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Transport in Granular Metals

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QUANTUM PHENOMENA AT LOW TEMPERATURES
ULTI USERS MEETING
Lammi Biological Station, 21.4. 2006 - 26.4. 2006

Outline

- Introduction: general features of granular conductors
- Metallic conductivity
- Insulating region – hopping conductance
- Hopping conductivity in superconductors

A. Gerber *et al*. PRL 78. 4277 (1997)

Granular metals: experiment

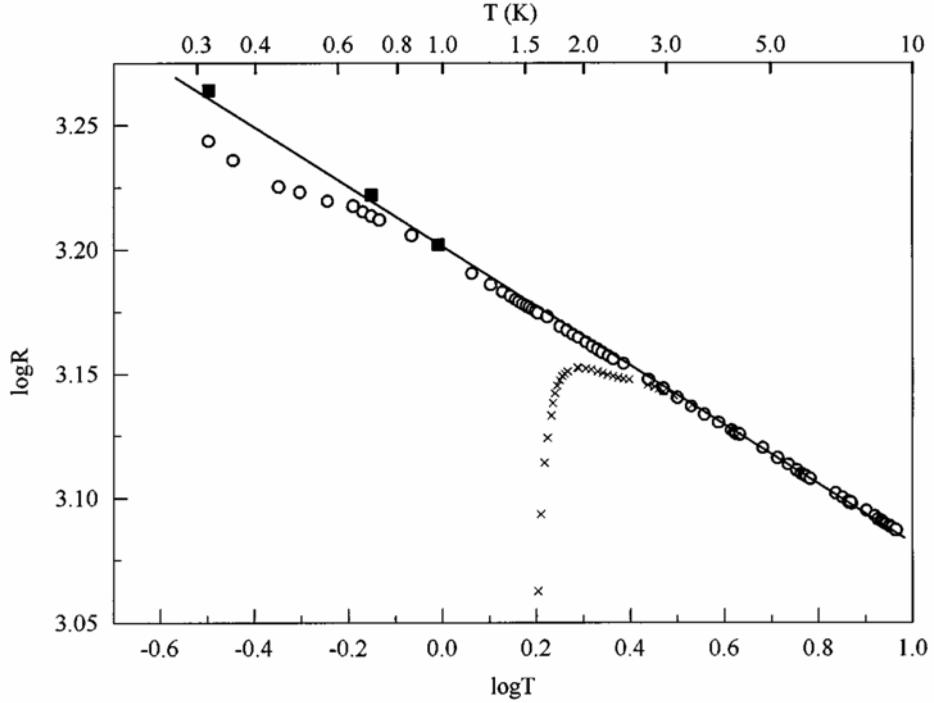


FIG. 3. Resistance of sample 3 as a function of temperature on a log-log scale, as measured at (zero) (×) and 100 kOe field (open circles). Open circles indicate resistance measured with a constant dc current $I = 10^{-5}$ A. Solid squares are zero bias resistances approximated from I-V measurements. Sample 3 room temperature resistance is 500 Ω.

Samples with larger coupling showed the behavior as

$$R \propto T^{-\alpha} \simeq 1 - \alpha \ln T \quad , \quad \alpha \approx 0.117$$

The samples are 3D, and the experiment was carried out in high magnetic field,

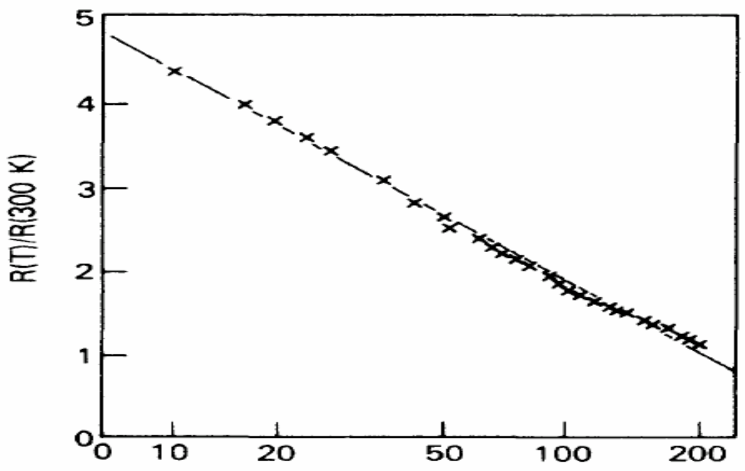
excluding thus the explanation of log as the weak localization behavior

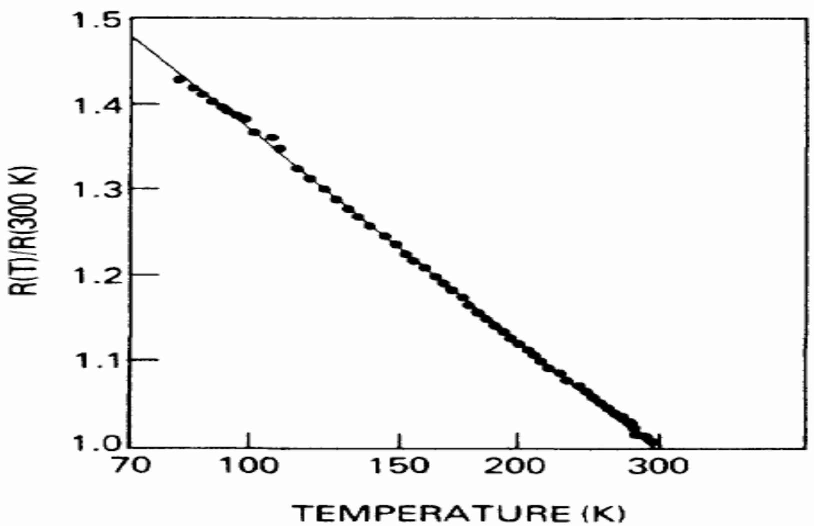
Granular metals: experiment.

R. W. Simon *at al*, PRL 36 (1987)

A. Gerber *et al*, PRL 78, 4277 (1997)

NbN in the insulating substrate





Samples (Al-Ge) with the high room temperature resistivity (weak tunneling) showed an exponential growth of resistivity as function of temperature.

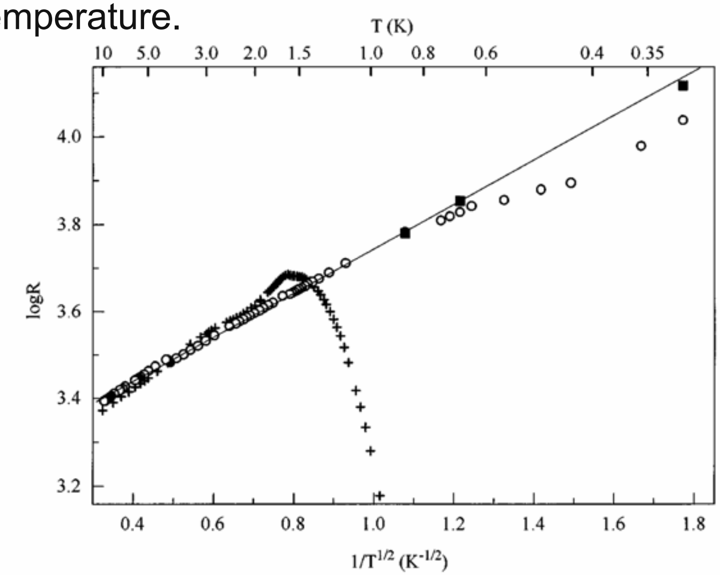


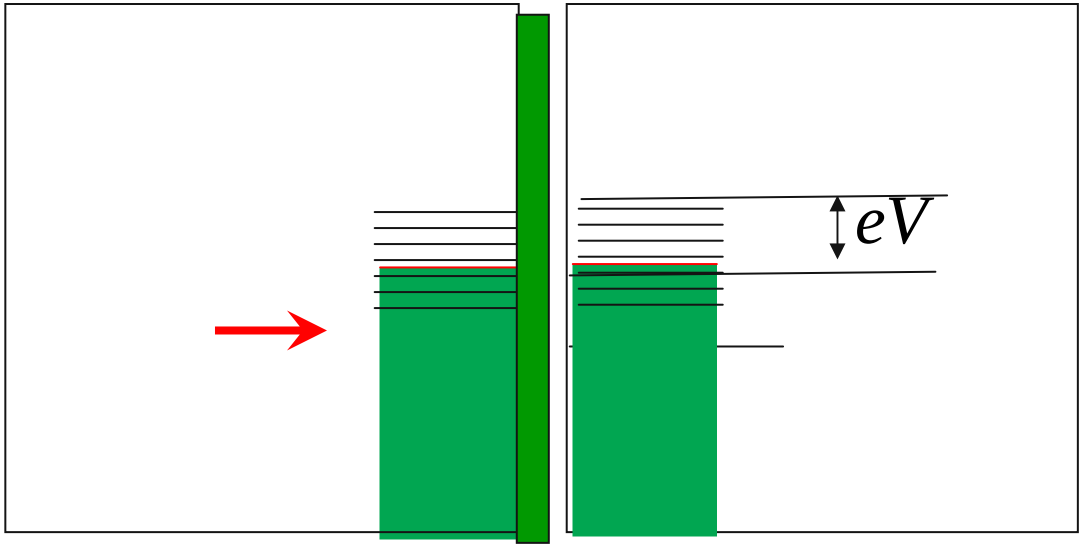
FIG. 2. Resistance of sample 2 measured at zero (crosses) and 100 kOe field (open circles) as a function of the inverse square root of the temperature. Open circles indicate resistance measured with a constant dc current $I = 10^{-6}$ A. Solid squares are zero bias resistances approximated from I-V measurements. Sample 2 room temperature resistance is 800 Ω.

$$\sigma \propto \exp\left[-\left(T_0/T\right)^{1/2}\right] \qquad ???$$

The problem: description of transport in granular metals

Let us recall first the approach to general properties of homogeneously disordered metals

Conductivity of disordered metals



Thouless arguments:

$$\frac{1}{\tau_L} = \frac{2\pi}{\hbar}\langle t^2 \rangle \nu_r(\varepsilon_F)$$

$$N = eV\nu_l(\varepsilon_F) \;\Rightarrow\; I = eN/\tau_L = e^2\nu_l(\varepsilon_F)V/\tau_L$$

$$G = \frac{e^2\nu_l(\varepsilon_F)}{\tau_L} = \frac{2\pi e^2}{\hbar}\langle t^2 \rangle \nu_l(\varepsilon_F)$$

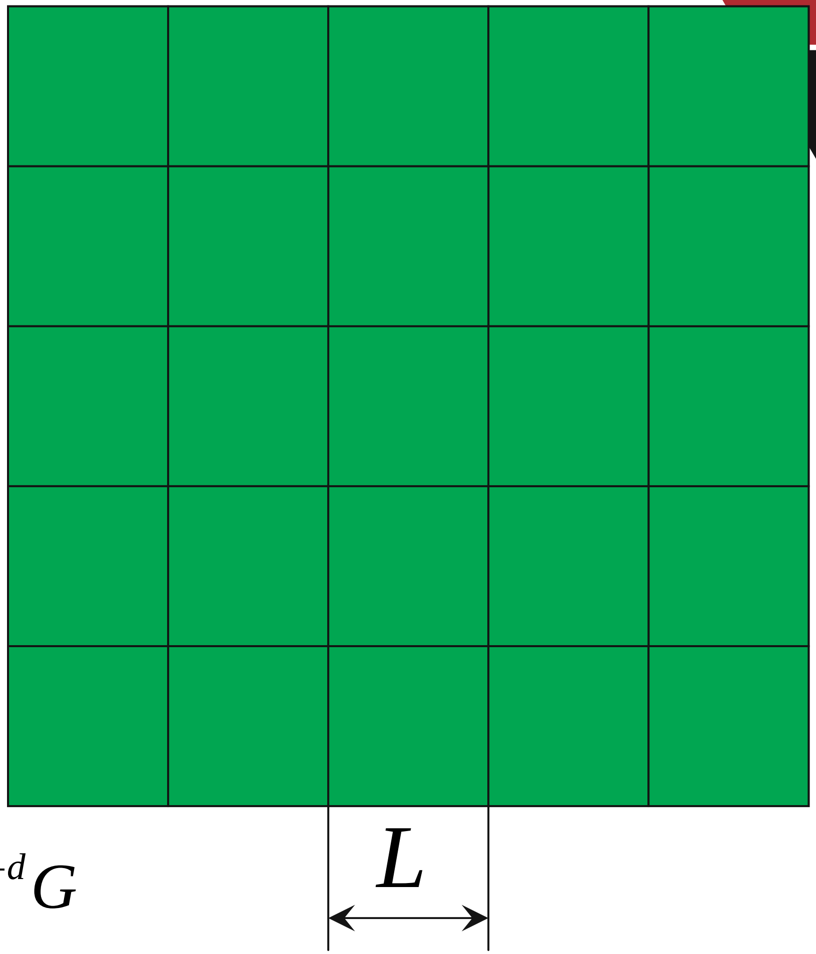
$$G = \frac{e^2 \nu_l(\varepsilon_F)}{\tau_L}$$

