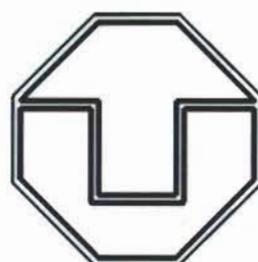


Surface-Wave Analogues of Black and White Holes

Ralf Schützhold

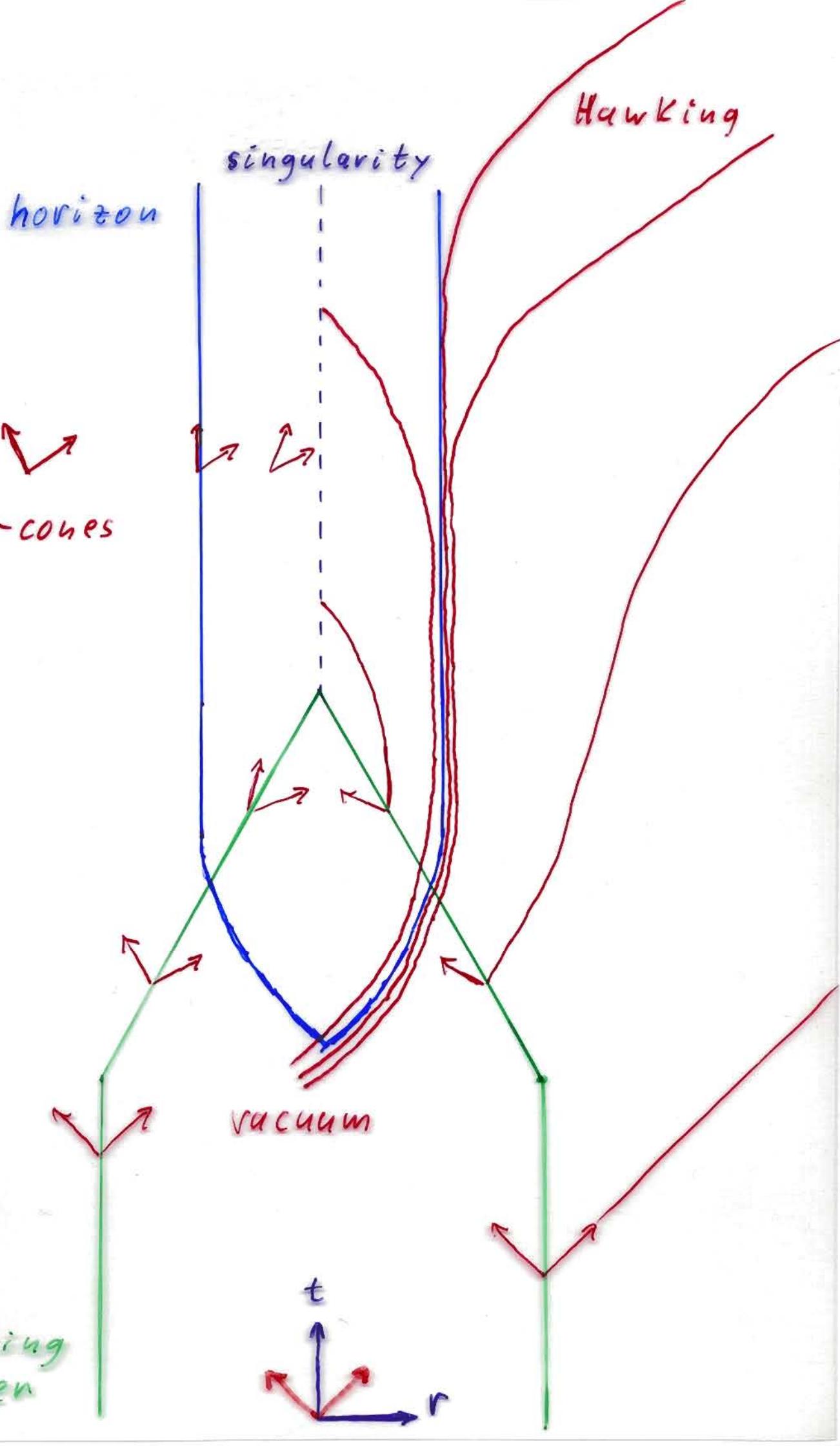
*Institute for Theoretical Physics,
Dresden University of Technology,
Germany*



