  
preferred reference frame

this modifies dispersion relation for ripplons

$$\omega = k \cdot (v_{sA} + v_{sB})/2 + \frac{\sqrt{kF + k^3\sigma} - k^2\rho(v_{sA} - v_{sB})^2/2 - i\Gamma k\omega}{\sqrt{2\rho}}$$

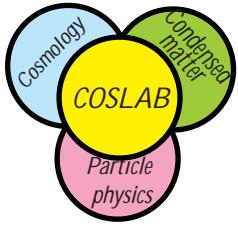
find **Re**  $\omega(k)$  and **Im**  $\omega(k)$

instability occurs when **Im**  $\omega(k)$  crosses 0 and becomes  $> 0$

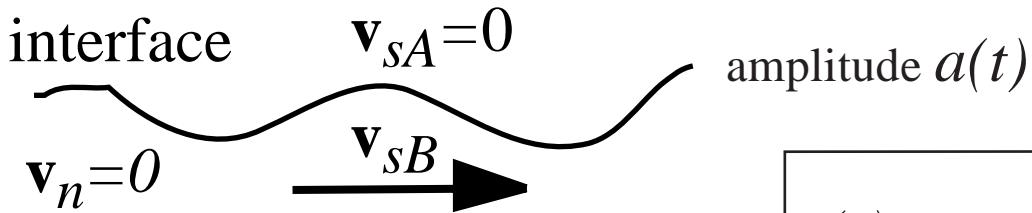
$$\rho(v_{sB} - v_n)^2 + \rho(v_{sA} - v_n)^2 = 2\sqrt{F\sigma}$$

Coincides with thermodynamic instability criterion

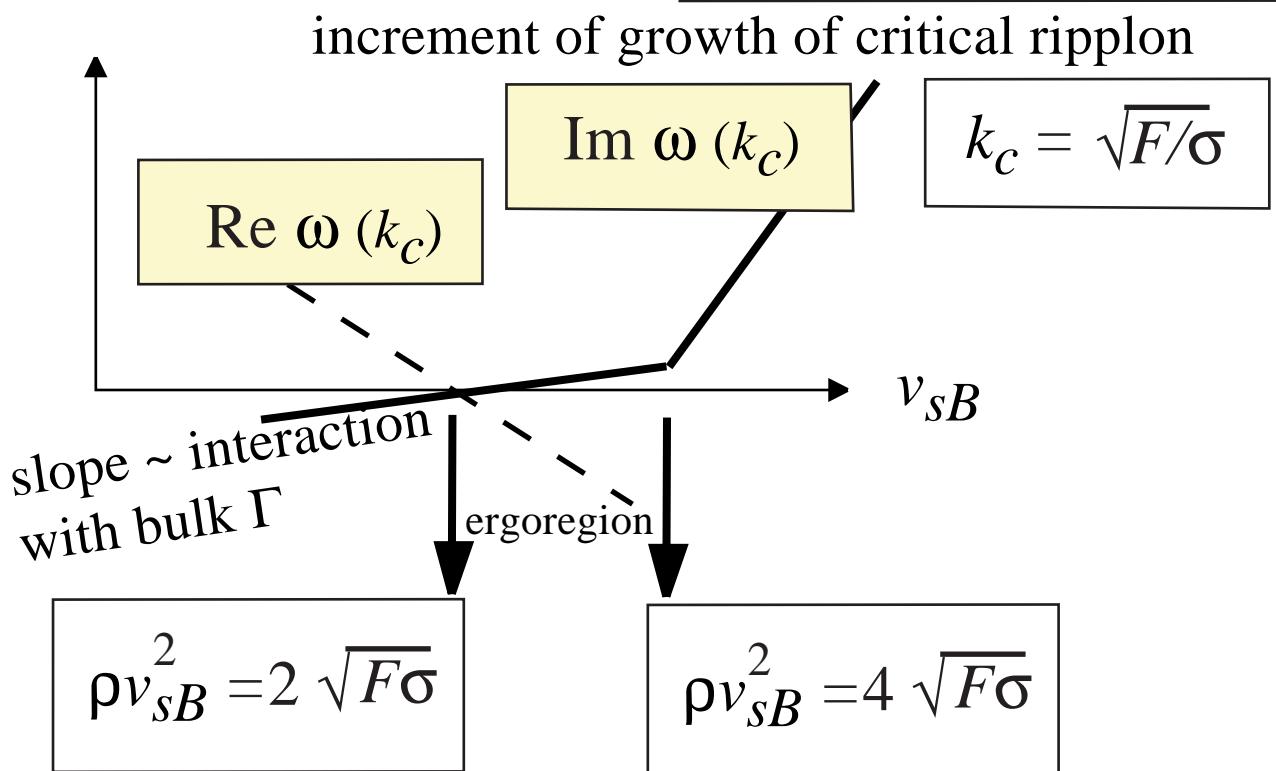
**Im**  $\omega(k)$  and **Re**  $\omega(k)$  cross 0 together