Divide the sample into blocks of size *L* and Introduce the energy associated With the lifetime within the block:

$$\Gamma_L = \frac{\pi\hbar}{\tau_L}$$

The electron diffusion over the step *L*:

$$D \sim L^2 / \tau_L$$



Now remembering that

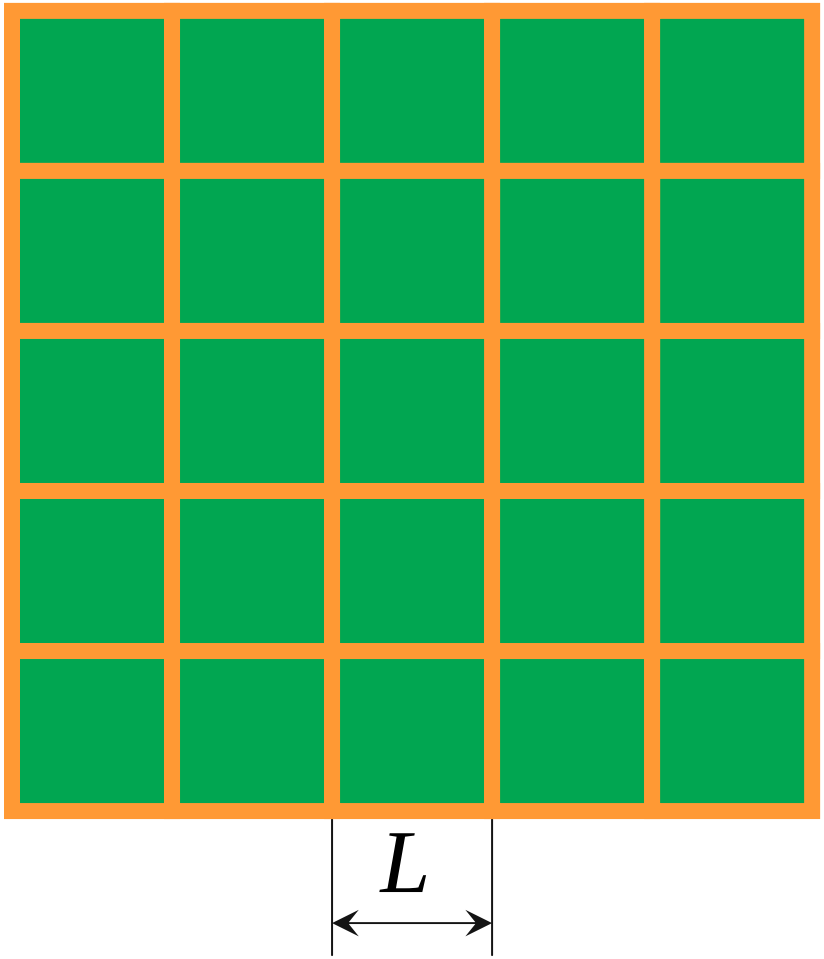
$$\sigma = e^2 \frac{dn}{dE} D, \; \nu_{\mathrm{L}}(E) = L^d \frac{dn}{dE}, \;\; \sigma = L^{2-d} G$$

$$g_L = \frac{\Gamma_L}{\delta_L}$$

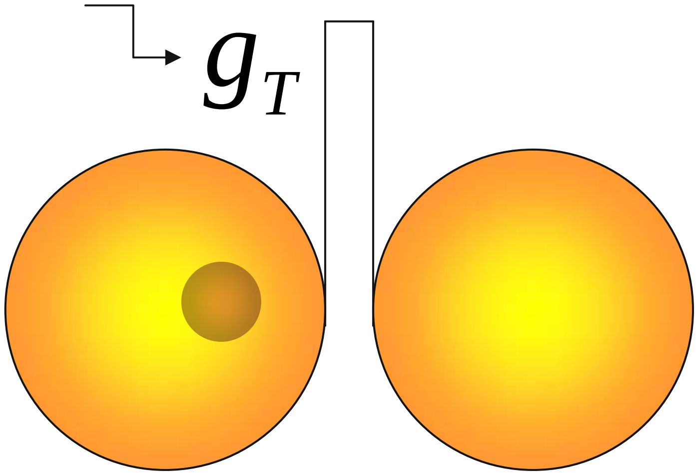
$$g_L = \frac{G}{e^2 / \pi\hbar}$$

$$\nu_L \sim 1/\delta_L$$

Now let us make our separation of the blocks not fictitious, but real



The key characteristic: tunneling conductance

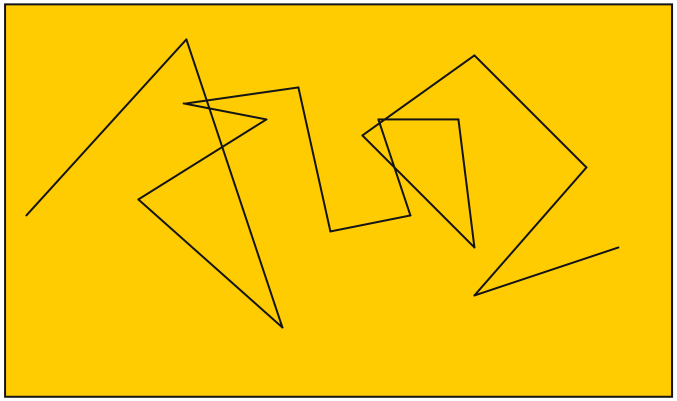


The tunneling conductance is measured in the units of the quantum conductance $e^2/2\hbar$

$g_T \gg 1$ metallic transport properties

$g_T \ll 1$ insulating behavior

Homogeneously disordered metal

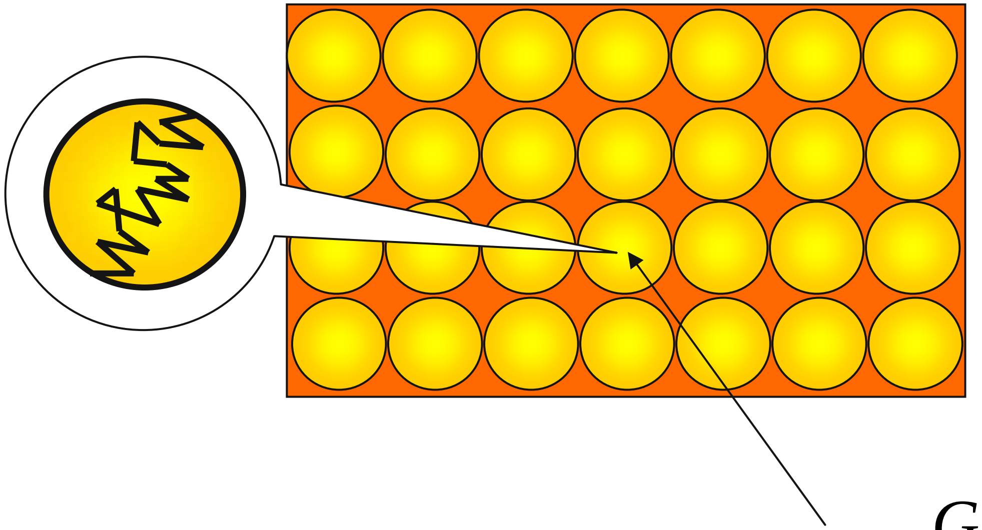


E_F : Fermi energy

$G = \dfrac{I}{V}$: conductance

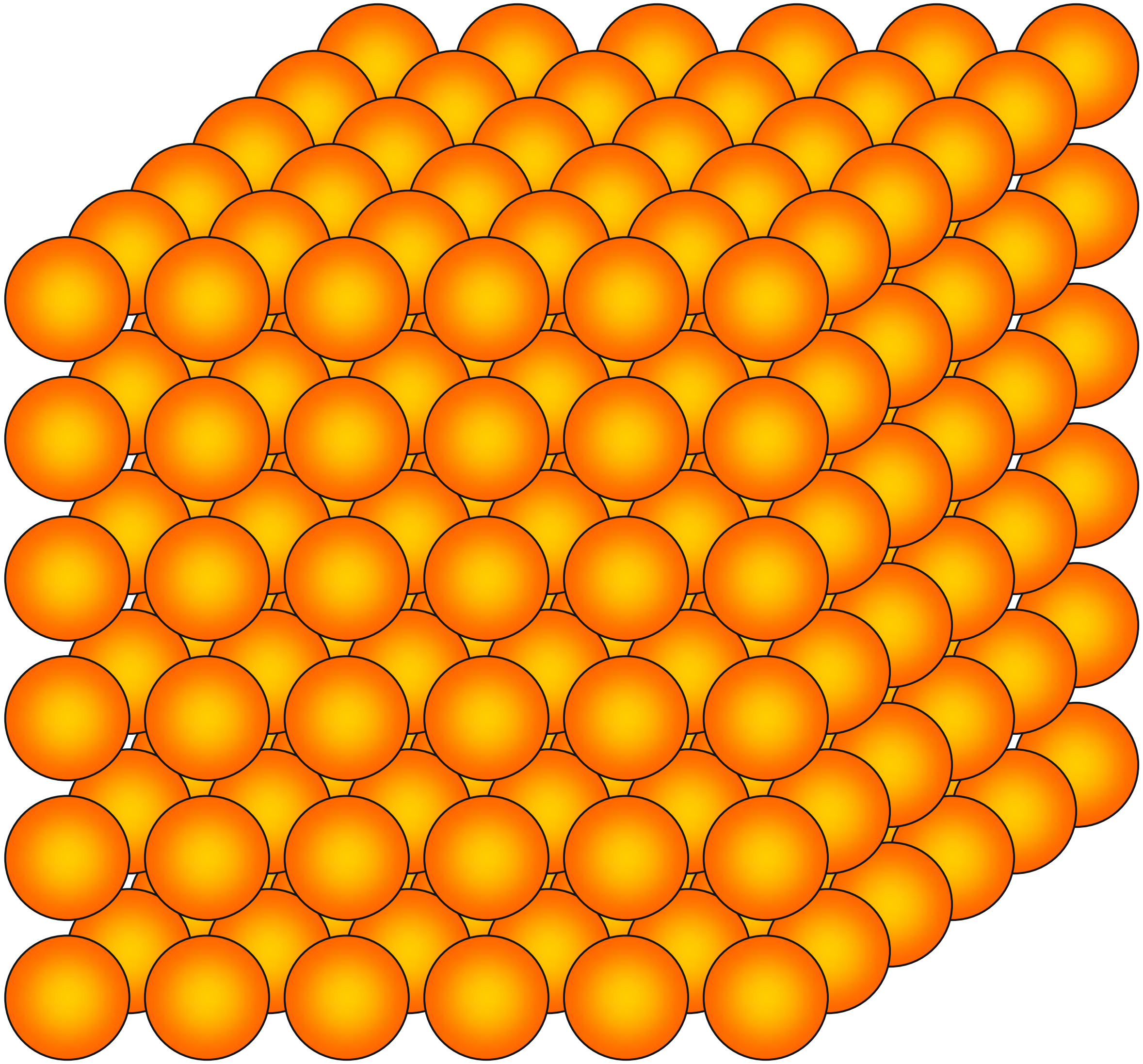
$$g_0 = \frac{G}{e^2 / \pi\hbar}$$

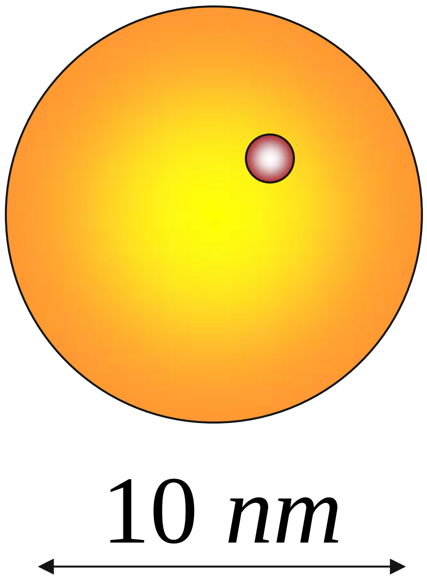
Granular metal

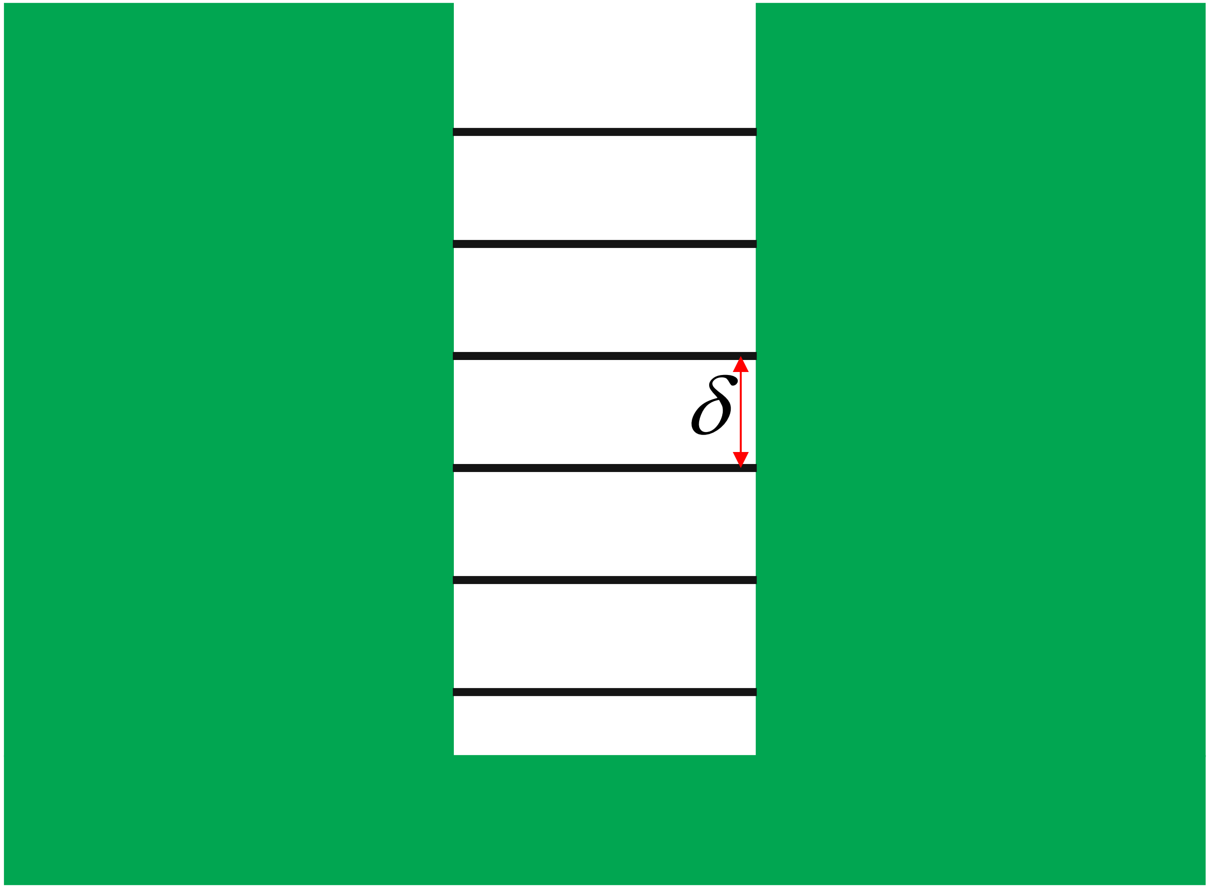


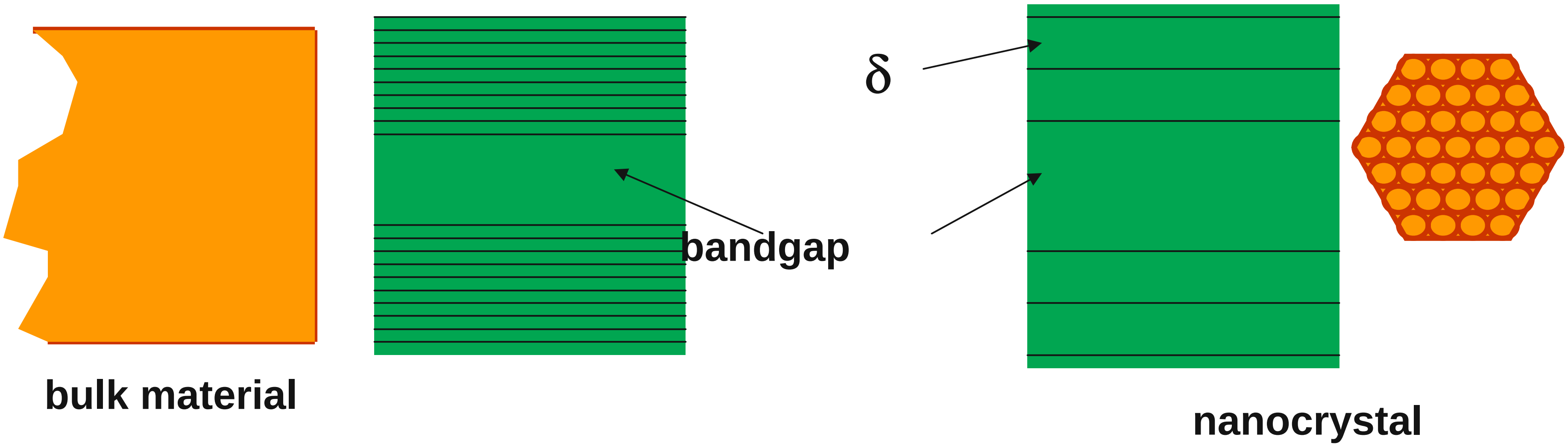
intragranule conductance $g_0 = \dfrac{G}{e^2 / \pi\hbar}$

