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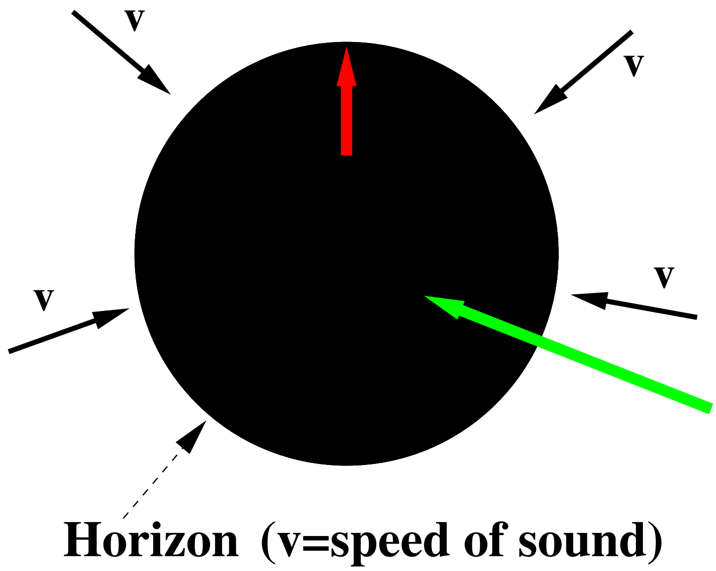
Quasiparticle Universes in Bose-Einstein Condensates

Motivation

Low-energy modes in condensed matter with linear dispersion mapped to relativistic quantum field theories in curved space-time (W. G. Unruh, M. Visser, G. E. Volovik)

Phenomena to expect: Unruh-Davies effect, Hawking radiation, Black and White Holes, Cosmic Strings vs. Vortices, Trans-Planckian Physics in the Laboratory



Comparison of Two Physical Systems

Two Quantum Fields on Curved Space-Times:

$\mathbf{I}$: Photons $\qquad\qquad$ $\mathbf{II}$: Phonons

$\mathbf{I} \Rightarrow \mathbf{II}$: Light Speed $c \Rightarrow$ Sound Speed c_s

$$\omega = ck \quad \Rightarrow \quad \omega = c_s k$$

$\mathbf{I}$: Propagates On Curved Space-Time
Background Determined
by Einstein Equations

$\mathbf{II}$: Propagates On **Effective** Curved
Space-Time Background in Nonrelativistic
Lab System (here: BEC)
Metric Determined by Euler and Continuity
Equations of Perfect Fluids

Advantage of Bose-Einstein Condensates:

$$\frac{c_s}{c} \simeq 10^{-11}$$

And Ultralow Temperatures Available

$\Rightarrow$ Classical and Quantum Effects $\propto 1/c_s$
Possible to Observe at Much Lower Energies

Identification of Space-Time Metric

Conventional Wave Equation

$$\left[-\frac{1}{c_s^2}\frac{\partial^2}{\partial t^2} + \Delta\right]\Phi = 0$$

for Inhomogeneous Media Reads

$$\frac{1}{\sqrt{-\mathrm{g}}}\partial_\mu(\sqrt{-\mathrm{g}}\,\mathrm{g}^{\mu\nu}\partial_\nu\Phi) = 0$$

$$[\mathrm{g} = \det(\mathrm{g}_{\mu\nu})]$$

Potential of Velocity Perturbations in Euler and Continuity Eqs

$$\delta \mathbf{v} = \nabla \Phi$$

Effective Space-Time Metric

$$\begin{aligned} ds^2 &= \mathrm{g}_{\mu\nu} dx^\mu dx^\nu \\ &= \frac{\rho}{c_s} \left[-(c_s^2 - \boldsymbol{v}^2) dt^2 - 2 v_i dx^i dt + dx^i dx_i \right] \end{aligned}$$