# Thermodynamic - Landau - ergoregion instability vs original Kelvin - Helmholtz instability



$$a(t) \sim \exp(t \operatorname{Im} \omega(k_c))$$



Thermodynamic-Landau-ergoregion  
instability condition

KH instability condition  
at  $\Gamma=0$

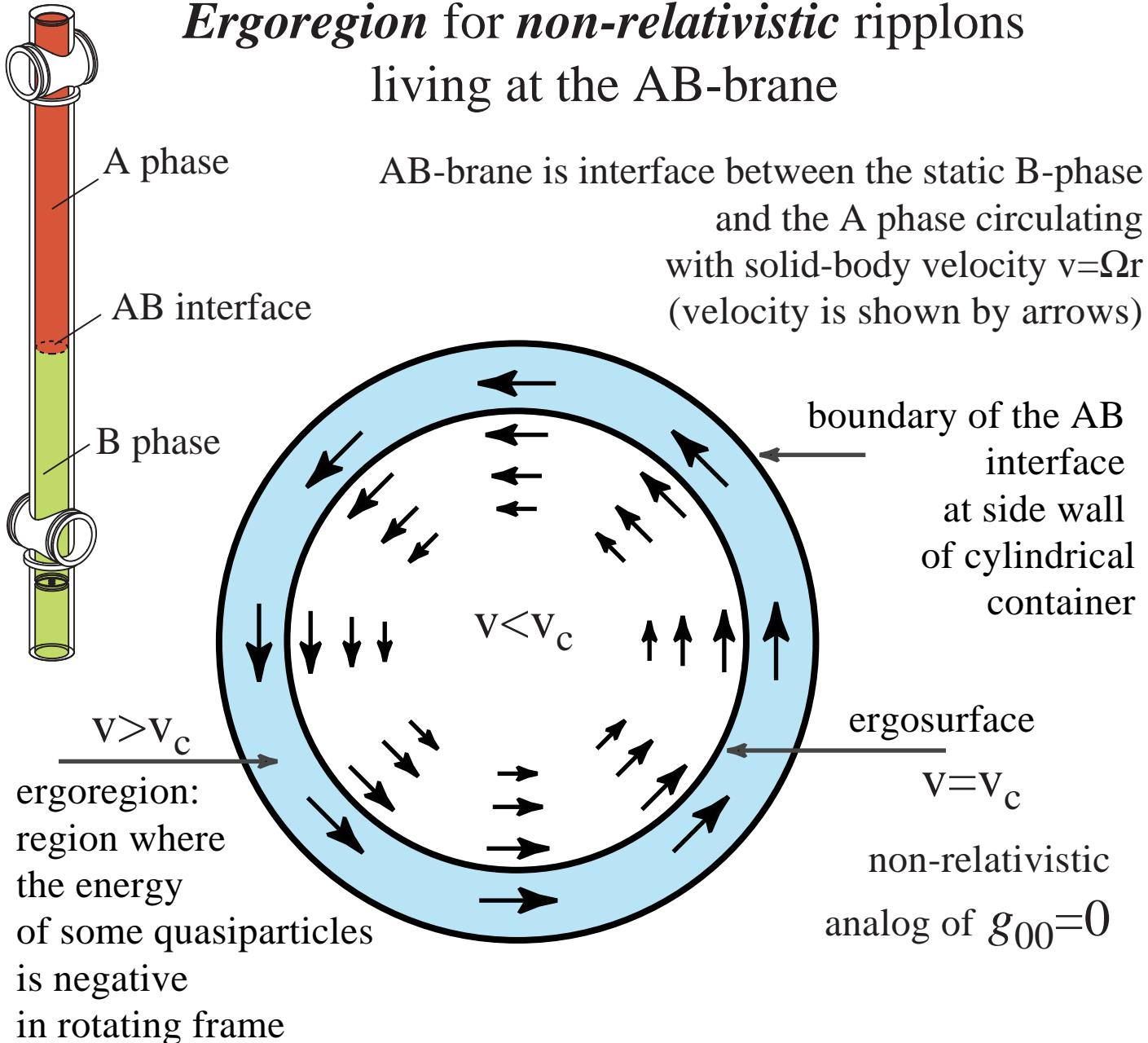
or at slow acceleration  
 $\dot{\Omega} \ll \lambda \Gamma$

or at fast acceleration  
 $\dot{\Omega} \gg \lambda \Gamma$

Result depends on observation time  
or on acceleration rate  $\dot{\Omega}$

$$\Gamma \sim T^3 \text{ (Kopnin, 1987)}$$

# *Ergoregion for non-relativistic ripplons living at the AB-brane*



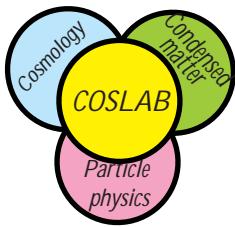
$$\omega(\mathbf{k}, \mathbf{v}_{sA}, \mathbf{v}_{sB}) < 0$$

non-relativistic

analog of  $g_{00} > 0$

## riplon spectrum

$$\omega(\mathbf{k}, \mathbf{v}_{sA}, \mathbf{v}_{sB}) = \frac{\mathbf{k} \cdot (\mathbf{v}_{sA} + \mathbf{v}_{sB})/2 + \sqrt{kF + k^3\sigma - k^2\rho(\mathbf{v}_{sA} - \mathbf{v}_{sB})^2/2}}{\sqrt{2\rho}}$$