William G. Unruh

*Department of Physics and Astronomy,
University of British Columbia,
Vancouver, Canada*





Black Hole Thermodynamics

No-hair theorem → only M , J (and Q)

Thermodynamics	Black holes
energy E	mass M
entropy S	surface area A
temperature T	surface gravity κ
pressure p	angular velocity at the horizon Ω_h
volume V	(minus) angular momentum J

- 0th law: κ constant (across horizon) in equilibrium
- 1st law: $dM = \kappa dA/(8\pi) + \Omega_h dJ$
- 2nd law: A always increases (energy conditions)
- 3rd law: one cannot reach $\kappa = 0$ (weak version)

Bekenstein's interpretation → real temperature?

Hawking effect

$$T_{\text{Hawking}} = \frac{\kappa}{2\pi} \rightarrow \frac{1}{8\pi M} \frac{\hbar c^3}{G_N k_B}$$

Gedanken experiment with box → consistency!

Nature seems to be telling us something here . . .

Entropy ↔ area (holography, AdS/CFT, etc.)

Trans-Planckian Problem

- standard model of particle physics: 10^3 GeV
- new physics (GUT, SuSy, ?): $10^3\text{--}10^{16}$ GeV
- Planck scale (quantum gravity?): 10^{19} GeV

Exponential gravitational red-shift at horizon

Origin of Hawking radiation → trans-Planckian modes!

Semi-classical treatment (QFT+gravity) breaks down

Does the Hawking effect depend on trans-Planckian physics?

→ Black Hole Analogues

- reproduce major features of black holes
- underlying physics is understood (in principle)
- emergent effective geometry/metric
- investigate influence of cut-off
- classical/quantum simulation of black hole physics

Black Hole Analogues: "Dumb Holes"

Sound in moving irrotational fluids $\delta \mathbf{v} = \nabla \phi$

$$\left(\frac{\partial}{\partial t} + \nabla \cdot \mathbf{v}_0 \right) \frac{\varrho_0}{c_s^2} \left(\frac{\partial}{\partial t} + \mathbf{v}_0 \cdot \nabla \right) \phi = \nabla \cdot (\varrho_0 \nabla \phi)$$

Effective geometry (curved space-time)

$$\square_{\text{eff}} \phi = \frac{1}{\sqrt{-g_{\text{eff}}}} \partial_\mu (\sqrt{-g_{\text{eff}}} g_{\text{eff}}^{\mu\nu} \partial_\nu \phi) = 0$$

Painlevé-Gullstrand-Lemaître metric

$$g_{\text{eff}}^{\mu\nu} = \frac{1}{\varrho_0 c_s} \begin{pmatrix} 1 & \mathbf{v}_0 \\ \mathbf{v}_0 & \mathbf{v}_0 \otimes \mathbf{v}_0 - c_s^2 \mathbf{1} \end{pmatrix}$$