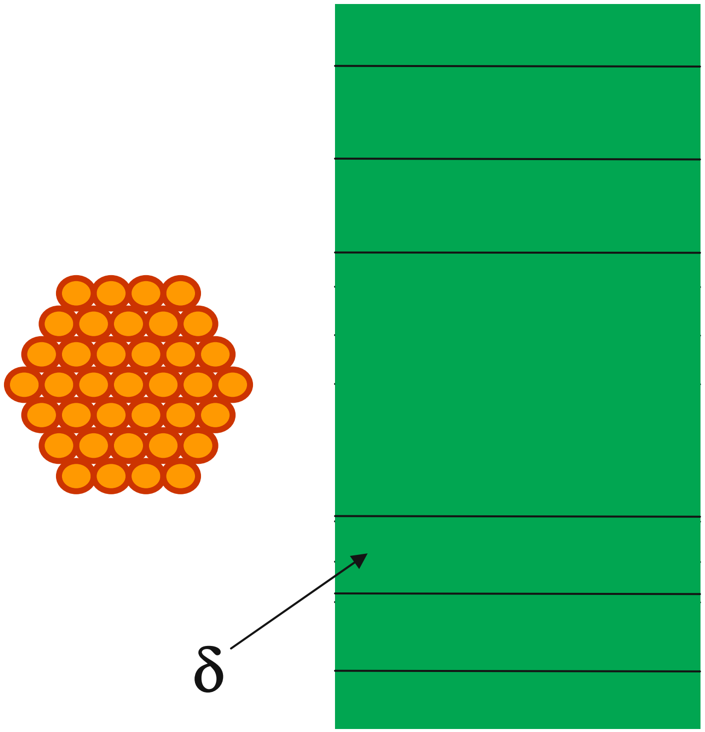
Granular sample




10 nm

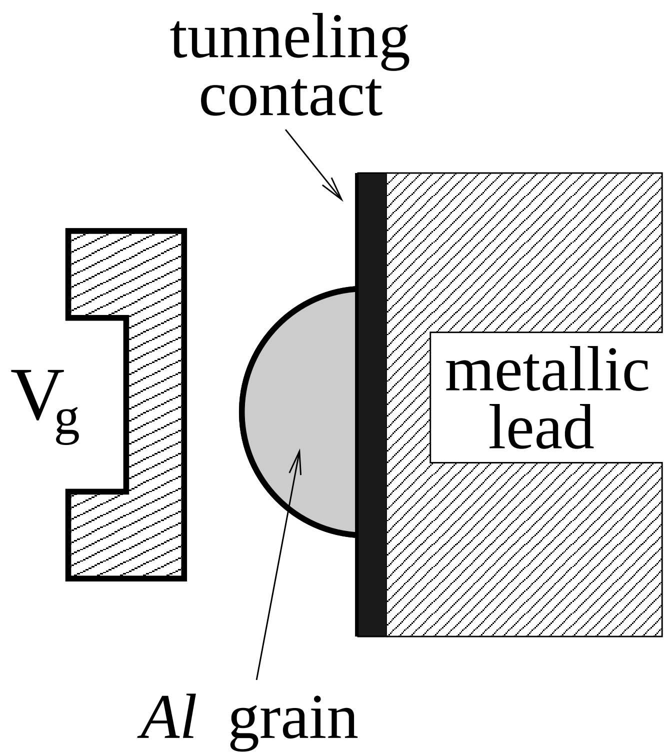

δ

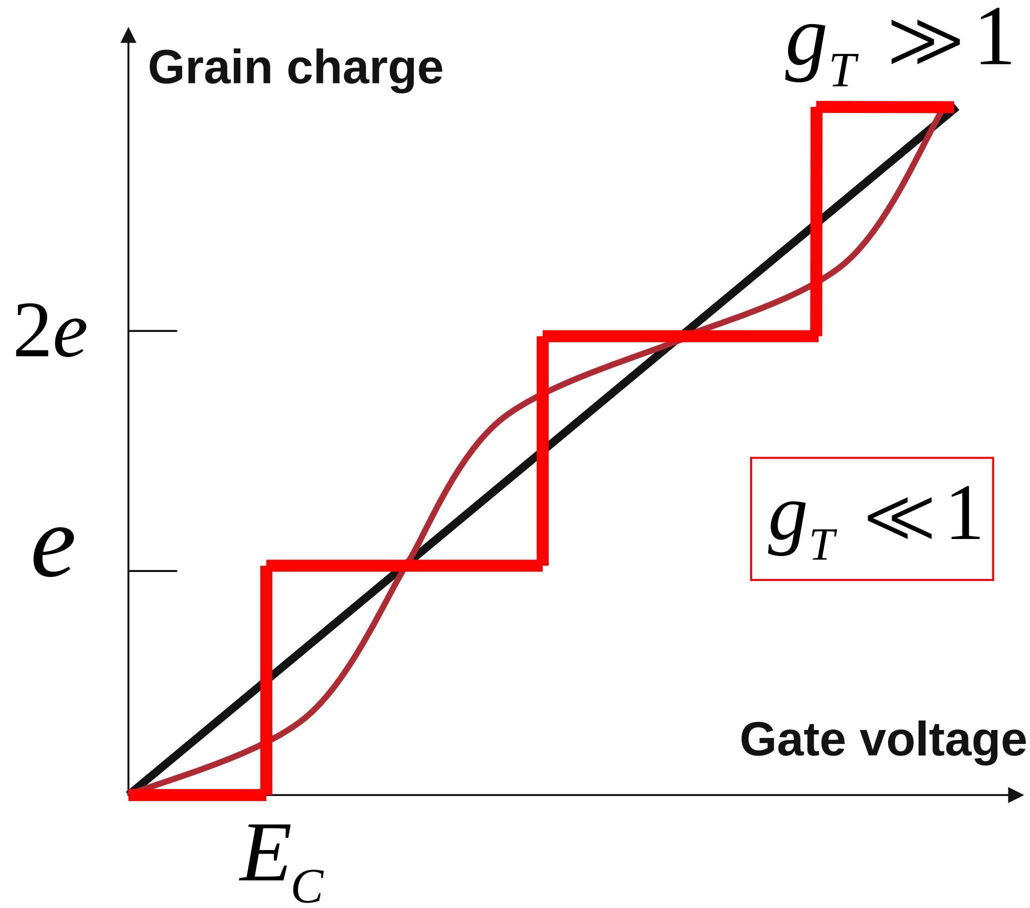

δ
bandgap
bulk material
nanocrystal



nanocrystal

Coulomb blockade



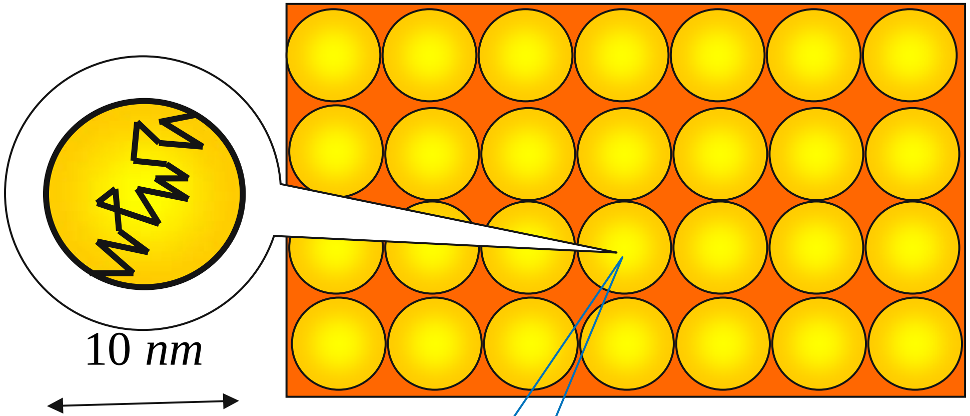


g_T << 1 - Coulomb blockade regime - charge quantization

g_T >> 1 - Charge quantization effects are exponentially small

In a granular system: metal-insulator transition at $g_T \approx 1$

Granular metal

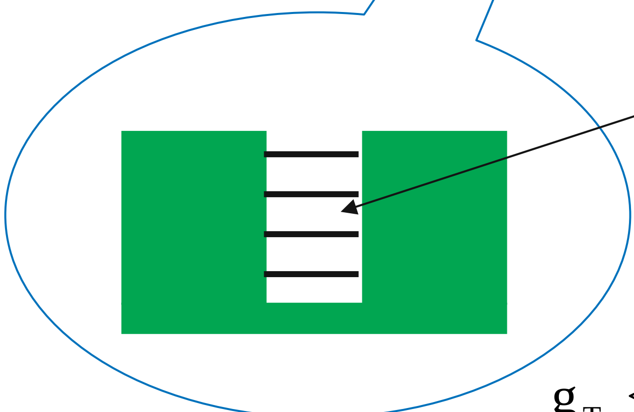


New energy scale:

$$\Gamma = g_T \delta$$

intragranule conductance $g_0 = \dfrac{G}{e^2/\pi\hbar}$

tunneling conductance $g_T \ll g_0$



δ : mean level spacing

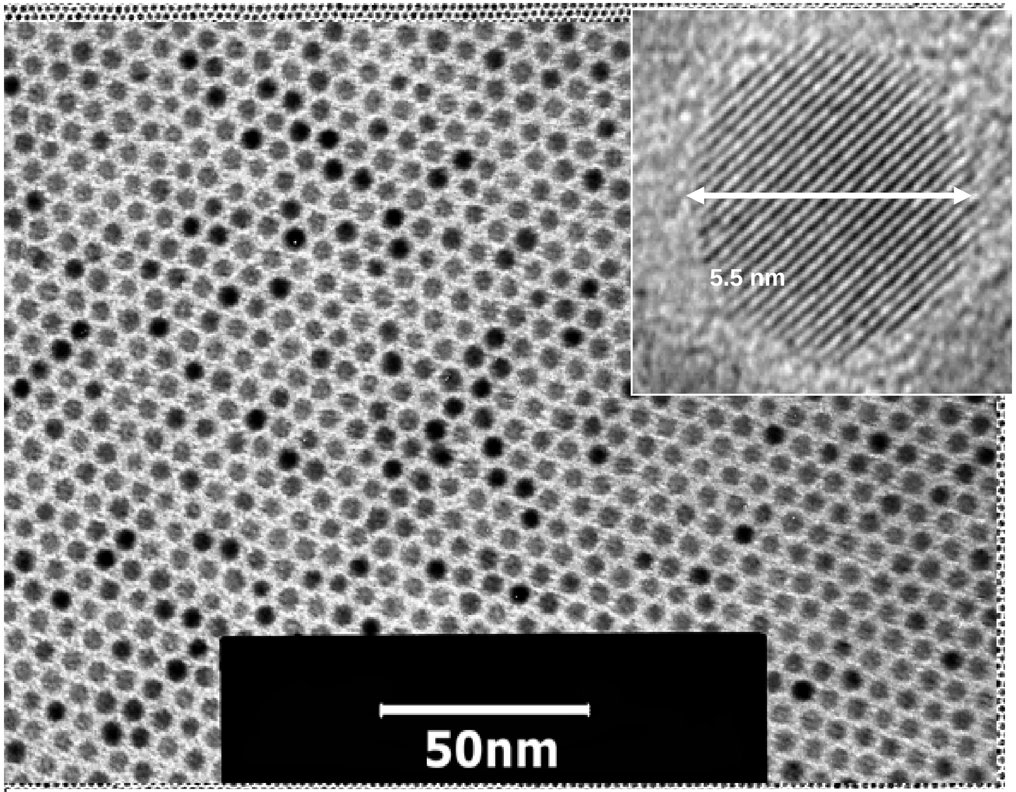
$E_C = e^2/\kappa a$: charging Coulomb energy of a single granule

g_T << 1 - Coulomb blockade regime - charge quantization

g_T >> 1 - Charge quantization effects are exponentially small

In a granular system: metal-insulator transition at $g_T \approx 1$

Granular conductors: a new class of artificial materials with tunable electronic properties controlled at the nanoscale and composed of close-packed granules varying in size from a few to hundred nanometers



X-M. Lin, H. Jaeger, *et al*

The granules are large enough to possess a distinct electronic structure, but sufficiently small to be mesoscopic in nature and exhibit effects of quantized electronic levels of confined electrons.