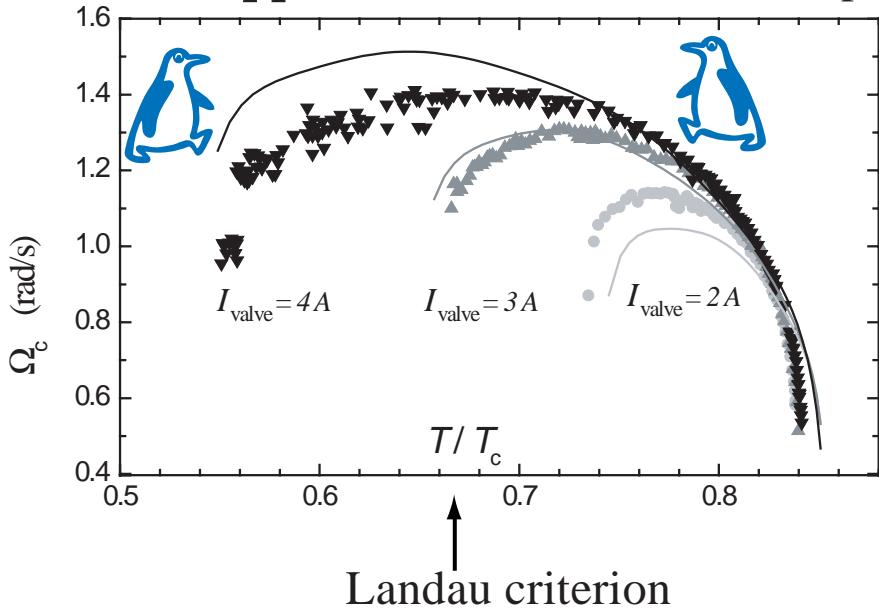


# Ergosurface & horizon on brane between two sliding superfluids



\* current experiments:

**riplons** (surface waves in deep water)



A-phase  
vacuum



Brane



B-phase  
vacuum

$$\rho_{sB}(v_{sB} - v_n)^2 + \rho_{sA}(v_{sA} - v_n)^2 = 2\sqrt{F\sigma}$$



Kelvin-Helmholtz instability

$$\rho_s (v_{sB} - v_{sA})^2 = 4\sqrt{F\sigma}$$



riplon

\* future experiments: riplons on surface of shallow water  
obey effective relativistic metric

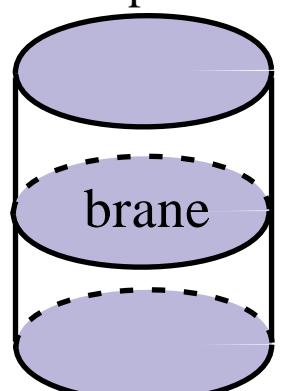
Landau criterion= black hole horizon



$$ds^2 = -dt^2 \frac{c^2 - W^2 - U^2}{c^2 - U^2} + dr^2 \frac{1}{c^2 - W^2 - U^2} + r^2 d\phi^2$$



A-phase



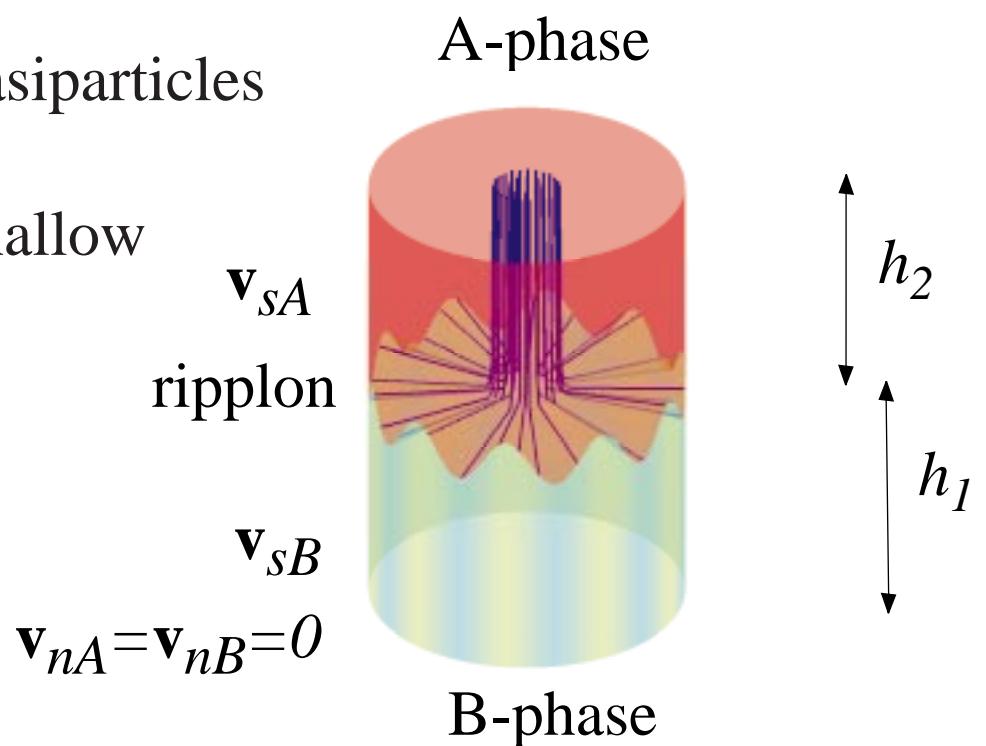
Kelvin-Helmholtz instability = physical singularity  
within black hole