Inhomogeneous Velocity and/or Density/Speed of Sound Profile

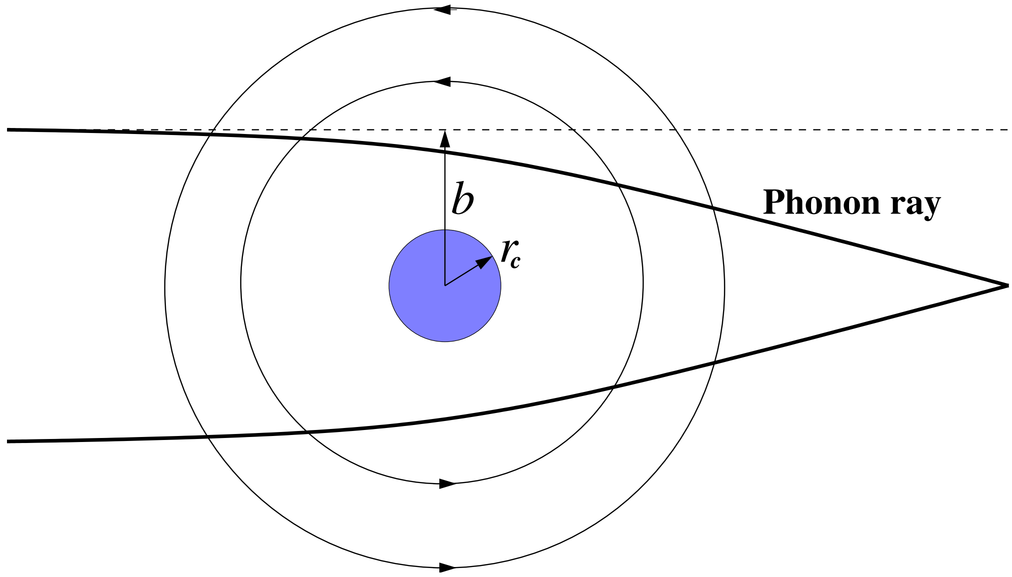
$\Rightarrow$ Phonons Moving in the Perfect Fluid "See" Effective Curved Space-Time

Ricci Curvature Scalar (Incompressible Fluid)

$$R = \mathrm{Tr}(\boldsymbol{D}^2)$$

[Deformation Tensor $\boldsymbol{D}_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i)$]

Example: **Vortex as Gravitational Lens**



Core Radius $\quad r_c = \frac{\Gamma}{2\pi c_s}$

Curvature $\quad R = \frac{2r_c^2}{r^4}$

Deflection Angle $\quad \Delta\phi \propto (r_c/b)^2 \propto 1/c_s^2$

$\Rightarrow$ The Slower the Sound the Better

[Phys. Rev. Lett. **88,** 110201 (2002)]

[Ann. Phys. (N.Y.) **304,** 22 (2003)]

[Europhys. Lett. **62,** 1 (2003)]

(With M. Visser)

The Analogy in Full Flight:

Quantum Field Theory in An Expanding Universe

Expanding (Contracting) Bose-Einstein Condensate under Variable Trapping and/or Particle Interaction

$\equiv$ **Expanding (Contracting) Universe**

Expected Phenomena:

- Cosmological Horizons (Robertson-Walker Cosmologies)
- Cosmological (Quasi-)Particle Production
- Static Horizon (de Sitter case): Gibbons-Hawking Effect

Nonuniqueness of the Particle Content of a Quantum Field

Unruh-Davies Effect: Constantly Accelerated Detector in Minkowski Vacuum

Detects Particles

Thermal State at Temperature $T_{\text{Unruh}} = \hbar a / c$

Generalization for Cosmological de Sitter Space-Time: Gibbons-Hawking Effect

de Sitter Temperature $\quad T_{\text{dS}} \propto \sqrt{\Lambda}$

Light: $\quad T_{\text{Unruh}} \simeq 1\,\text{K}$ for $a = 10^{20} g_{\oplus}$!!

BEC: $\quad T_{\text{dS}} \simeq 10 - 100$ nK, but $T_c \simeq 1\ \mu$K

Important: Need Detector with
Frequency Standard in Units of de Sitter Time

Facts About Expanding BECs

Solution by Scaling Ansatz

$$\rho(\boldsymbol{r},t) = \frac{\rho(\boldsymbol{r}/b)}{b^3} \qquad \boldsymbol{v}_s = \frac{\dot{b}}{b}r \qquad b = b(t)$$