Conductivity of a granular sample

1. Metallic regime (strong coupling between the grains $g_T >> 1$)

$$D_{eff} \sim a^2 g_T \delta / \hbar = a^2 \Gamma / \hbar = a^2 / \tau_L$$

Interaction time: $\tau_T \sim \hbar / T$

$$L_T = \sqrt{D_{eff} \tau_T} = a\sqrt{\Gamma / T}$$

$$L_T > a: \quad \Gamma > T$$

$$L_T < a: \quad \Gamma < T$$

Usual behavior of a disordered metal

incoherent electron tunneling:
Temperature dependence of conductivity is controlled by electron tunneling between the neighboring grains

High temperature conductivity: $T > \Gamma$

Conductivity: $\sigma = \sigma_0 + \delta\sigma$ $\delta\sigma$ - interaction correction

$$\delta\sigma \approx -(e^2/\pi\hbar)\ln(\tau_\varphi/\tau_C), \quad \tau_C = \hbar/E_C$$

$$\tau_\varphi = g_T\hbar/T: \text{ the dephasing time}$$

Perturbation theory in $1/g_T$ results in the correction to conductivity

$$\frac{\delta\sigma}{\sigma_0} = -\frac{1}{2\pi d g_T}\ln\left(\frac{g_T E_C}{T}\right)$$

Efetov, Tschersich (2003)

Can be understood as the renormalization of the tunneling conductance between the neighboring grains

$$\tilde{g}_T = g_T - \frac{1}{2\pi d}\ln\left(\frac{g_T E_C}{T}\right), \quad \text{valid as long as } \tilde{g}_T \gg 1.$$

Conductivity depends logarithmically on temperature for in all dimensions !

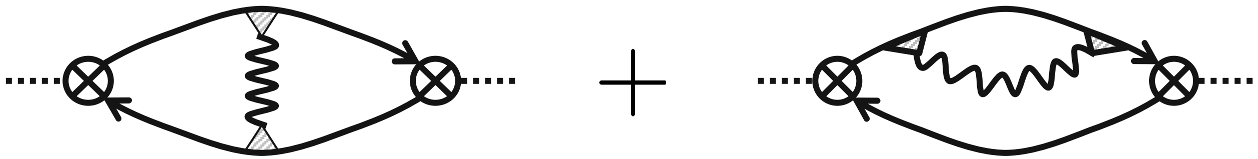
Low temperature conductivity: $T < \Gamma$

$$\sigma = \sigma_0 + \delta\sigma \quad \text{where} \quad \delta\sigma = \delta\sigma_1 + \delta\sigma_2$$

$\delta\sigma_1$ comes from the large energies, $\varepsilon > g_T\delta$, where the granular structure of the array dominates the physics. The fact that this correction is independent of the dimensionality *d* means that the tunneling of electrons is incoherent

$\delta\sigma_2$ is similar to AA correction for homogeneously disordered metals. This is the contribution from the low energies, $\varepsilon < g_T\delta$, the behavior is dominated by coherent electron motion on large scales.

Large energy (small scales) correction

$\delta\sigma_1 =$ 

$$\frac{\delta\sigma_1}{\sigma_0} = -\frac{1}{2\pi d g_T}\,\mathrm{Im}\sum_{\mathbf{q}}\int d\omega\,\gamma(\omega)\varepsilon_{\mathbf{q}}\tilde{V}(\omega,\mathbf{q})$$

$$\gamma(\omega) = \frac{d}{d\omega}\,\omega\coth\frac{\omega}{2T},\ \varepsilon_q = 2g_T\sum\nolimits_{\mathbf{a}}(1-\cos\mathbf{qa})$$

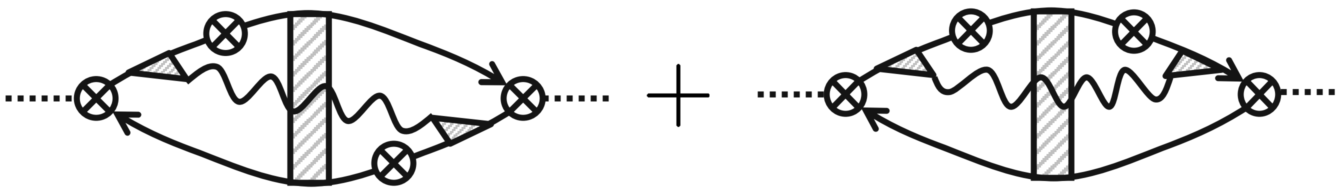
$$\tilde{V}(\omega,\mathbf{q}) = \frac{2E_C(\mathbf{q})}{(\varepsilon_{\mathbf{q}}\delta - i\omega)[4\varepsilon_{\mathbf{q}}E_C(\mathbf{q}) - i\omega]}$$

$$\frac{\delta\sigma_1}{\sigma_0} = -\frac{1}{2\pi d g_T}\ln\left[\frac{g_T E_C}{\max(T, g_T\delta)}\right]$$

Efetov & Tschersich 2003

Large scales correction

$$\delta\sigma_2 =$$

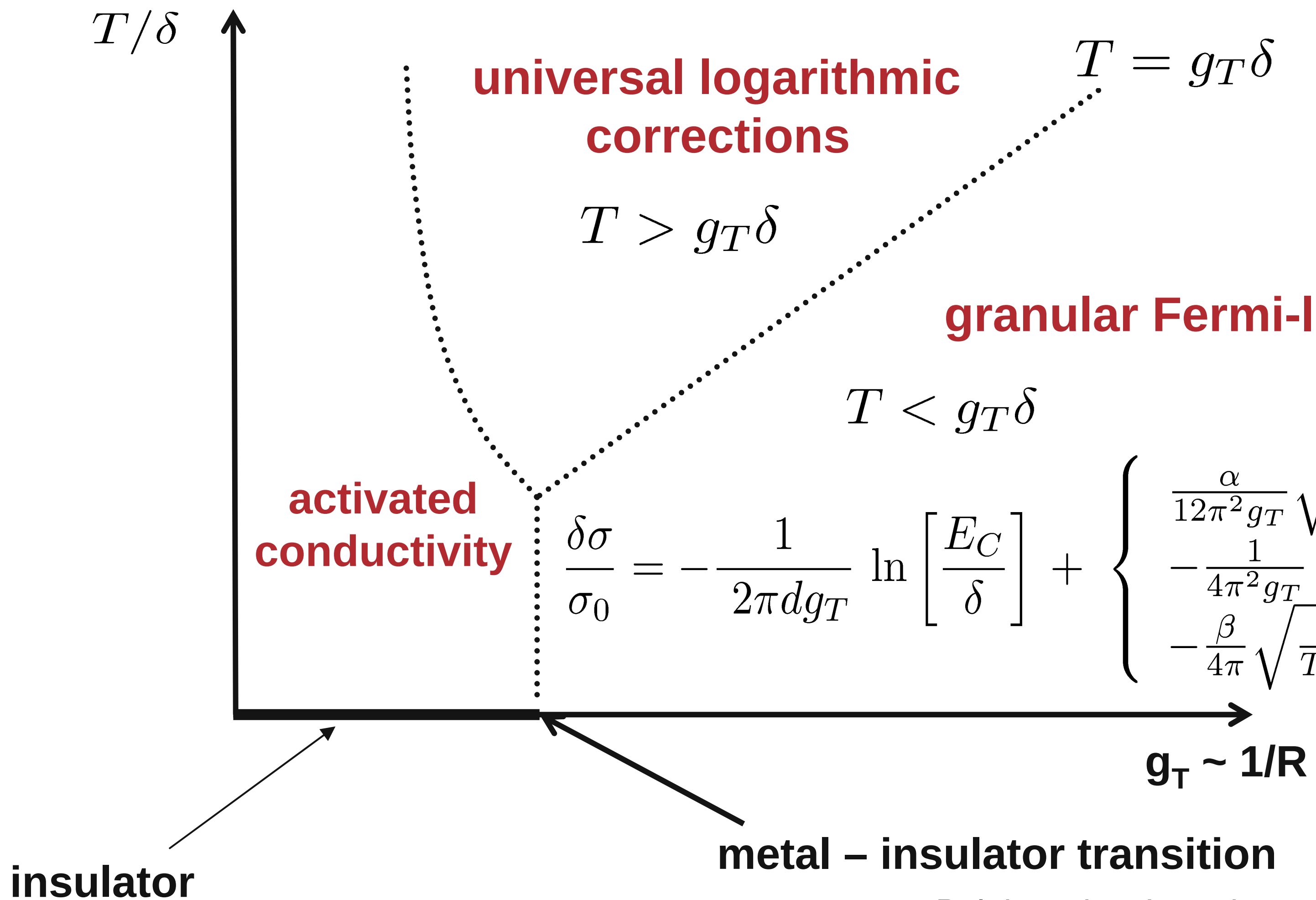


$$\frac{\delta\sigma_2}{\sigma_0} = -\frac{2g_T\delta}{\pi d}\sum_{\mathbf{q}}\int d\omega\,\gamma(\omega)\,\mathrm{Im}\frac{\tilde{V}(\omega,\mathbf{q})\sum_{\mathbf{a}}\sin^2(\mathbf{qa})}{\varepsilon_{\mathbf{q}}\delta - i\omega}$$

$$\frac{\delta\sigma_2}{\sigma_0} = \begin{cases} \frac{\alpha}{12\pi^2 g_T}\sqrt{\frac{T}{g_T\delta}} & D=3 \\ -\frac{1}{4\pi^2 g_T}\ln\frac{g_T\delta}{T} & D=2 \\ -\frac{\beta}{4\pi}\sqrt{\frac{\delta}{Tg_T}} & D=1 \end{cases}$$

$$\frac{\delta\sigma}{\sigma_0} = -\frac{1}{2\pi d g_T} \ln\left[\frac{E_C}{\delta}\right] + \begin{cases} \frac{\alpha}{12\pi^2 g_T}\sqrt{\frac{T}{g_T \delta}}, & D = 3 \\ -\frac{1}{4\pi^2 g_T} \ln \frac{g_T \delta}{T}, & D = 2 \\ -\frac{\beta}{4\pi}\sqrt{\frac{\delta}{T g_T}}, & D = 1 \end{cases}$$

Phase Diagram of Granular Metallic Systems



$$\frac{\delta\sigma}{\sigma_0} = -\frac{1}{2\pi d g_T}\ln\left[\frac{E_C}{\delta}\right] + \begin{cases} \frac{\alpha}{12\pi^2 g_T}\sqrt{\frac{T}{g_T\delta}}, & D=3 \\ -\frac{1}{4\pi^2 g_T}\ln\frac{g_T\delta}{T}, & D=2 \\ -\frac{\beta}{4\pi}\sqrt{\frac{\delta}{T g_T}}, & D=1 \end{cases}$$

δ - mean level spacing

Beloborodov, Lopatin and Vinokur (2004)

2. Insulating phase

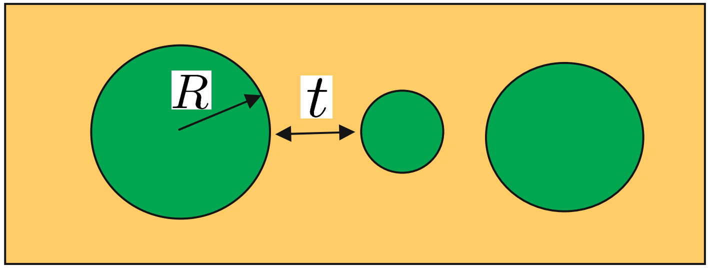
Typical experimental dependence: $\sigma \sim e^{-A/T^p}, \quad p \approx 1/2$

B. Abeles, P. Sheng, M. D. Coutts, and Y. Arie, Adv. Phys. 24, 407 (1975).

Earlier attempts to explain the conductivity temperature dependence were based on

ASCA phenomenological model:

Thickness of the insulating layer between two grains is proportional to grain sizes $t \sim R$



Coulomb energy $E_c \sim e^2/R$

Tunneling probability $P \sim e^{-2t/L}$ L – localization length of the insulating layer

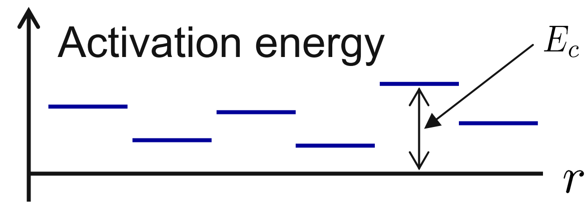
Optimization of $e^{-e^2/RT-2t/L}$ under constraint $R \sim t$

results in **p=1/2** dependence

ASCA model **?????????** **M. Pollak, C. Adkins (1992), R. Zhang, B.I. Shklovskii PRB (2004)**

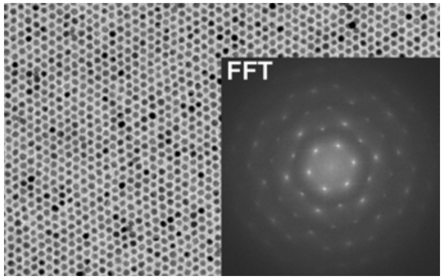
1. Capacitance disorder cannot remove the Coulomb gap completely

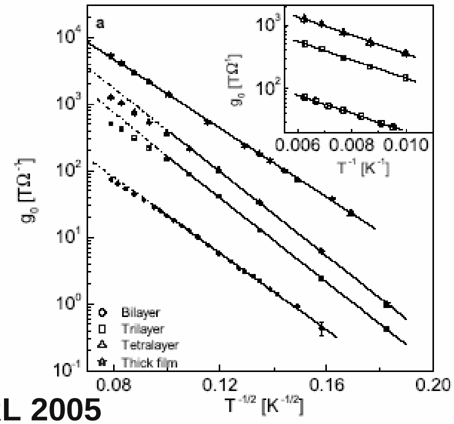
→ ASCA model cannot explain the observed behavior at low temperature



2. Recent experiments showed the p=1/2 law for periodic arrays.

   2d array of gold particles of size ~ 5.5 nm.
   Particle sizes are controlled within 5% accuracy.