B-phase

Ripplons on surface of shallow water are relativistic  
 Schützhold & Unruh  
 'Gravity wave analogs of black holes'  
 PRD **66** (2002) 044019

Relativistic quasiparticles  
 living on brane  
 between two shallow  
 superfluids



### Relativistic spectrum of ripplons

$$\alpha_1(\omega - \mathbf{k} \cdot \mathbf{v}_{sA})^2 + \alpha_2(\omega - \mathbf{k} \cdot \mathbf{v}_{sB})^2 = c^2 k^2 \quad \alpha_1 + \alpha_2 = 1$$

'Speed of light' - maximum attainable speed for ripplons

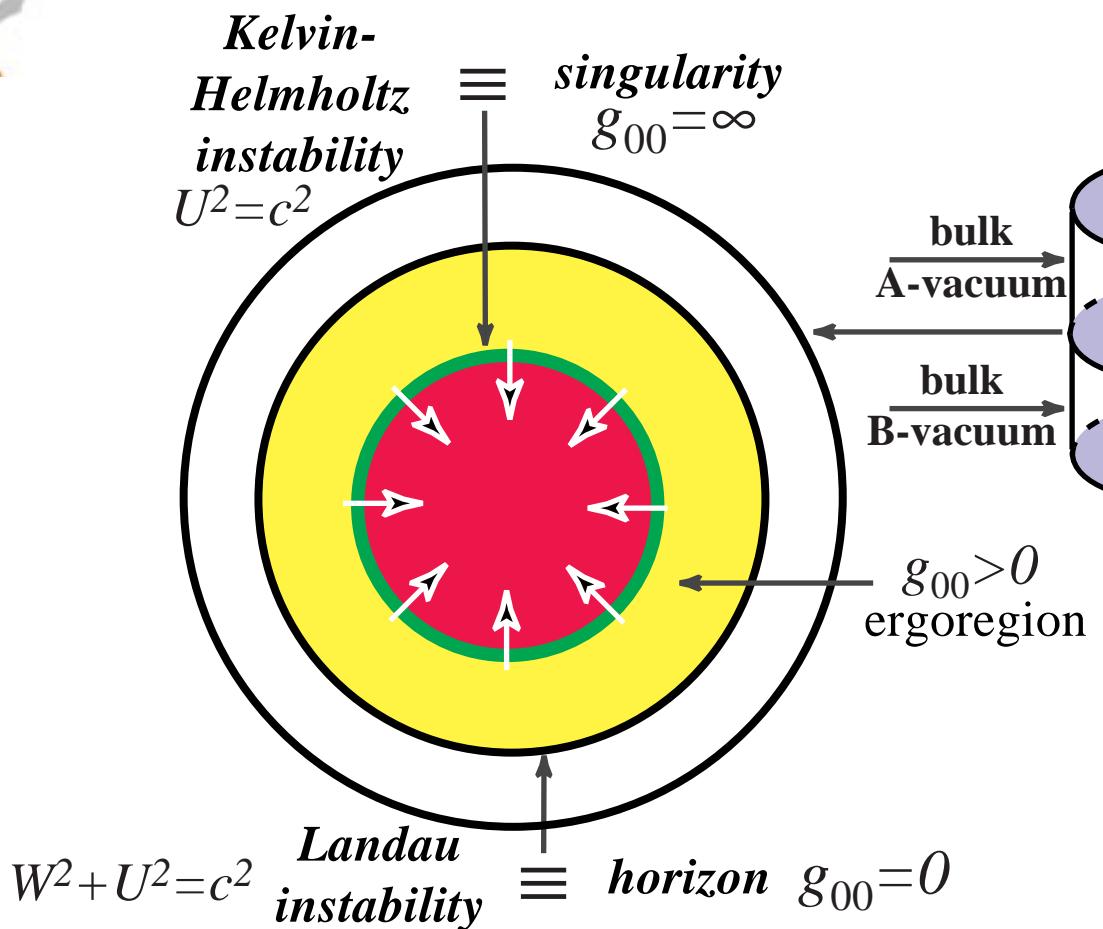
$$c^2 = (F/\rho_s) h_1 h_2 / (h_1 + h_2)$$