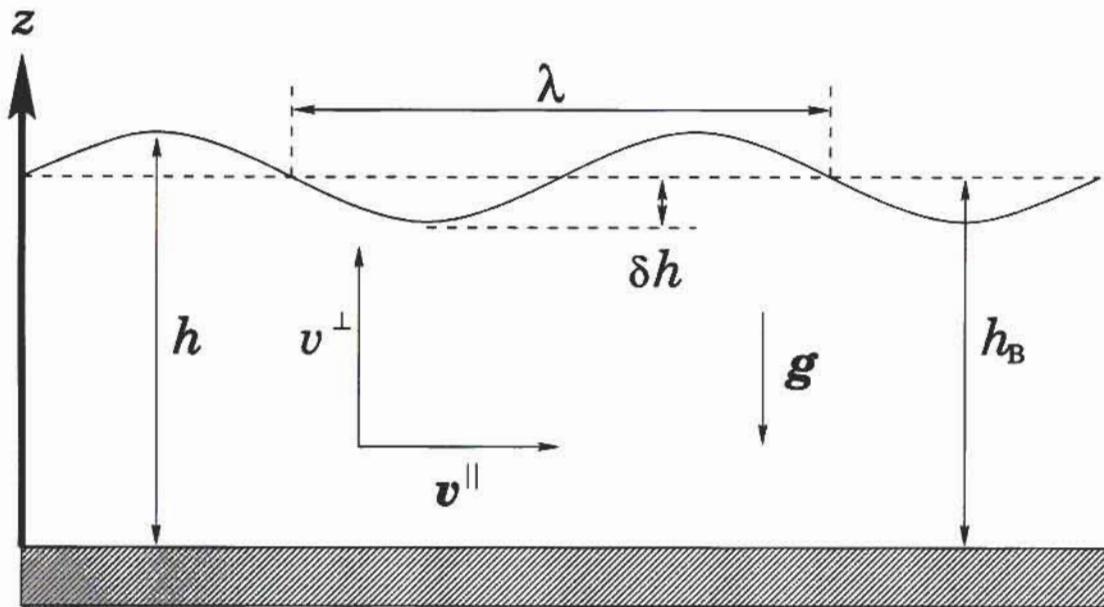
Horizon if $v_0^\perp = c_s$ – exact correspondence!

- Bose-Einstein condensates
- super-fluids (Helium)
- gravity (surface) waves
- photons in dielectrics (Gordon metric)
- etc.

Hawking temperature

$$T_{\text{Hawking}} = \frac{\hbar}{2\pi k_B} \left| \frac{\partial}{\partial r} (v_0^\perp - c_s) \right|$$

Basic Idea



- shallow water waves $\lambda \gg h$
- small amplitude $\delta h \ll h$
- no viscosity/friction
- incompressible, irrotational (laminar) flow

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_B^{\parallel} \cdot \nabla_{\parallel} \right)^2 \Phi - gh_B \nabla_{\parallel}^2 \Phi = 0$$

Effective Painlevé-Gullstrand-Lemaître type metric

$$g_{\text{eff}}^{\mu\nu} = \begin{pmatrix} 1 & \mathbf{v}_B^{\parallel} \\ \mathbf{v}_B^{\parallel} & \mathbf{v}_B^{\parallel} \otimes \mathbf{v}_B^{\parallel} - gh_B \mathbf{1} \end{pmatrix}$$

Arbitrary Bottom and Height

$$d\mathbf{r}^2 = dz^2 + \eta_{ij} dx^i dx^j$$

Intrinsic 2-D metric $\eta_{ij}(z) = \eta_{ij}^{(0)} + z\eta_{ij}^{(1)} + \dots$

$$\mathfrak{g}_{\text{eff}}^{\mu\nu} = \frac{1}{h_B^2} \begin{pmatrix} 1 & v_B^i \\ v_B^j & v_B^i v_B^j - \tilde{g} h_B \eta_{(0)}^{ij} \end{pmatrix}$$