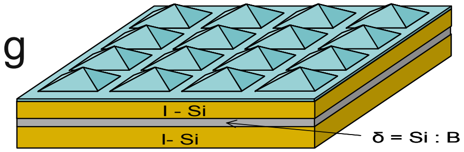
Parthasarathy, X.-M. Lin, K. Elteto, T. F. Rosenbaum, H. M. Jaeger PRL 2004

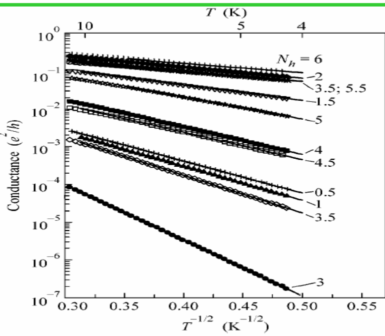
D. Yu, C. Wang, B. L. Wehrenberg, P. Guyot-Sionnest PRL 2004

T.B. Tran, et al, PRL 2005

ES law was also observed in the nanocrystal arrays of semiconducting quantum dots

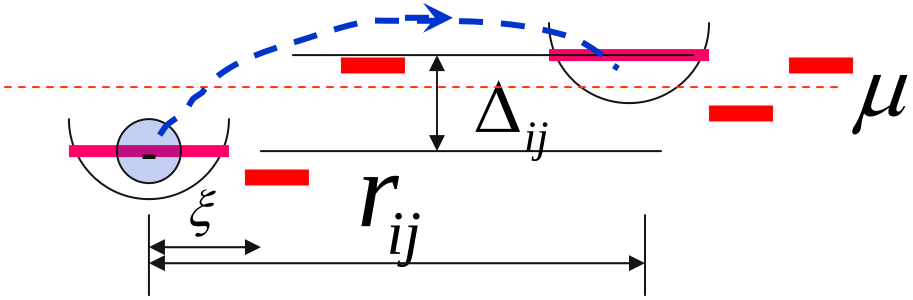
Yakimov, et al, JETP Lett. 2003





Hopping conductivity.

Optimization of tunneling probability: variable range hopping

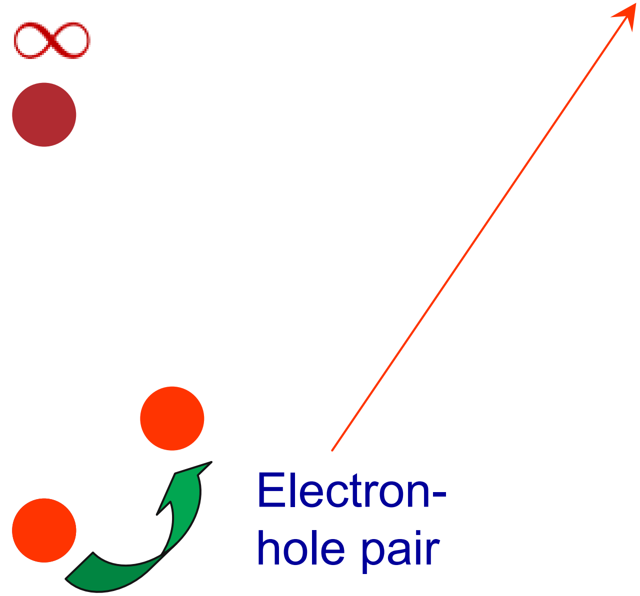


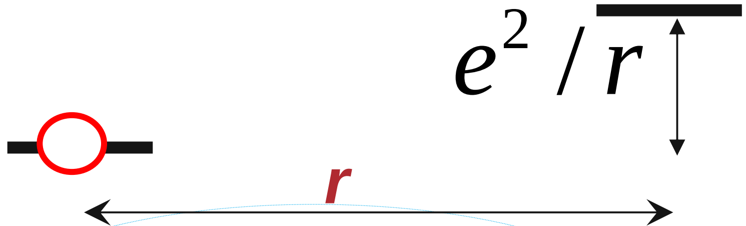
$$\Gamma_{ij}^{0} = \gamma_{ij}^{(0)} \exp\left(-2r_{ij} / \xi\right) \exp\left(-\varepsilon_{ij} / T\right)$$

In the presence of Coulomb interaction the site energies are renormalized due to interactions with the surrounding sites.

Denoting the renormalized energies as $\tilde{\varepsilon}_i$, let us estimate the energy cost for an electron transfer from a filled, i, to an empty donor, j:

$$\Delta_i^j = \tilde{\varepsilon}_j - \tilde{\varepsilon}_i - e^2/\kappa r_{ij}$$





$$\sigma \propto \exp(-r/\xi)\ \exp(-e^2/\kappa r T)$$

Tunneling probability

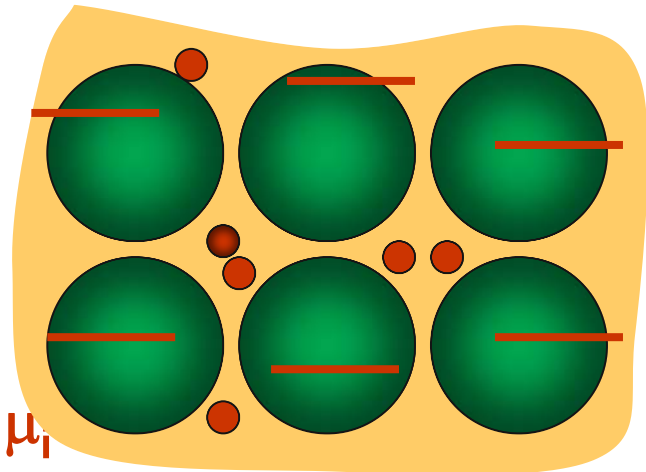
Probability to overcome Coulomb barrier

Optimizing with respect to *r*, one finds:

$$\sigma \propto \exp[-(T_{ES}/T)^{1/2}]$$

$$T_{ES} = \frac{e^2}{\kappa \xi}$$

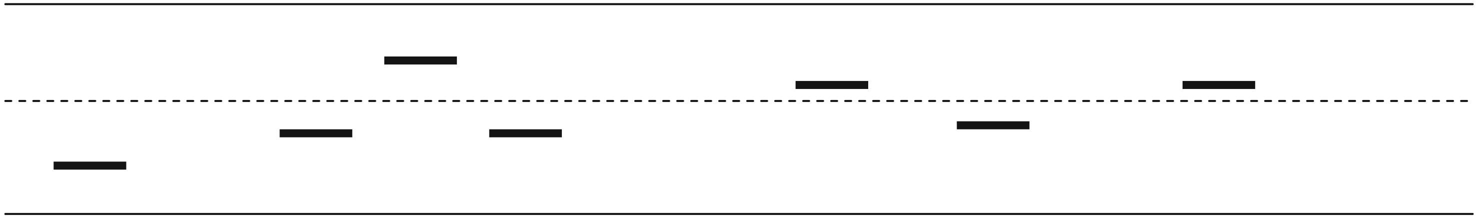
Coulomb interactions and hopping conductivity in granular arrays



$$H = \sum_i \mu_i n_i + \sum_{ij} n_i\, E^c_{ij} n_j$$

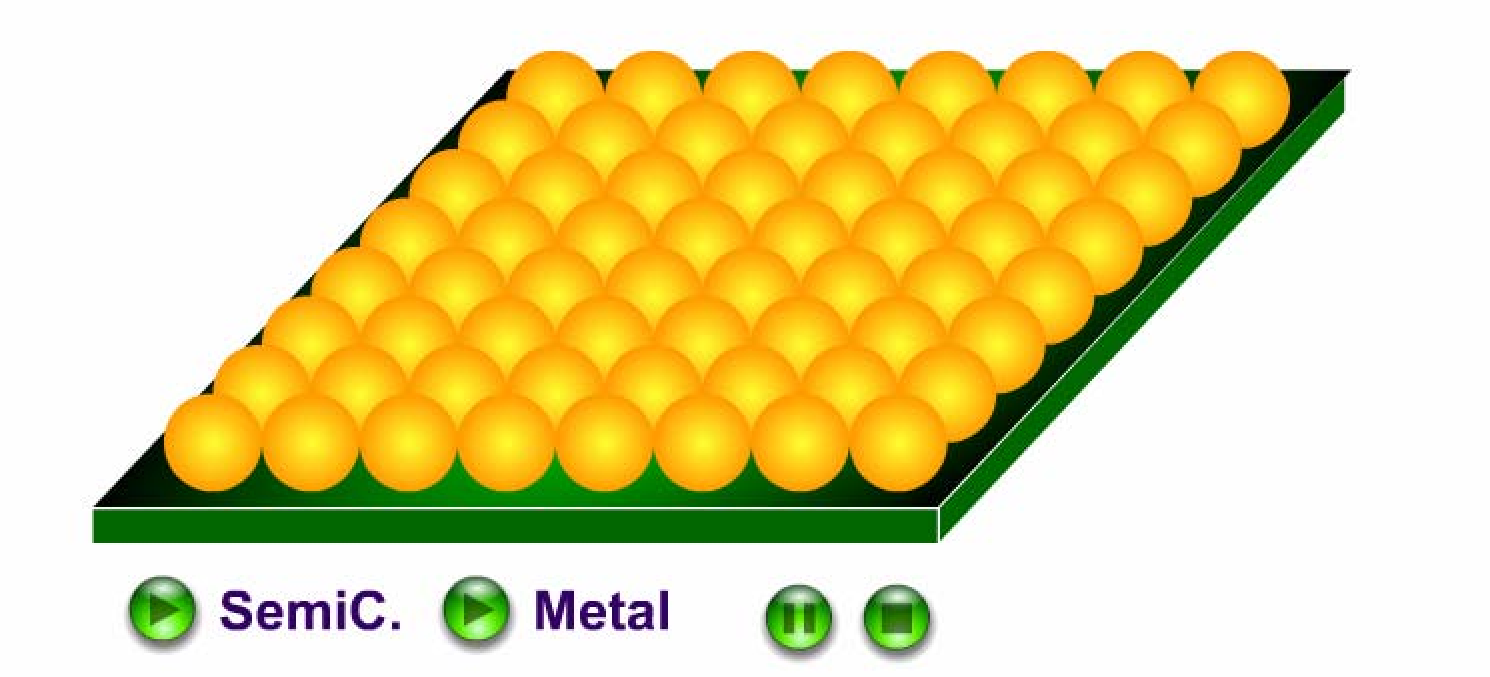
$$E_i^{\pm} = E_i^c \pm \mu_i$$

This looks exactly like the impurity levels in doped semiconductors:

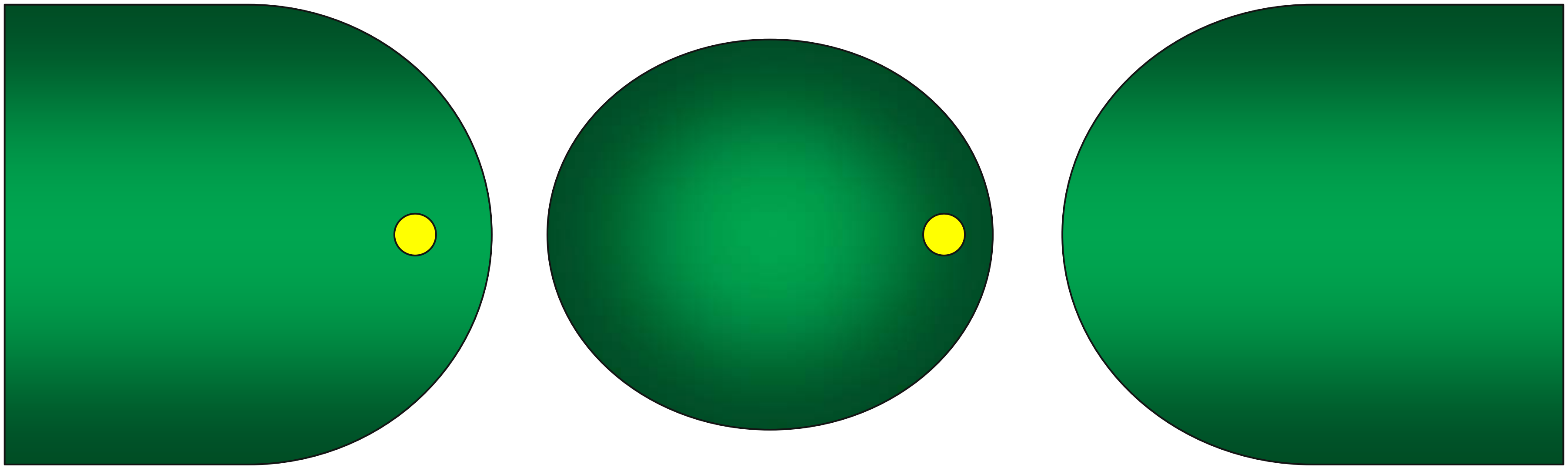


VRH?

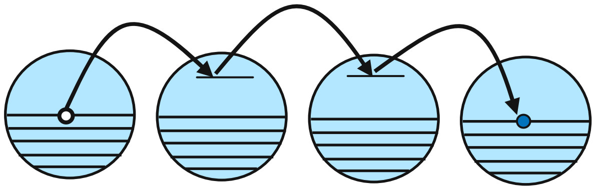
The puzzle of tunneling



COTUNNELING

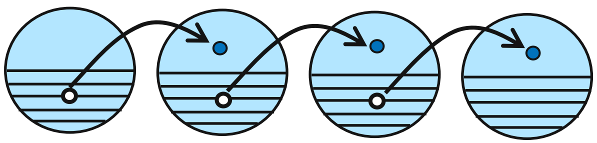


tunneling via virtual states of intermediate grains



Elastic cotunneling mechanism

$$T < \sqrt{E_0^c \delta}$$



Inelastic cotunneling mechanism

$$T > \sqrt{E_0^c \delta}$$

Cotunneling allows for charge transport through several junctions at a time by cooperative electron motion.

At low temperature the sequential tunneling is exponentially suppressed by the Coulomb blockade.
In this case, a higher-order tunneling process transferring electron charge coherently through two junctions can take place. The excess electron charge at the grain exists only virtually.

$$\Gamma_{i\to f} = \frac{2\pi}{\hbar}\left|\sum_{\psi}\frac{\langle i|\mathcal{H}_{\text{int}}|\psi\rangle\langle\psi|\mathcal{H}_{\text{int}}|i\rangle}{E_\psi - E_i}\right|^2 \delta(E_i - E_f)$$

1. There are 2 channels which add coherently
2. The leads have macroscopic number of electrons. Therefore, with the overwhelming probability the outgoing electron will come from a different state than the one which the incoming electron occupies → After the process an electron-hole excitation is left in the grain.