### Effective metric for ripplons in radial flow

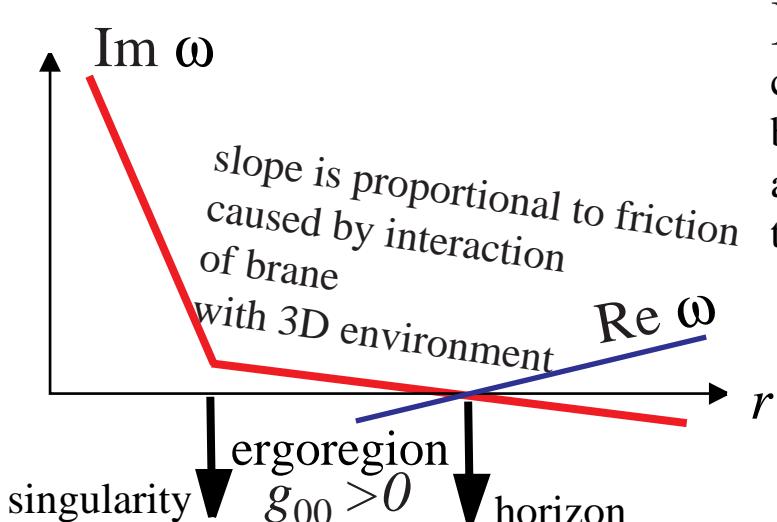
$$ds^2 = -dt^2 \frac{c^2 - W^2 - U^2}{c^2 - U^2} + dr^2 \frac{1}{c^2 - W^2 - U^2} + r^2 d\phi^2$$

$$U^2 = \alpha_1 \alpha_2 (\mathbf{v}_{sA} - \mathbf{v}_{sB})^2 \quad \mathbf{W} = \alpha_1 \mathbf{v}_{sA} + \alpha_2 \mathbf{v}_{sB}$$

# Artificial black hole for ripplons at AB-brane



$$ds^2 = -dt^2 \frac{c^2 - W^2 - U^2}{c^2 - U^2} + dr^2 \frac{1}{c^2 - W^2 - U^2} + r^2 d\phi^2$$

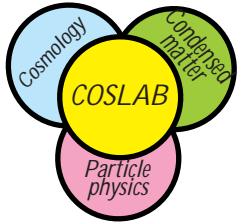


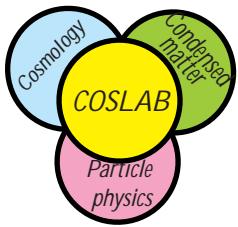
Im  $\omega$  & Re  $\omega$   
cross 0 simultaneously:  
behind horizon  
attenuation transforms  
to amplification

**lesson**

**from AB-brane:**

Black hole  
may collapse  
due to  
vacuum instability





# Thermodynamic - Landau - ergoregion instability of vacuum inside black holes



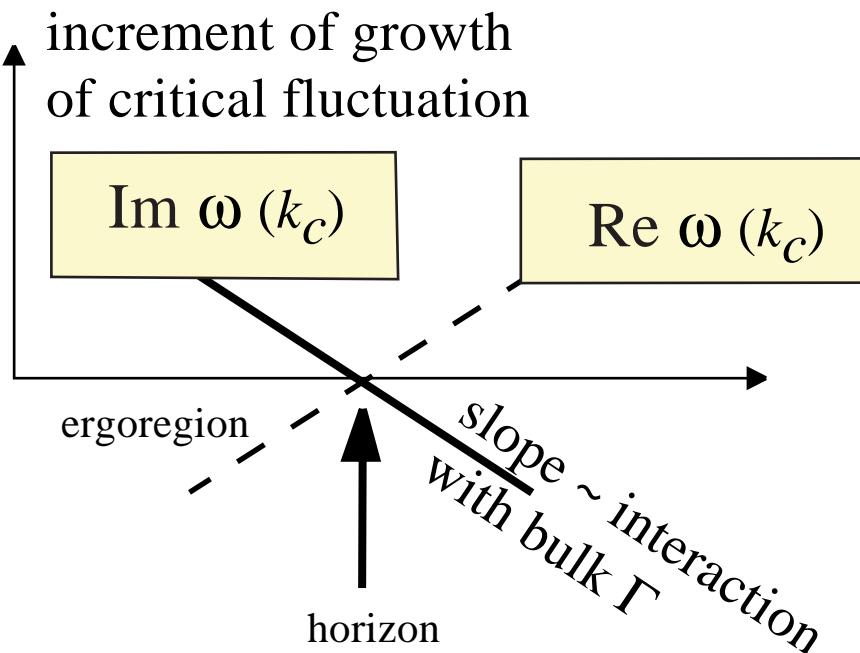
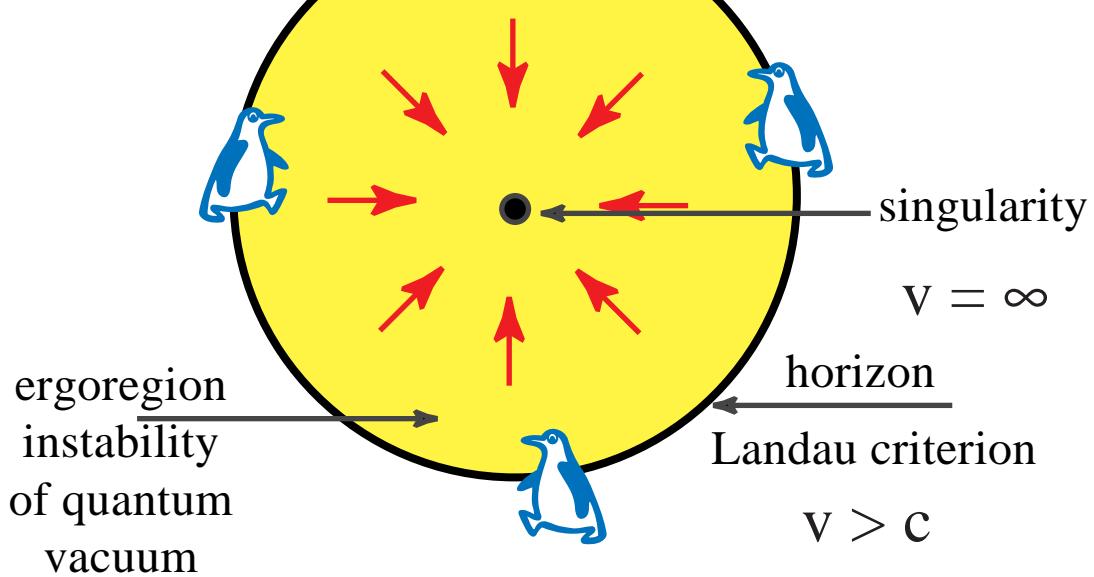
black hole in Painleve-Gulstrand metric