Evolution of Scaling Parameter

Changing Trap Frequency $\omega(t)$, Coupling $g(t)$

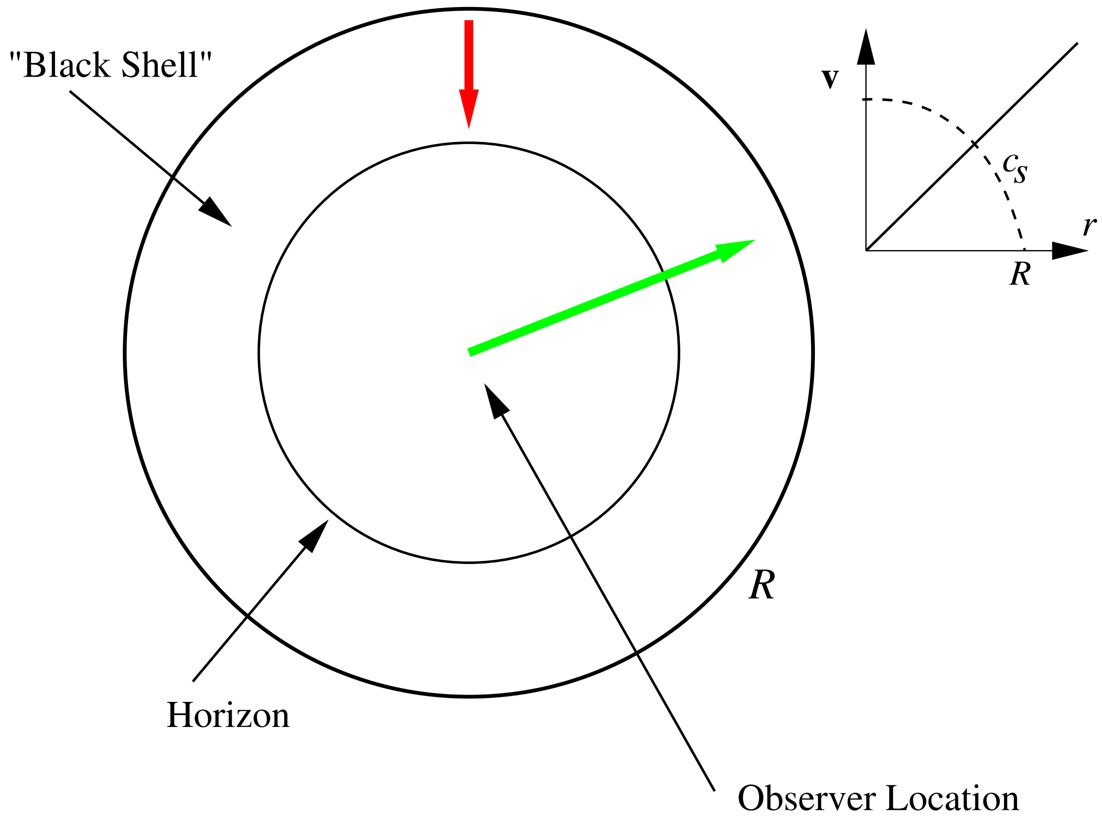
$$\ddot{b} + \omega^2(t)b = \frac{[g(t)/g(0)]\omega_0^2}{b^4}$$

Scaling Solution in Thomas-Fermi

$$\rho(\boldsymbol{r},t) = \frac{\rho_0}{b^3}\left(1 - \frac{r^2}{b^2 R_{\mathrm{TF}}^2}\right)$$

Point Where $|\boldsymbol{v}_s| = c_s = \sqrt{g(t)\rho(t)}$

$\Longrightarrow$ Horizon for Phonons



Robertson-Walker Universe if $g(t)/g(0) = b^3$:

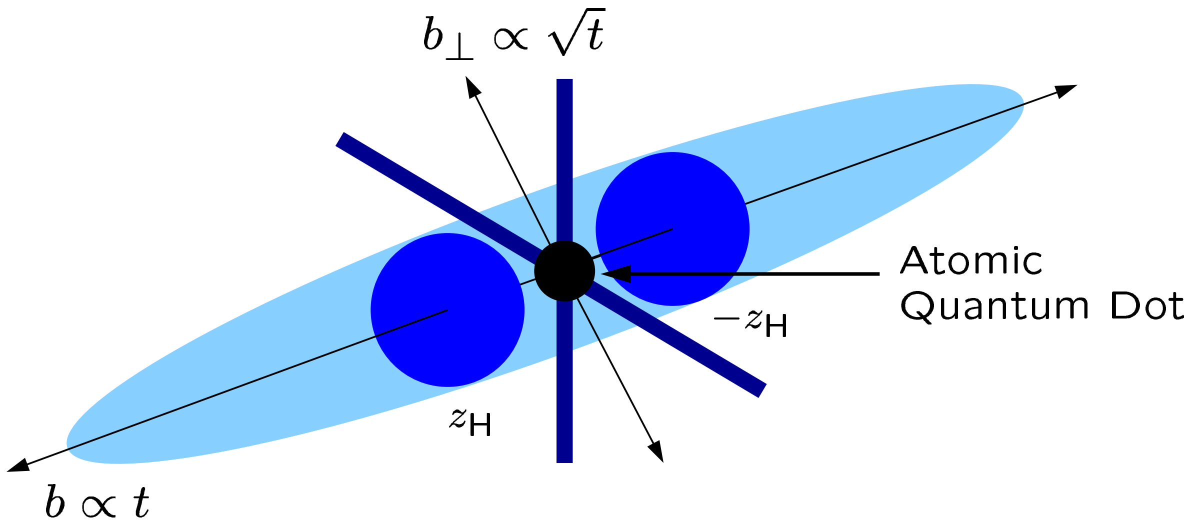
$$ds^2 \propto -c^2 dt^2 + b^2 dr_b^2 + r_b^2 d\Omega^2$$

$r_b = r/b$: Scaling Coordinate

$\Rightarrow b(t)$: Scale Parameter of "Universe"

$H = \dot{b}/b$: Hubble Parameter

1+1 D de Sitter Universe



Adiabatic Separation

$$\Phi(r,z,t) = \sum_n \phi_n(r)\chi_n(z,t)$$

Cigar at initial equilibrium: Dispersion Relation

$$\epsilon_{nk}^2 = 2\omega_\perp^2 n(n+1) + c_0^2 k^2$$

Has one **Massless** ($n = 0$) and Sequence of Massive Excitations

Action for Expanding Cigar (Massless Branch)

$$S = \int dtdz \frac{b_\perp^2 C_0}{2g} \left[-(\dot{\chi}_0 - v_z \partial_z \chi_0)^2 + \frac{c_0^2}{b_\perp^2 b} (\partial_z \chi_0)^2 \right]$$

$$\equiv \int dtdz \sqrt{-\mathrm{g}} \mathrm{g}^{\mu\nu} \partial_\mu \chi_0 \partial_\nu \chi_0$$

Choice For Expansion Parameters

$$\frac{b_\perp}{\sqrt{b}} = B = \text{const.} \qquad b(t) = \dot{b}t$$

$\Rightarrow$ De Sitter Space-Time Metric Reproduced

$$ds^2 = -c_0^2 \left(1 - \Lambda z^2\right) d\tau^2 + \left(1 - \Lambda z^2\right)^{-1} dz^2$$

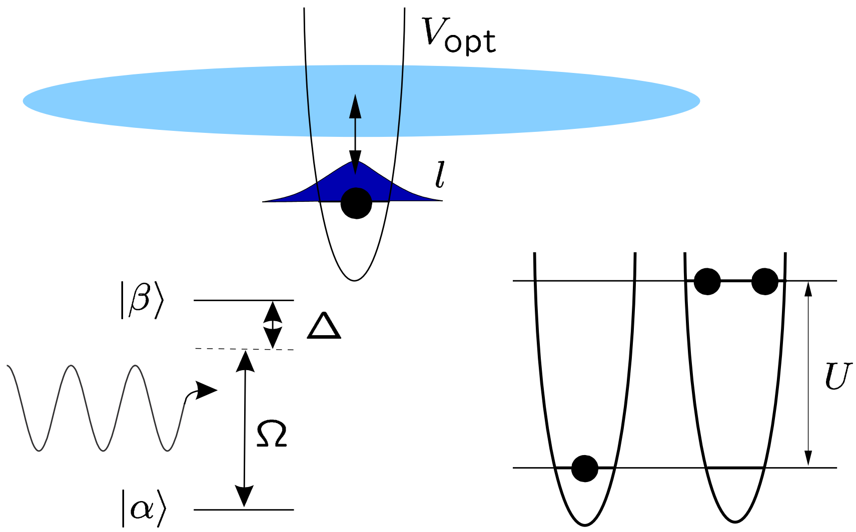
$$\Lambda = B^2 \dot{b}^2 / c_0^2$$

With De Sitter Time Interval

$$d\tau = dt/Bb \qquad \Longleftrightarrow \qquad t/t_0 = \exp[B\dot{b}\tau]$$

De Sitter and Laboratory Time
Exponentially Different !!