D. V. Averin and A. A. Odintsov, Phys. Lett. A 140, 251 (1989)

D. V. Averin and Y. V. Nazarov, Phys. Rev. Lett. 65, 2446 (1990).

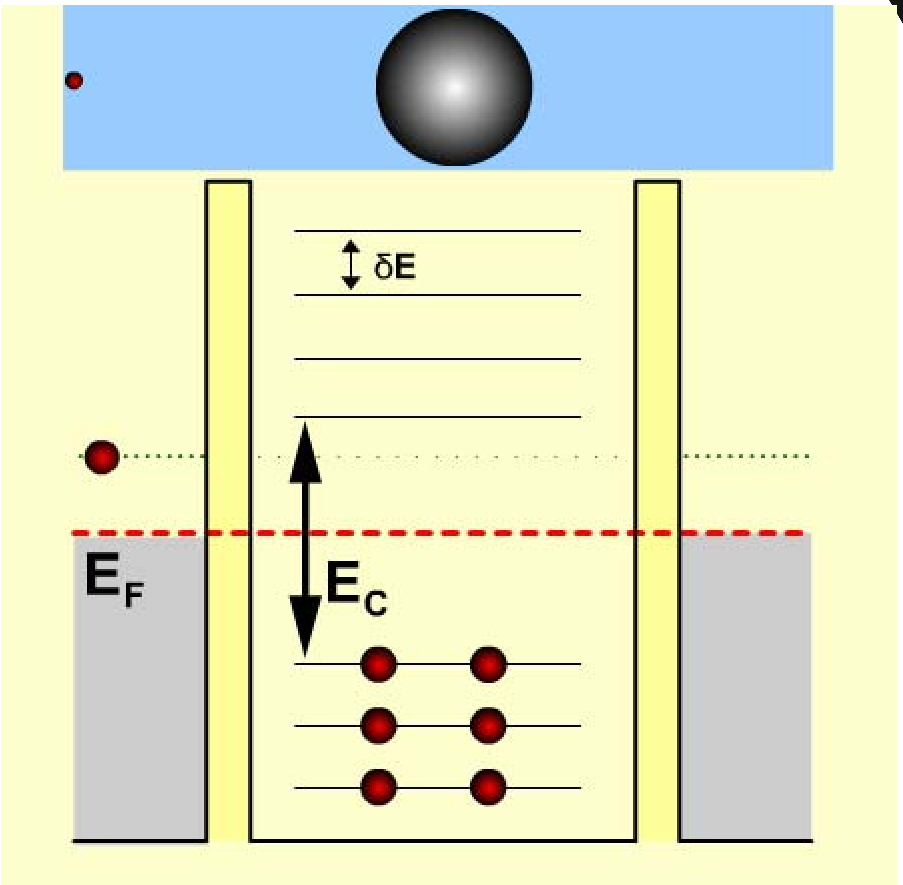
Elastic co-tunneling mechanism

$$P_{el} \sim e^{-2s/\xi_{el}}$$

$$\xi_{el} = \frac{2}{\ln(\bar{E}\,\pi/c\bar{g}\,\bar{\delta})}$$

$$\ln \bar{E} = \frac{1}{N}\sum_{k=1}^{N} \ln \tilde{E}_k$$

$$\tilde{E}_k = 2\left(1/E_k^+ + 1/E_k^-\right)^{-1}$$



$$T < \sqrt{E_0^c \delta}$$

$$\text{hopping probability} \propto \exp[-(2s/\xi_{el}) - (e^2/\tilde{\kappa}Tas)]$$

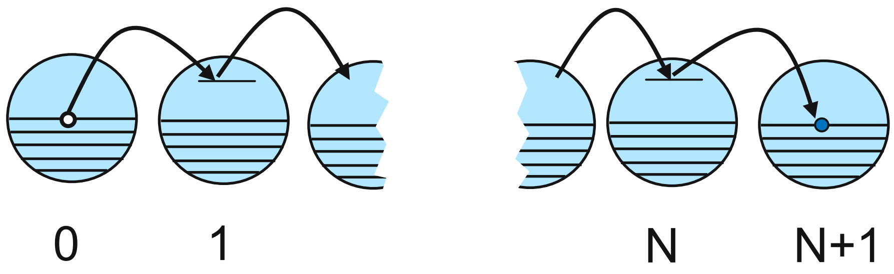
Elastic co-tunneling mechanism

Tunneling through a chain of grains

Model: Short range on-site interaction:

Electron (hole) excitation energies

$$E_i^{\pm} = E_i^c \pm \mu_i$$



Tunneling probability is a product $P_{el} = \delta(\xi_{N+1} - \xi_0)\, g_0 \prod_{k=1}^{N} P_k$

of elementary probabilities $P_k = \frac{g_k\, \delta_k}{\pi \tilde{E}_k}$ $\tilde{E}_k = 2\,\left(1/E_k^+ + 1/E_k^-\right)^{-1}$

g_k - conductance between k-th and k+1 - st grains

In terms of geometrical averages along the tunneling path the probability is

$$P_{el} = \bar{g}^{N+1}\left(\frac{\bar{\delta}}{\pi \bar{E}}\right)^N \delta(\xi_{N+1} - \xi_0) \quad \ln \bar{E} = \frac{1}{N}\sum_{k=1}^N \ln \tilde{E}_k$$

$P_{el} \sim e^{-2s/\xi_{el}}$ s – distance along the path

Effective localization length:

$$\xi_{el} = \frac{2\,a}{\ln(\,\bar{E}\,\pi/\bar{g}\,\bar{\delta})}$$

Hopping conductivity in the regime of elastic cotunneling

Variable range hopping: Phonon assisted tunneling:

(Granular metals: electrons also contribute to the energy relaxation)

$$I \sim e^{-2r/\xi_{el} - \varepsilon/T}$$

$r^d \int_0^\varepsilon d\varepsilon' \nu_g(\varepsilon') \sim 1$

E.S. DOGS $\quad \nu_g(\varepsilon) \sim (\tilde{\kappa}/e^2)^d \, |\varepsilon|^{d-1} \quad \longrightarrow \quad r\,\varepsilon\,\tilde{\kappa}/e^2 \sim 1$

Minimization results in the E.S. law:

$$\sigma \sim e^{-(T_0/T)^{1/2}} \qquad T_0 \sim e^2/\tilde{\kappa}\xi_{el} \qquad \xi_{el} = \frac{2\,a}{\ln(\,\bar{E}\,\pi/\bar{g}\,\bar{\delta})}$$

Hopping distance r within the energy shell ε is given by

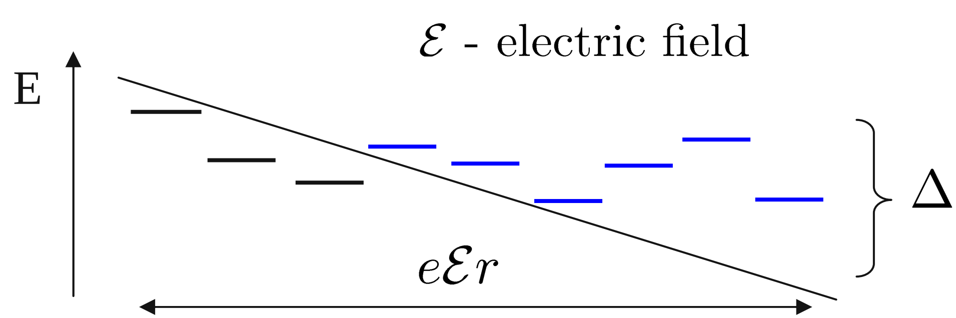
Nonlinear conductivity at strong electric fields

Hopping distance r within the energy shell Δ:

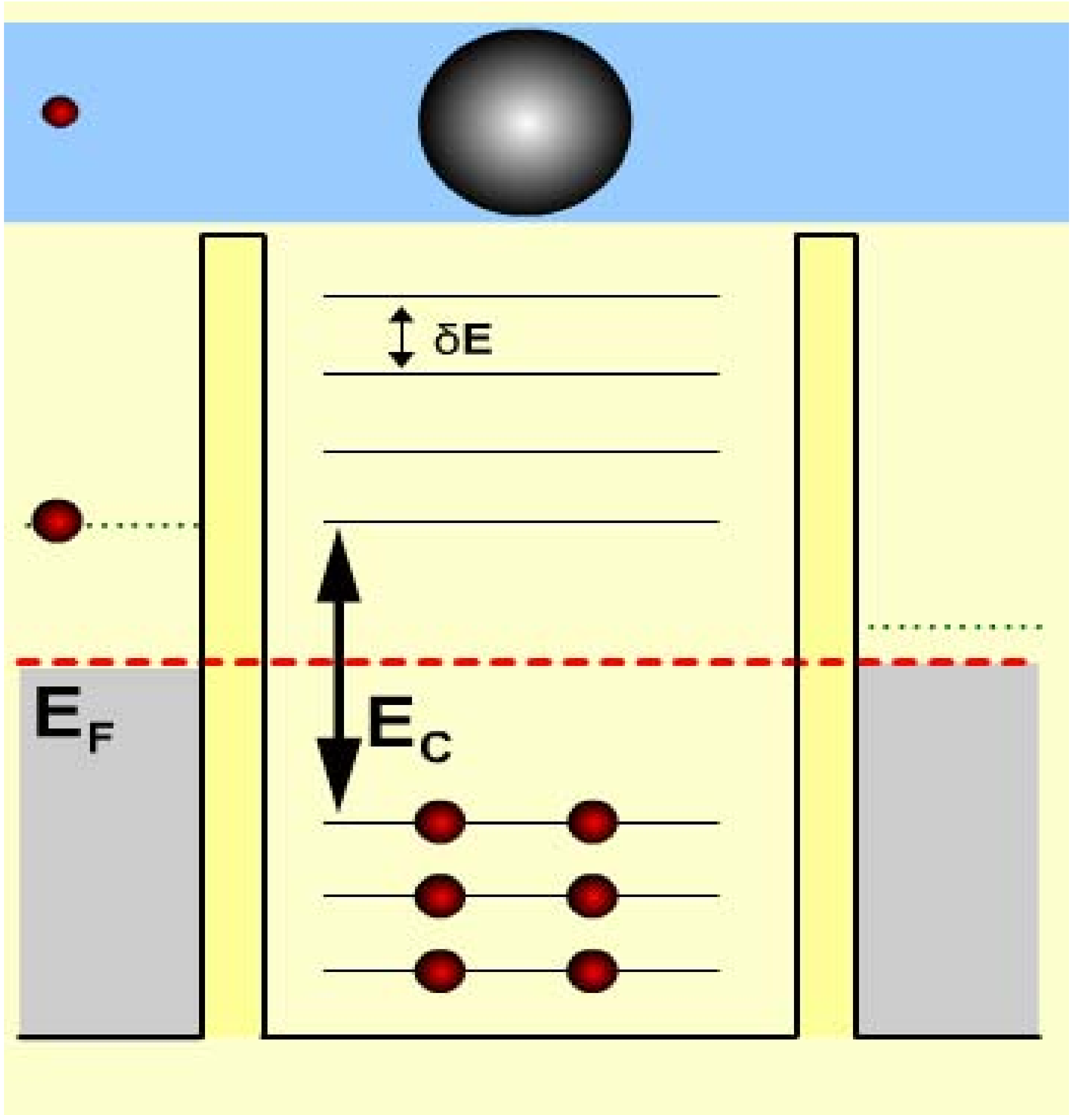
Shklovskii 1973

$$\left.\begin{aligned} e\mathcal{E}r &\sim \Delta \\ r\,\Delta\,\tilde{\kappa}/e^2 &\sim 1 \end{aligned}\right\} \quad r \sim \sqrt{e/\tilde{\kappa}\mathcal{E}}$$

$$j \sim j_0\, e^{-(\mathcal{E}_0/\mathcal{E})^{1/2}}, \qquad \mathcal{E}_0 \sim T_0/e\,\xi_{el}$$

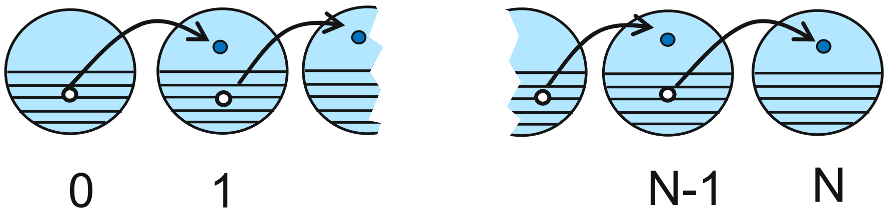


Inelastic cotunneling: single granule



Hopping conductivity: inelastic cotunneling

Hopping through a chain of grains via inelastic cotunneling



$$P_{in} = \frac{1}{4\pi T}\frac{\bar{g}^{N+1}}{\pi^{N+1}}\left[\frac{4\pi T}{\bar{E}}\right]^{2N}\frac{|\Gamma(N+\frac{i\Delta}{2\pi T})|^2}{\Gamma(2N)}e^{-\frac{\Delta}{2T}}$$

$\Delta = \xi_N - \xi_0$ - difference of the energies of initial and final states

Low electric field (linear regime)

Optimization under constraint $$Na\tilde{\kappa}\Delta/e^2 \sim 1, \quad (N \gg 1),$$

results in the ES law:

$$\sigma \sim e^{-(T_0(T)/T)^{1/2}},$$

$$T_0(T) \sim e^2/\tilde{\kappa}\,\xi_{in}(T)$$

$$\xi_{in}(T) = \frac{2\,a}{\ln[\,\bar{E}^2/16\pi T^2\bar{g}\,]}$$

Crossover temperature between elastic and inelastic regimes

$$\xi_{in} = \xi_{el} \longrightarrow T = \sqrt{\delta E_c}$$

Hopping conductivity via inelastic cotunneling: strong fields

Low temperatures T→ 0:

$$P_{in}(T=0) = \frac{2^{2N}\pi}{(2N-1)!}\frac{|\Delta|^{2N-1}}{\bar{E}^{2N}}\left(\frac{\bar{g}}{\pi}\right)^{N+1}$$

Hopping distance can be found as in the case of elastic cotunneling

$$\left.\begin{array}{l} e\mathcal{E}r \sim \Delta \\ r\,\Delta\,\tilde{\kappa}/e^2 \sim 1 \end{array}\right\} \longrightarrow \quad r \sim \sqrt{e/\tilde{\kappa}\mathcal{E}}$$

Using that $N \sim r/a, \quad N \gg 1$

$$j \sim j_0\, e^{-(\mathcal{E}_0/\mathcal{E})^{1/2}},$$

$$\mathcal{E}_0(\mathcal{E}) \sim \frac{e}{\tilde{\kappa}\, a^2}\ \ln^2[\bar{E}^2/e^2\mathcal{E}^2a^2\bar{g}]$$

Applicability:

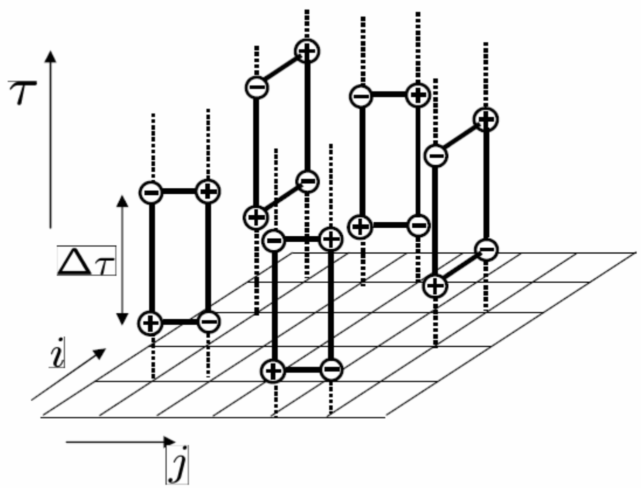
Nonlinear regime: $\mathcal{E}ea \gg T$

Inelastic cotunneling dominates elastic one: $\mathcal{E}ea \gg \sqrt{\delta E_c^0}$

Mapping onto the classical Coulomb gas

Phase action: $S_{AES} = -\frac{1}{2e^2}\sum_{ij}\int d\tau\, \dot\phi_i C_{ij} \dot\phi_j + S_t[\phi],$

$$S_t[\phi] = \frac{1}{2\pi}\sum_{<ij>} g_{ij} \int_{-\infty}^{+\infty} d\tau_1\, d\tau_2 \frac{e^{i\phi_{ij}(\tau_1) - i\phi_{ij}(\tau_2)}}{(\tau_1-\tau_2)^2}$$



Mapping onto the Coulomb gas:

Expand partition function in S_t :

$$Z = Z_0 \sum_{N=1}^{\infty} \langle\, S_t^N[\phi]\, \rangle \,/\, N!\,.$$

Averaging over the Coulomb action gives rise to the classical charges:

$$\left\langle e^{i\sum_n \phi_i(\tau_n) e_n} \right\rangle = e^{-U^c}, \qquad U^c = \frac{1}{2}\sum_{n_1,n_2} E^c_{i_{n_1} i_{n_2}} |\tau_{n_1} - \tau_{n_2}|\, e_{n_1} e_{n_2}$$

1d Coulomb interaction along the "time" axis.