$$ds^2 = -dt^2(c^2-v^2) + 2v dr dt + dr^2 + r^2 d\Omega^2$$

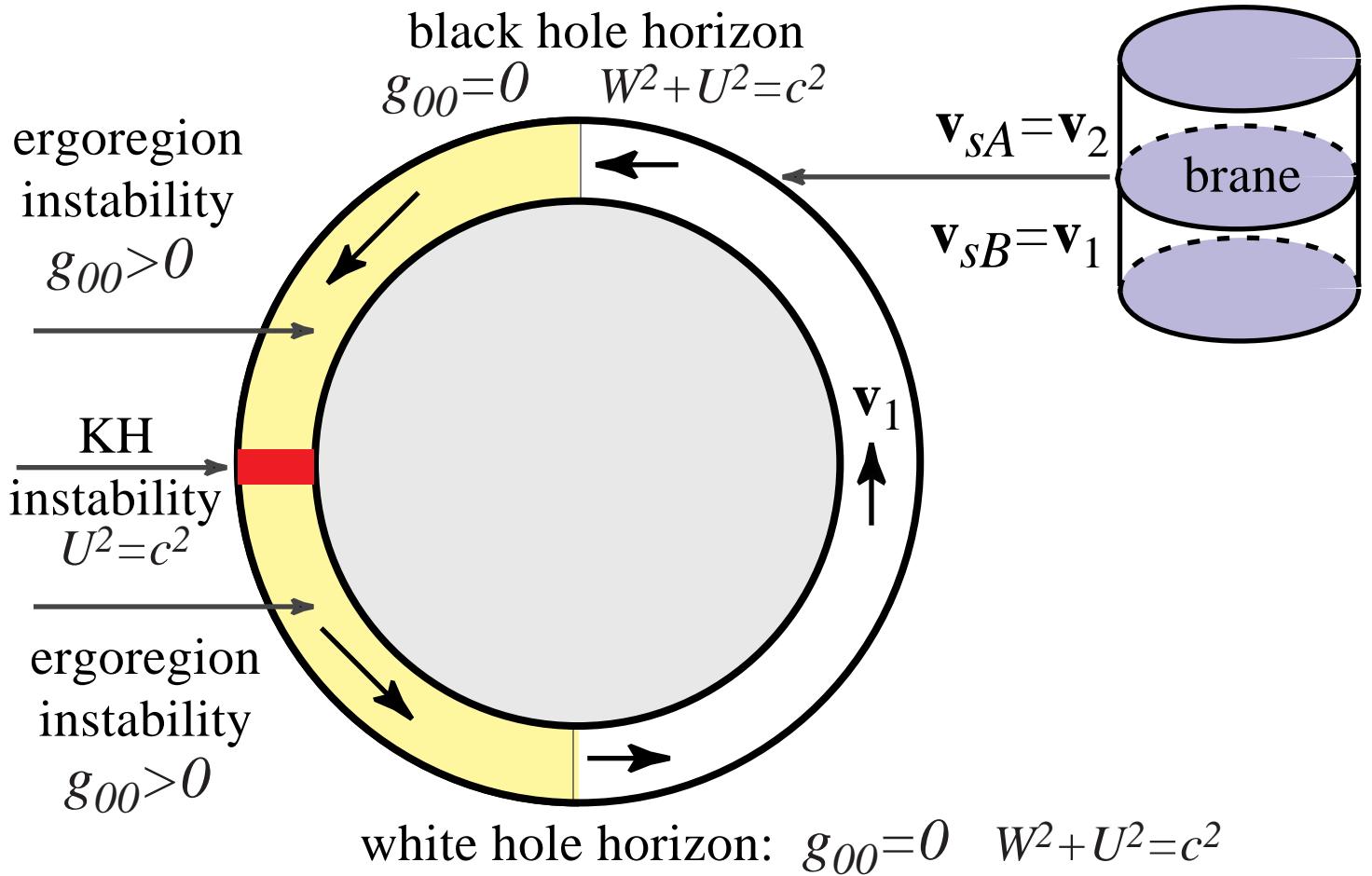
$$\begin{matrix} \uparrow \\ g_{00} \end{matrix} \quad \begin{matrix} \uparrow \\ g_{0r} \end{matrix}$$