The Phonon Detector: An Atomic Quantum Dot



Effective Two-Level System Occupied by
Maximally One β Atom

Detector couples to Cigar like $\sqrt{\rho_0} \propto 1/b \propto d\tau/dt$

Equations of Motion

$$i\frac{d\psi_\beta}{d\tau} = \frac{\omega_0}{2}\psi_\alpha + \left\{\delta V + bB[-\Delta + \rho_0(g_{\alpha\beta} - g)]\right\}\psi_\beta$$

$$i\frac{d\psi_\alpha}{d\tau} = \frac{\omega_0}{2}\psi_\beta \qquad\qquad \delta V \propto \delta\rho$$

At Detuning Compensated Point:

Detector Measures in de Sitter time τ !!

Detector Response

Integral of Two-Point Correlation Function
$\langle \delta\hat{V}(\tau)\delta\hat{V}(\tau')\rangle$

Gives Probability for Detector to "Click":
Excitation (+) or De-Excitation (−)

$$\frac{dP_\pm}{d\tau} = \lim_{\mathcal{T}\to\infty} \frac{1}{\mathcal{T}} \int^{\mathcal{T}}\!\!\int^{\mathcal{T}} d\tau d\tau' \langle \delta\hat{V}(\tau)\delta\hat{V}(\tau')\rangle e^{\mp i\omega_0(\tau-\tau')}$$

Result : $$\frac{dP_+/d\tau}{dP_-/d\tau} = \frac{n_{\mathrm{B}}}{1+n_{\mathrm{B}}}$$

$$n_{\mathrm{B}} = \frac{1}{\exp[\omega_0/T_{\mathrm{dS}}]-1} \qquad T_{\mathrm{dS}} = \frac{B\dot{b}}{2\pi}$$

$\Rightarrow$ Thermal State at de Sitter Temperature

Leads to Damping of Rabi Oscillations between Detector States

Note: Background in Highly Nonstationary Motional State

Two-Component Case: Simulating Inflation And Galaxy Formation

Close to phase separation point of equal intraspecies repulsion $g_{11} = g_{22} \equiv g_{\text{diag}}$ and interspecies repulsion $g_{12} \equiv g_{\text{off}}$

One hard (density) and one soft (spin) mode

Propagation speeds $c_{\pm}^2 = \rho(g_{\text{diag}} \pm g_{\text{off}})$.

Bi-metric case when velocities are equal

Adjusting (for $\boldsymbol{v} = 0$)

$$g_{\text{soft}}(t) = \frac{g_0}{H^4 t^4}$$

$\Rightarrow$ soft spin mode has de Sitter metric !

In Conformal Time $\eta = -e^{-H\tau}/H = -1/(H^2 t)$
Inflaton Wave Equation

$$\left(\frac{\partial^2}{\partial \eta^2} - \frac{2}{\eta}\frac{\partial}{\partial \eta} + [c_0 \boldsymbol{k}]^2\right)\phi_{\boldsymbol{k}}(\eta) = 0$$