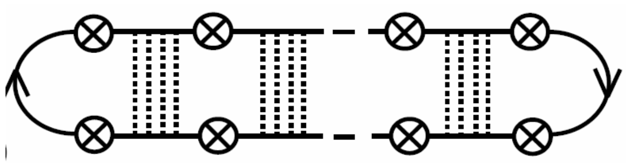
Internal interaction:

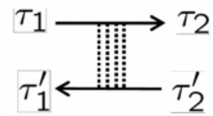
$$\frac{g}{2\pi}\,\frac{1}{(\tau_n - \tau_n')^2} = e^{-\mathcal{U}_n^q(\tau_n - \tau_n')}, \qquad \mathcal{U}_n^q = \ln[2\pi(\Delta\tau_n)^2/g]$$

Total classical energy (internal +Coulomb parts) : $U^q = \sum_{n=1}^{N} \mathcal{U}_n^q + U^c_{4N}$

Elastic cotunneling is beyond the AES approach

Feynman diagram

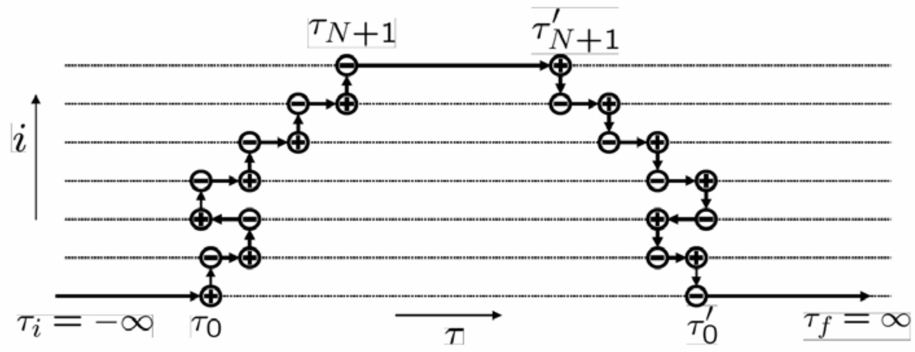




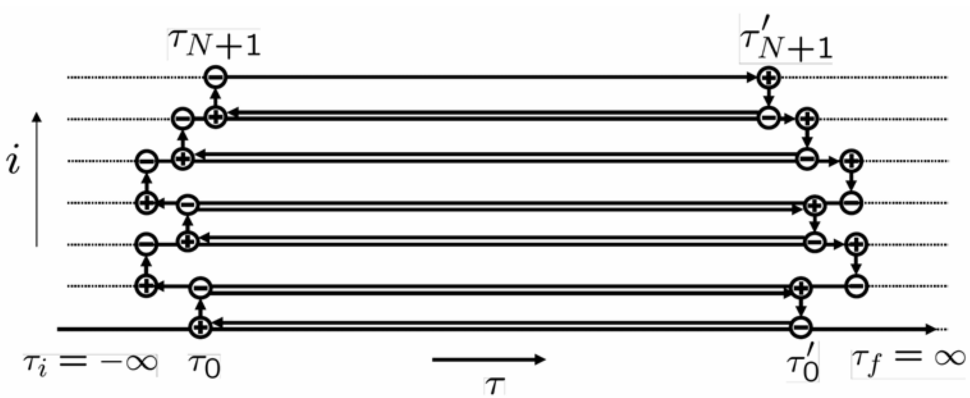
$$= \delta^{-1} \frac{\theta\left[(\tau_1 - \tau_2)(\tau_2' - \tau_1')\right]}{|\tau_1 - \tau_2| + |\tau_1' - \tau_2'|}$$

maps onto the Coulomb gas:

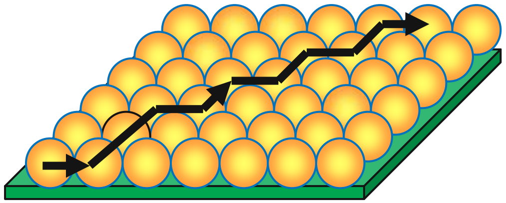
The electron world line representing the probability of elastic cotunneling from the 0th to the N+1st grain.

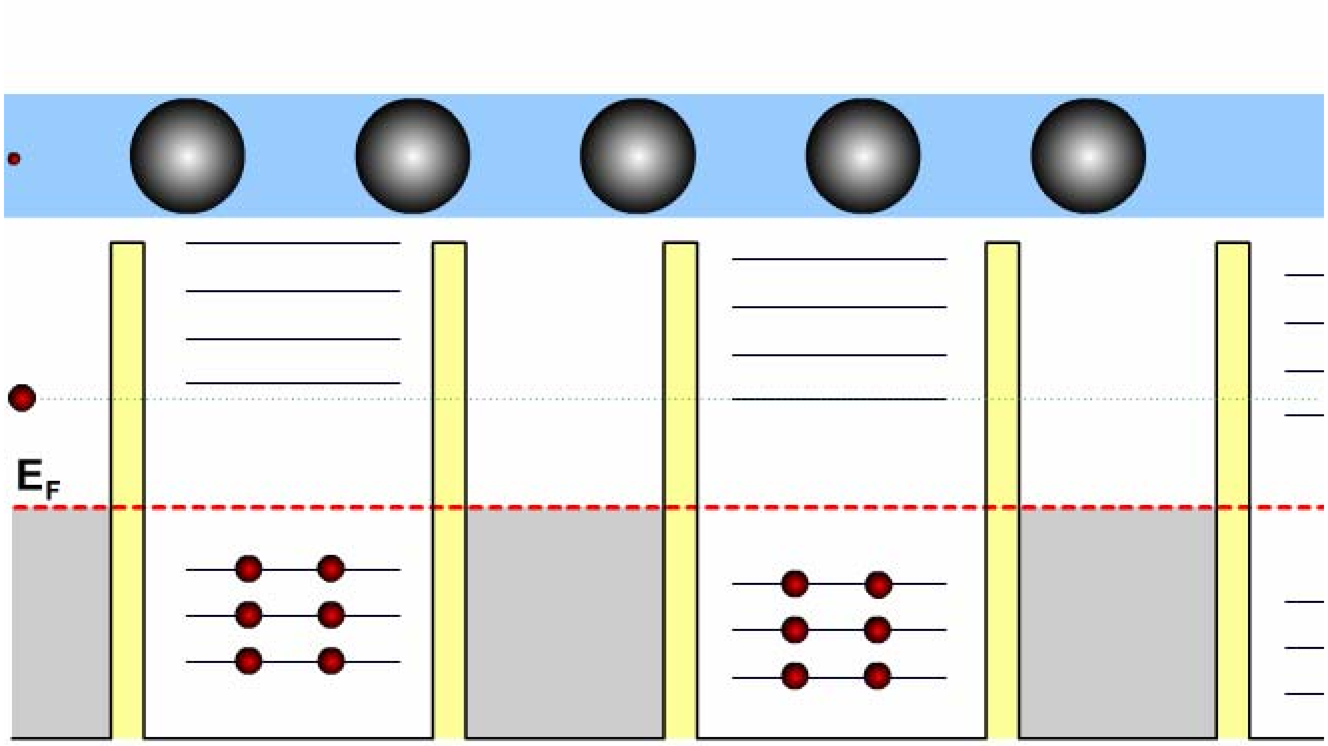


Inelastic cotunneling is described within the AES approximation



General process: tunneling through the granule chain

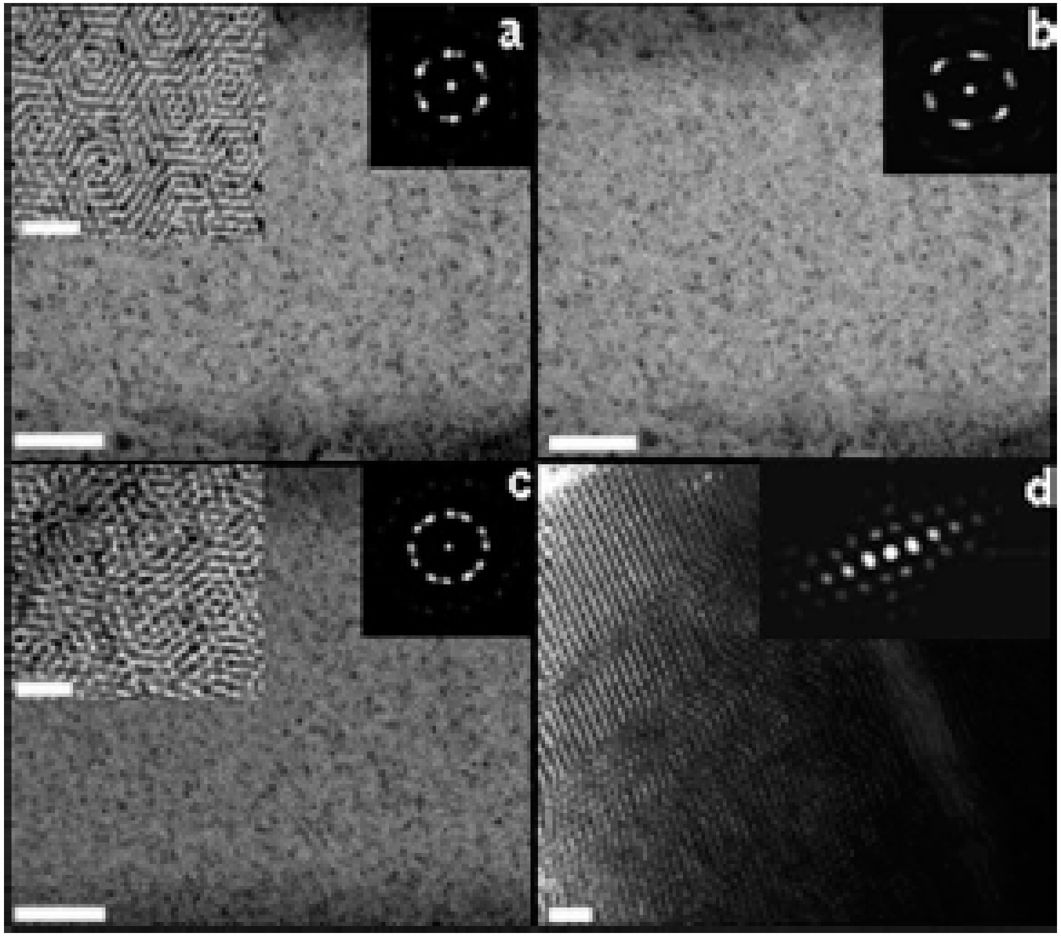


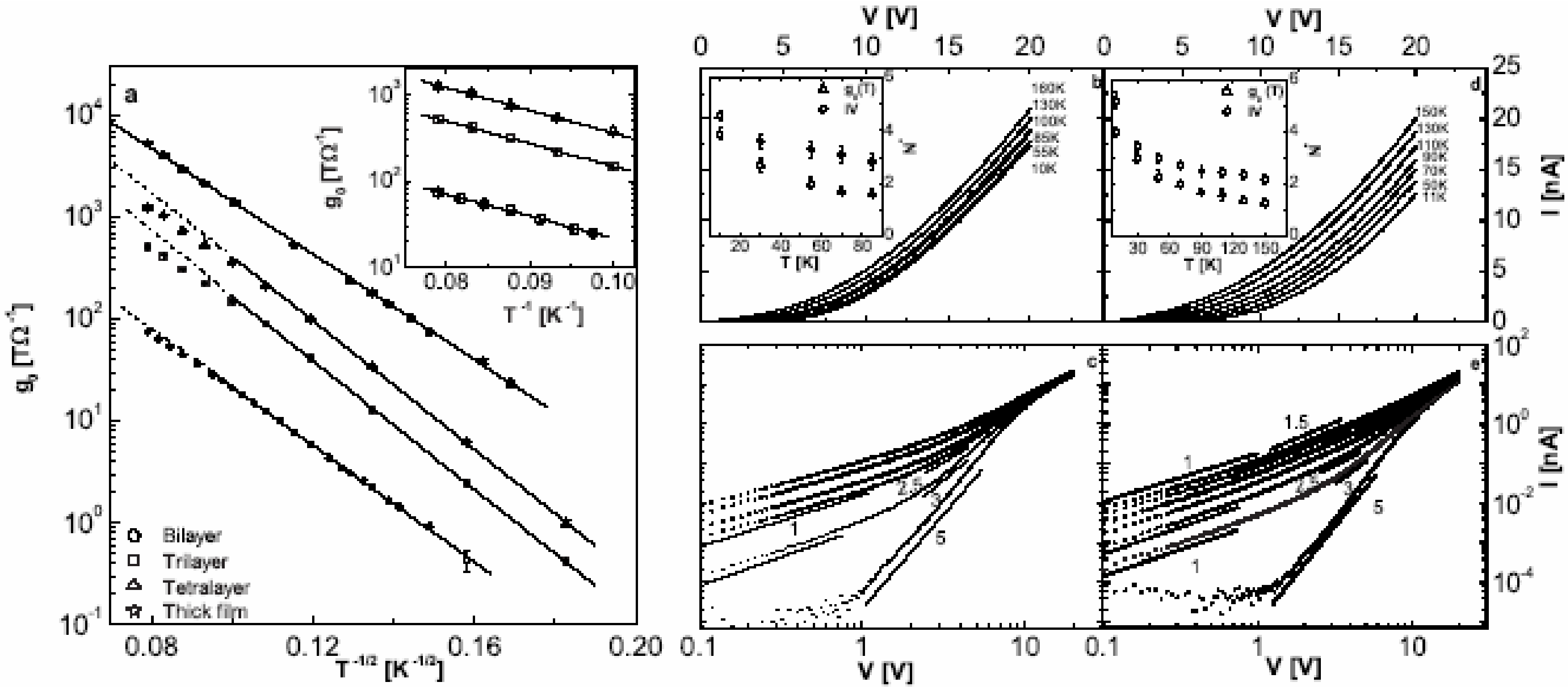


Experiment

H. Jaeger's group
at the University of Chicago

Transmission electron micrographs showing the region between the in-plane electrodes for a) bilayers, b) trilayers, c)tetralayers and d) thick films. The darker regions on top and bottom of a-c are the electrodes. The insets on the right sides are diffraction patterns computed by fast fourier transform. The insets on the left sides of panels a&c are the zoomed-in images. The scale bars correspond to 200nm (a-c) and 40nm (d, all insets).