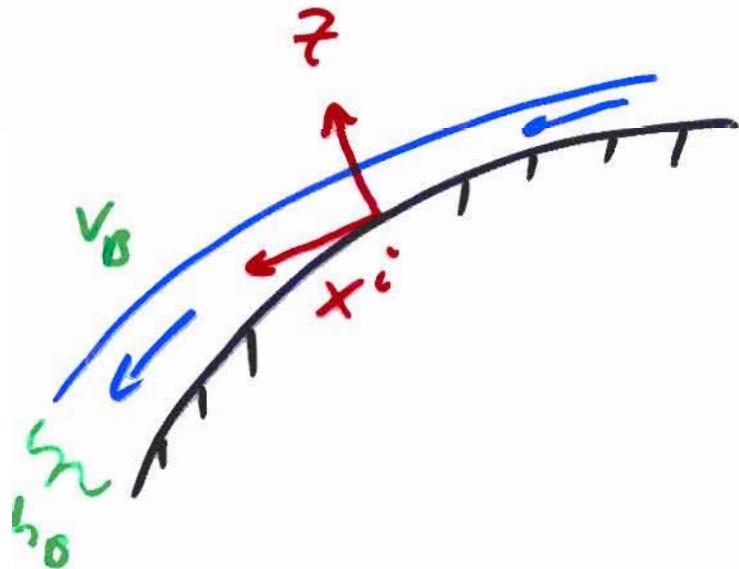
Modified gravitational acceleration

$$\tilde{g} = g_z + \frac{1}{2} \eta_{(1)}^{ij} v_i^B v_j^B$$

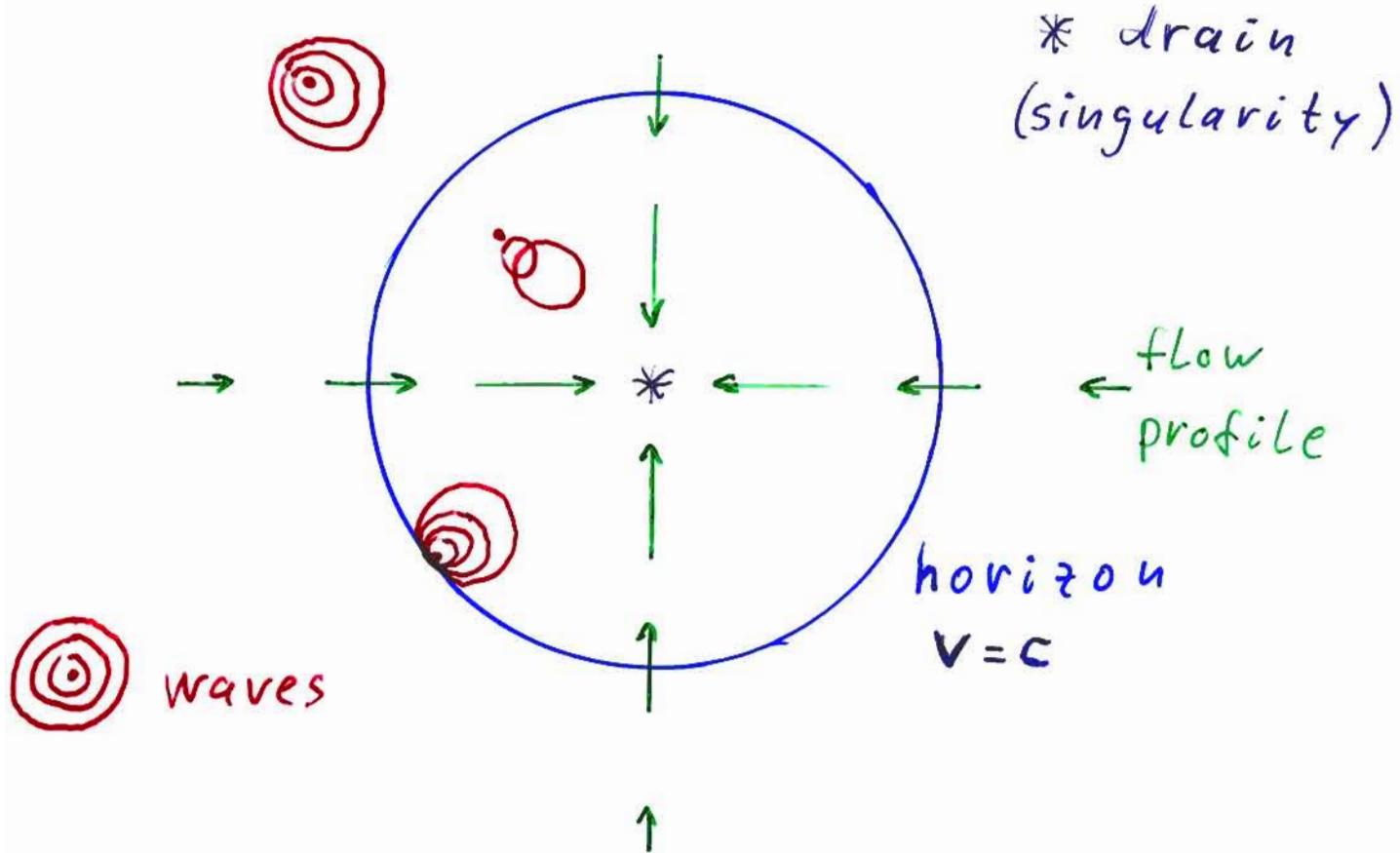
Eigenvalues κ give curvature of the surface