Solution for Inflaton Quantum Field

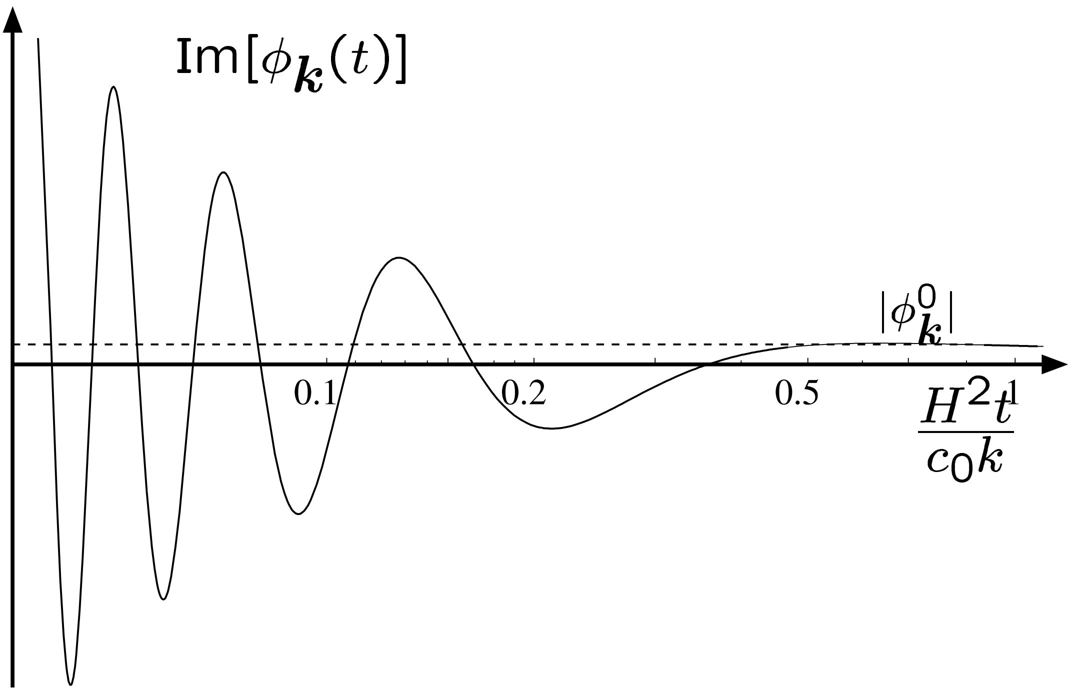
$$\hat{\phi}(\boldsymbol{r},\eta) = H\sqrt{\frac{g_0}{2Vc_0^3}}\sum_{k}\frac{i - c_0 k\eta}{\sqrt{k^3}}\,e^{i\boldsymbol{k}\cdot\boldsymbol{r} - ic_0 k\eta}\hat{a}_{\boldsymbol{k}} + \text{H.c.}$$

$\Rightarrow$ At late times $t \to \infty, \eta \to 0$

After Cosmological Horizon Crossing Damping Wins; Quantum Fluctuations Frozen In:

$$\langle \hat{\phi}^{\dagger}_{\boldsymbol{k}} \hat{\phi}_{\boldsymbol{k}} \rangle = \frac{H^2 g_0}{2Vc_0^3 k^3} \equiv |\phi^0_{\boldsymbol{k}}|^2$$

Supposed to be Seeds of Galaxy Formation



Problem: Quantum Fluctuations Are Weak

Maximal Phase-Phase Correlations

$$(\Delta k)^3 C_\phi^{\text{max}}(\boldsymbol{k}) \sim \sqrt{\frac{g_0}{g_{\text{final}}}} \sqrt{\varrho_0 g_{\text{final}}^3} \left(\frac{\Delta k}{k_{\text{final}}^{\text{Planck}}}\right)^3$$

Suppressed by Final Diluteness Parameter

$$\sqrt{\varrho_0 g_{\text{final}}^3} \sim 10^{-2} \cdots 10^{-4}$$

Solution: Drive System into Unstable Regime of Small **Negative** g_{soft}

$\Rightarrow$ Amplification of Phase Fluctuations

$$\phi_{\boldsymbol{k}}^{\text{out}}(t) = A_{\boldsymbol{k}} \exp\left\{+\sqrt{\frac{\varrho_0 |g_{\text{soft}}| \boldsymbol{k}^2}{m}}\, t\right\}$$

$$+ B_{\boldsymbol{k}} \exp\left\{-\sqrt{\frac{\varrho_0 |g_{\text{soft}}| \boldsymbol{k}^2}{m}}\, t\right\}$$

Freezing implies $A_{\boldsymbol{k}} \approx B_{\boldsymbol{k}}$:

$\Rightarrow$ Frozen Value of Fluctuations Grows

Summary

- A Bose-Einstein Condensate Can Model Large Variety of Cosmological and Other Effective Space-Times

- Observer Dependence of Particle Content of Quantum Fields Can Be Verified for the First Time

- Aspects of Cosmology Can be Turned into Laboratory Science !!

[Phys. Rev. Lett. **91,** 240407 (2003)]

[Phys. Rev. D **69,** 064021 (2004)]

[Phys. Rev. A **69,** 033602 (2004)]

(With P. O. Fedichev)

[Mod. Phys. Lett. A **19**, 1789-1812 (2004)]
(Review Paper)

[cond-mat/0406470] (With R. Schützhold)