Schwarzschild black hole

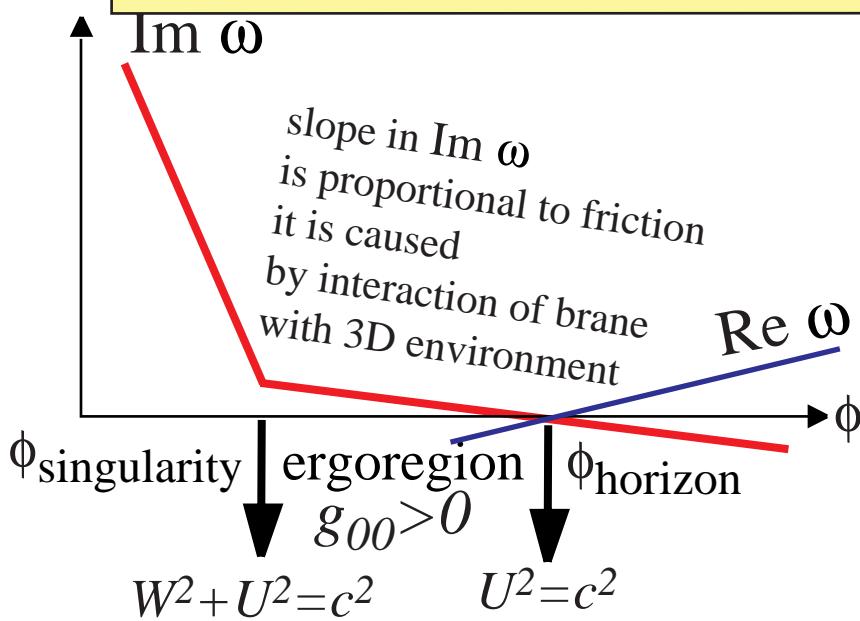
$$v^2(r) = \frac{2GM}{r} = c^2 \frac{r_h}{r}$$



# Artificial black hole for ripplons within AB-brane (azimuthal flow)



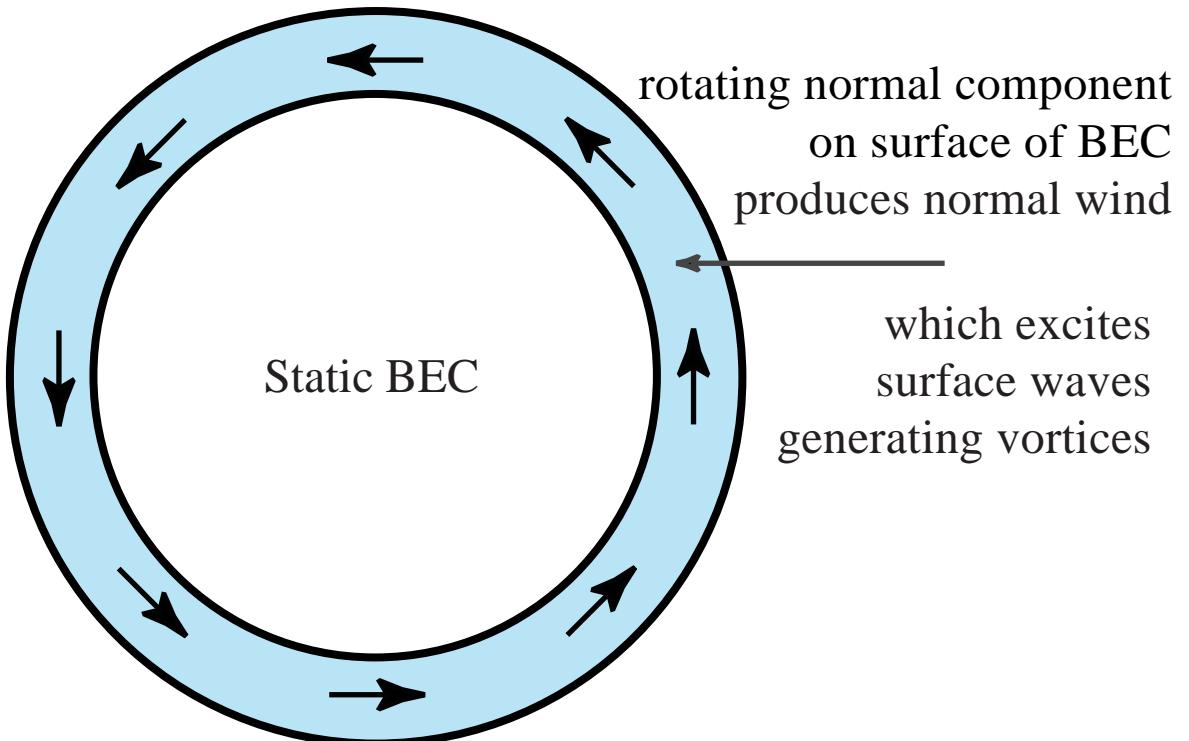
$$ds^2 = -dt^2 \frac{c^2 - W^2 - U^2}{c^2 - U^2} + r^2 d\phi^2 \frac{1}{c^2 - W^2 - U^2} + dr^2$$