$$\eta_{(1)}^{ij} x_j = \kappa \eta_{(0)}^{ij} x_j$$

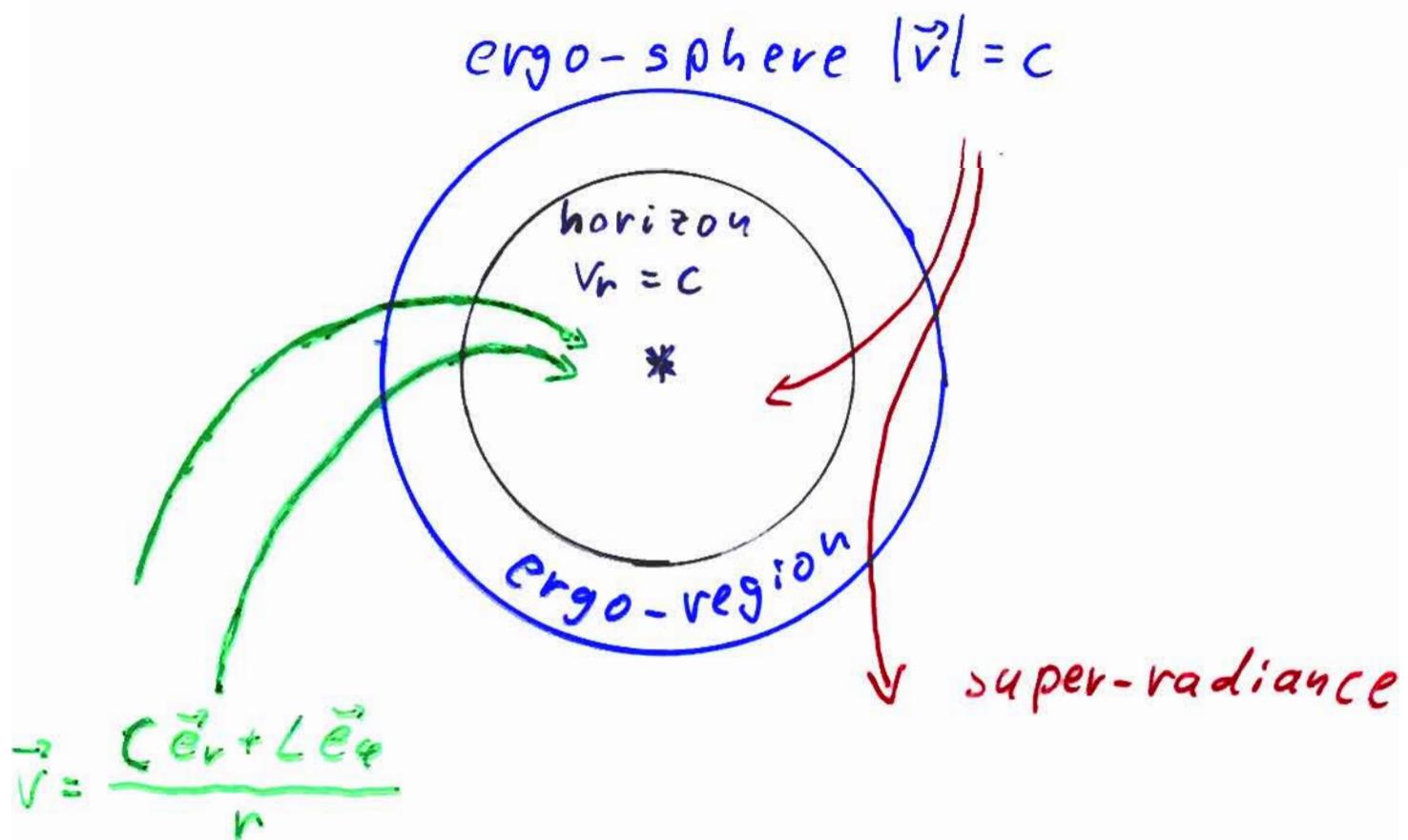
\Rightarrow centrifugal force



Kitchen-sink black-hole analogue



Analogue for Kerr metric (rotating BH)



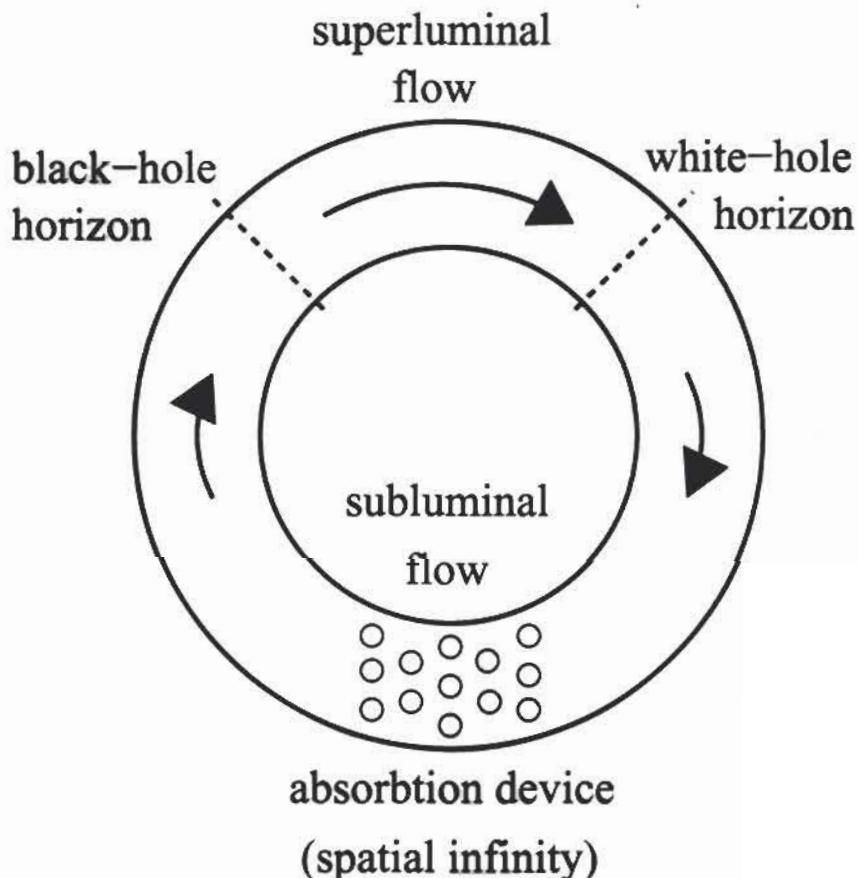
Black Hole Analogue

$$g_{00}^{\text{eff}} = 1 - \left(\frac{v_B^{\parallel}}{\sqrt{gh_B}} \right)^2$$

Horizon (or static limit/ergo-sphere) at $|v_B^{\parallel}| = \sqrt{gh_B}$

$$ds_{\text{eff}}^2 = \left(1 - \frac{C^2}{r^2} \right) dt^2 + 2 \frac{C}{r} dt dr - dr^2 - r^2 d\varphi^2$$

Radial flow profile, or



Effective Energy – Miles Instability

$$E = \int d\Sigma_\mu T^{\mu\nu} \xi_\nu = \int d^2r T_0^0$$

Killing vector $\xi^\mu = \partial/\partial t$ (Noether theorem)

$$T_0^0 = \frac{1}{2} \left[\left(\frac{\partial \phi}{\partial t} \right)^2 + gh_B (\nabla_{||}\phi)^2 - (\mathbf{v}_B^{\parallel} \cdot \nabla_{||}\phi)^2 \right]$$

Negative parts for $|\mathbf{v}_B^{\parallel}| > \sqrt{gh_B}$

$$(\omega + \mathbf{v}_B^{\parallel} \cdot \mathbf{k})^2 = gh_B \mathbf{k}^2 - i\omega\Gamma$$

Miles instability

$$\Im(\omega) = -\frac{\Gamma}{2} \left(1 \pm \frac{\mathbf{v}_B^{\parallel} \cdot \mathbf{k}}{\sqrt{gh_B \mathbf{k}^2}} \right)$$

- wind-generated water waves
- Landau instability in plasma, etc.
- brane-world scenarios ?

Super-radiance

$$v_B^{\parallel} = \frac{Ce_r + Le_{\varphi}}{r}$$

Kerr type metric for vortex velocity profile (e.g., sink)

$$\begin{aligned} ds_{\text{eff}}^2 = & \left(1 - \frac{C^2 + L^2}{r^2}\right) dt^2 + 2 \frac{C}{r} dt dr + 2L dt d\varphi \\ & - dr^2 - r^2 d\varphi^2 \end{aligned}$$

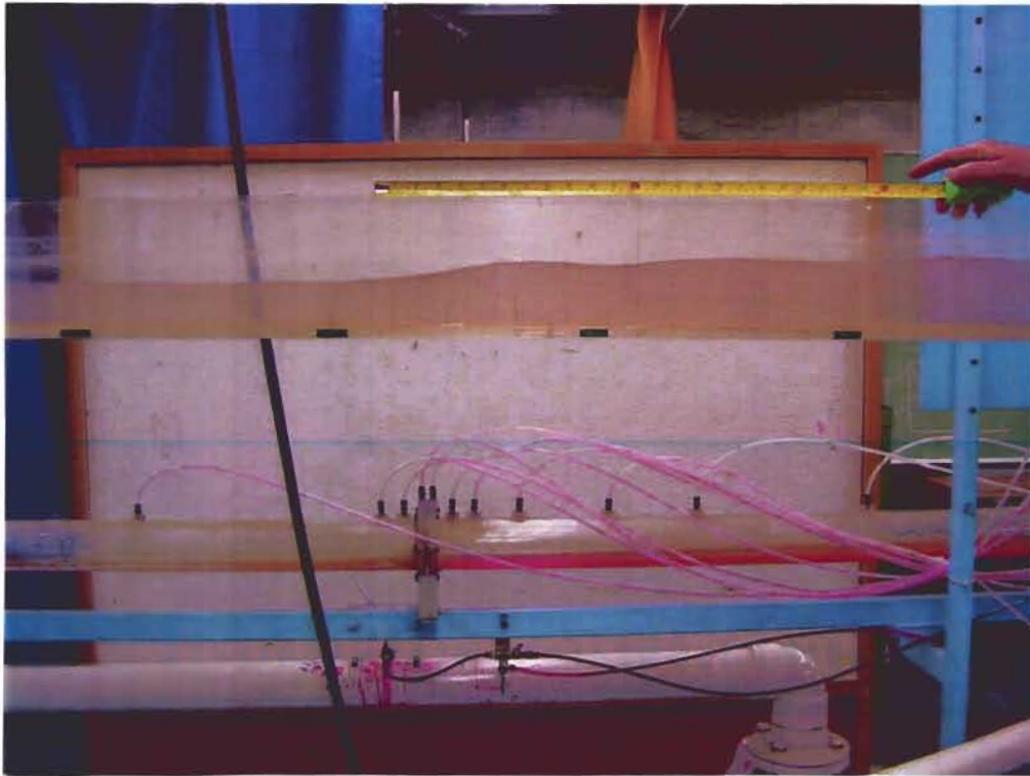
Wronskian conserved ($\Omega_H = L/C^2$)

$$1 - |\mathcal{R}_{\omega m}|^2 = \frac{\omega - m\Omega_H}{\omega} |\mathcal{T}_{\omega m}|^2$$