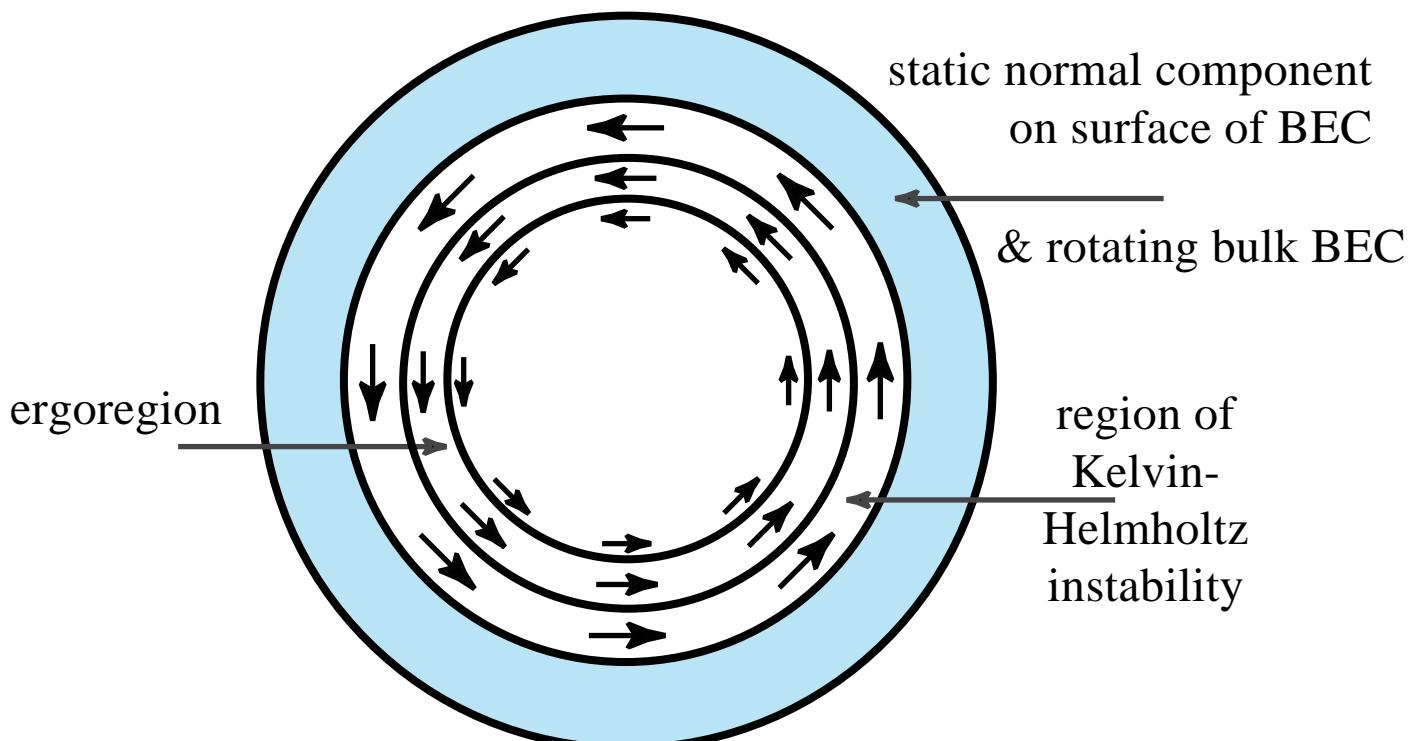
$\text{Im } \omega$  and  $\text{Re } \omega$   
cross 0 simultaneously:  
in ergoregion  
attenuation transforms  
to amplification

# *Ergoregion in rotating BEC & vortex formation*

## *Laboratory frame*



## *Rotating frame*



## *Conclusion*

- \* Brane physics can be simulated in  ${}^3\text{He}$
- \* Horizon can be constructed at AB-brane
- \* Vacuum in ergoregion can be unstable due to interaction with extra-dimensional environment  
*(was probed at AB brane)*
- \* Physical singularity can be constructed at AB-brane
- \* All possible 2+1 metrics can be constructed at AB-brane

**We need a lot of theoretical and experimental work  
and  
contribution from:  
hydrodynamic/gravity/brane/hep communities**