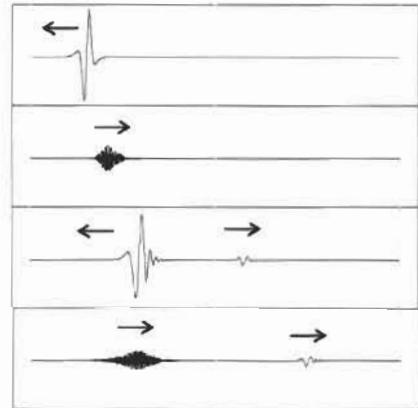
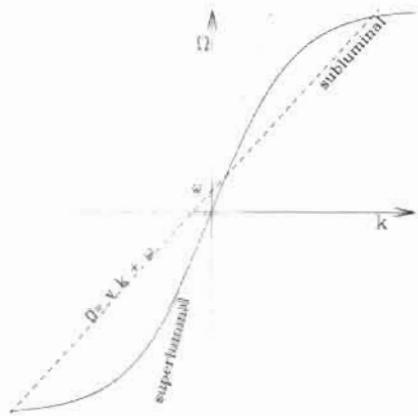
Reflection and transmission coefficients



White Hole Horizon Instability



Mode mixing (“Bogoliubov” coefficients)



$$k \left(v_B^{\parallel} \right)^2 = g \tanh(k h_B)$$

Summary

- analogy to gravitation (but no Einstein Eqs...)
- helps understanding (e.g., pedagogical)
- interdisciplinary know-how transfer
- geometrical concepts, e.g., negative energy
- super-radiance, mode mixing, Hawking radiation?

E.g., third sound in Helium (gravity → van der Waals):

$$h = \mathcal{O}(10 \text{ \AA}), c_{\text{sound}} = \mathcal{O}(10 \text{ m/s})$$

→ Hawking temperature $\mathcal{O}(1 \text{ K})$?

Acknowledgements

- DFG (Emmy-Noether program)
- Humboldt foundation (Feodor-Lynen fellowship)
- ESF (COSLAB program)
- CIAR, NSERC, EU-IHP ULTI 3
- many fruitful discussions...

Black-Hole Information Paradox

Hawking effect ($T \leftrightarrow \kappa$) black-hole entropy ($S \leftrightarrow A$)

$$dM = \frac{\kappa}{8\pi} dA + \dots \leftrightarrow dE = T dS + \dots$$

Surface area of horizon A in Planckian units ℓ_{Planck}

$$S = \frac{k_B A}{4 \ell_{\text{Planck}}^2}$$

E.g., micro-canonical ensemble

$$S = k_B \ln \Omega$$

Measure for number of fundamental degrees of freedom?

- evaporation until $S = \mathcal{O}(1)$
- Hawking radiation thermal, i.e., no correlations
- no information (e.g., sub-luminal dispersion)

Loss of information – non-unitary evolution?

Second Law of (Black Hole) Thermodynamics
Versus
Unitarity – Conservation of Information

Hawking's Talk in Dublin

Disclaimer: very simplified and as I understand it...

'Old' argument: conjectured AdS-CFT duality
Boundary (CFT) unitary, therefore...

'New': Scattering process with Euclidean path integral

- only observables at infinity (weak fields)
- horizon – non-trivial topologies (Wick rotation)
- quantum superposition $|\text{horizon}\rangle + |\text{no horizon}\rangle$
- event and apparent horizon (slicing-dependent)

Is this the final solution (how exactly does it work)?

Violation of the Second Law of Thermodynamics?