a) Zero-bias conductance g_0 versus inverse temperature $T^{-1/2}$ for multilayer and thick film data. Inset: For the high-temperature range, where the multilayer data in the main panel deviate form the dotted lines, g_0 has been replotted as a function of T^{-1}, indicating Arrhenius behavior from 100-160K(b-e) Evolution of the I – V characteristics with temperature for bilayers (b,c) and thick films (d,e). Panels (c) and (e) are log-log plots of the data shown in the plots above them. The straight solid lines are guide to the eye, indicating power law behavior. Insets to b&d: Temperature dependence of the hopping distance N obtained from g_0(T) and the I-V power-law exponents obtained from panels c&e in the range 2V < V < 7V .

Determining $T_{0\,l}$ in the Efros-Shklovski hopping formula we find (at T = 10K), N = 4 for multilayers and N= 4-5 for the thick films.

$$I_{in} \sim V \left[\frac{g_T}{h/e^2}\right]^j \left[\frac{(eV)^2 + (k_B T)^2}{E_C^2}\right]^{j-1}$$

N=4 or j=3 implies $I \sim V^5$. This is what was experimentally observed

Unresolved questions (*or what we do not know*):

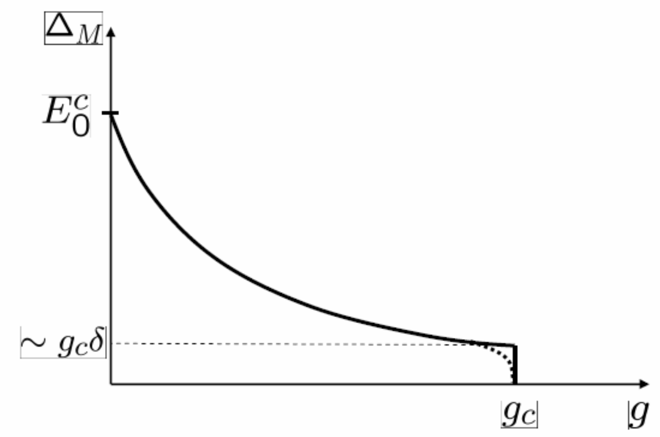
- R~logT behavior
- Transport in the arrays of quantum dots (semiconductors)
- Theory of Hall effect
- Phononless (hopping) transport
- ……………………

1. Periodic granular array:

Activation conductivity $\sigma \sim e^{-\Delta_M(g)/T}$

T=0: Insulator to metal transition occurs at

$$g_c = \frac{1}{\pi z} \ln(E_c/g\delta)$$



2. Arrays with electrostatic disorder:

Linear regime:

$$\sigma \sim e^{-(T_0/T)^{1/2}}, \quad T_0 \sim e^2/\tilde{\kappa}\,\xi, \quad \xi \sim \begin{cases} \dfrac{2\,a}{\ln(\,\bar{E}\,\pi/\bar{g}\,\bar{\delta})}, & T < \sqrt{E_0^c\,\delta} \quad \text{elastic} \\ \dfrac{2\,a}{\ln[\,\bar{E}^2/16\pi T^2 \bar{g}\,]}, & T > \sqrt{E_0^c\,\delta} \quad \text{inelastic} \end{cases}$$

Nonlinear regime:

$$j \sim j_0\, e^{-(\mathcal{E}_0/\mathcal{E})^{1/2}}, \qquad \mathcal{E}_0 \sim \begin{cases} \frac{e}{\tilde{\kappa}\,a^2}\ \ln^2[\bar{E}\pi/\delta\bar{g}], & \mathcal{E}ea < \sqrt{\delta E_c^0} \quad \text{elastic} \\ \frac{e}{\tilde{\kappa}\,a^2}\ \ln^2[\bar{E}^2/e^2\mathcal{E}^2a^2\bar{g}], & \mathcal{E}ea > \sqrt{\delta E_c^0} \quad \text{inelastic} \end{cases}$$

Hopping conductivity in granular superconductors

Weak coupling regime g_T<<1

Simplest Model:
Coulomb energy +
Josephson couplings

$$H = 4\sum_{ij} \hat{n}_i E_{ij} \hat{n}_j + \sum_{ij} J_{ij} \cos(\phi_i - \phi_j)$$

$\phi - \text{Cooper pair phase}$ $\hat{n} = -i\partial/\partial\phi$ - Cooper pair number operator

Anderson-Abeles criterion for the global superconductivity development $J > E_c$

Applicable at g<<1 - as long as the charge renormalization effects may be neglected.

We assume g<<1 g Δ << Ec → No global coherence

In the presence of electrostatic disorder the transport can be mediated by Cooper pair hopping

Questions:

1. **Electron** hopping transport in the presence of the superconducting gap - ?
2. Multiple cotunneling in the presence of the gap - ?
3. Conductivity temperature dependence - ?

Density of states: Single grain model

Energy of a single superconducting grain.

$$E = n^2 E_c - Vn + P(n+p)\,\Delta$$

Charging energy — Random potential — Parity term.

n - number of excessive electrons, counted with the respect to N_0 – the total charge of the neutral state

Total number of electrons $N=N_0 + n$.

Parity effect: A state with odd number of electrons has an extra energy Δ.

Matveev, Averin Nazarov 1992

Parity function: $$P(n) = \begin{cases} 0 - \text{even n} \\ 1 - \text{odd n} \end{cases}$$

p=1,2 – the total charge N of the neutral state can be even or odd.

The energy E is at minimum with respect to electron number n for a given potential V.

→ **Excitation energies to add or remove an electron are all positive for any V !**

Same for the pair creation and annihilation processes.

Single grain model: Electron occupation number

Electron excitation energy:

$$\mathcal{E}_\pm \equiv E(n+1) \pm E(n)$$
$$= (\pm 2n+1)E_c \mp V + \Delta \cos\pi(n+p)$$

Occupation number n jumps $n \to n \pm 1$ at $\mathcal{E}_\pm = 0$

Pair excitation energy:

$$\mathcal{E}_{2\pm} \equiv E(n+2) - E(n)$$
$$= 4(\pm n+1)E_c \mp 2V$$

Occupation number n jumps $n \to n \pm 2$ at $\mathcal{E}_{2\pm} = 0$

Depending on the mutual relation of E_C and Δ one finds qualitatively different dependences n(V):

1. Charging energy dominates $E_C>\Delta$.

Modified Coulomb staircase:

Occupation number changes by one

n→n+1 at

$$V_n = (2n+1)E_c + \Delta\cos\pi(n+p)$$

Pair excitations are gapped for all V!

$$\mathcal{E}_{2\pm} > 2(E_c - \Delta)$$

2. Parity term dominates $\Delta>E_C$.

Usual staircase but for Cooper pairs:

Occupation number changes by two

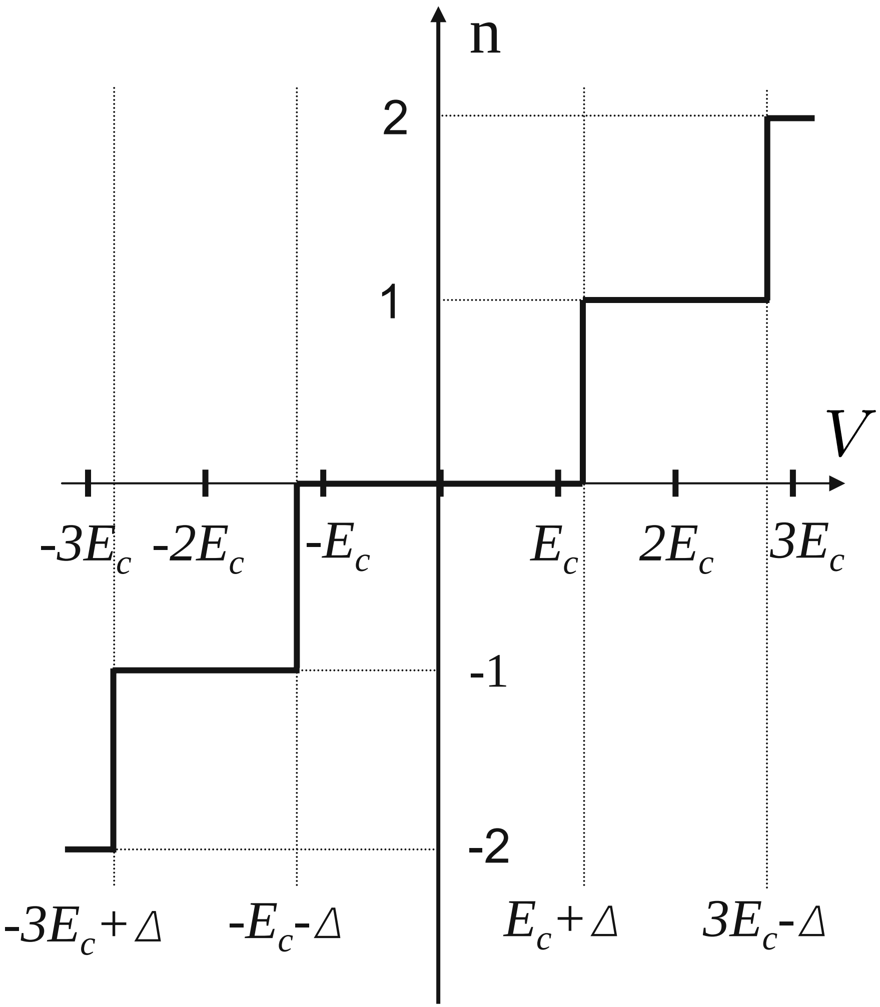
n→n+2 at $V_n = E_c(4n+2)$

Electron excitations are gapped for all V!

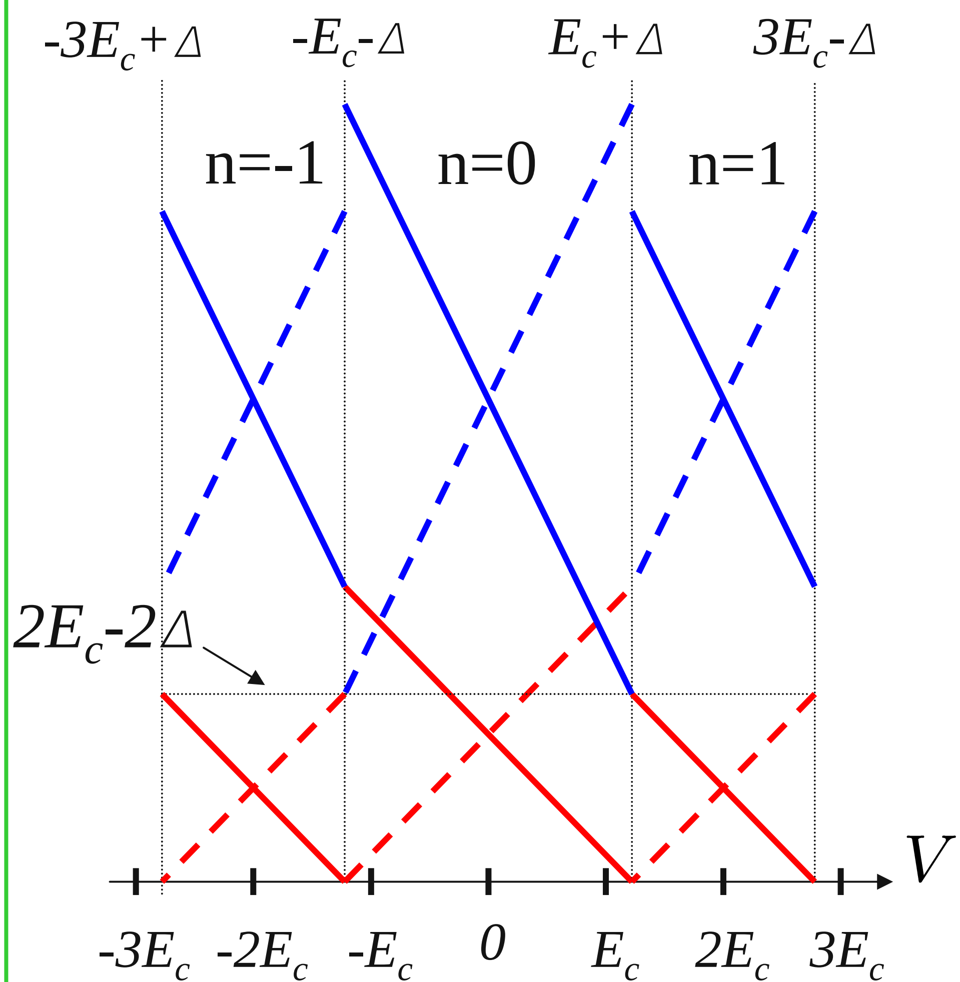
$$\mathcal{E}_\pm > \Delta - E_c$$

Electron number and excitation energies as functions of V, $E_C>\Delta$

Electron occupation number: n(V)



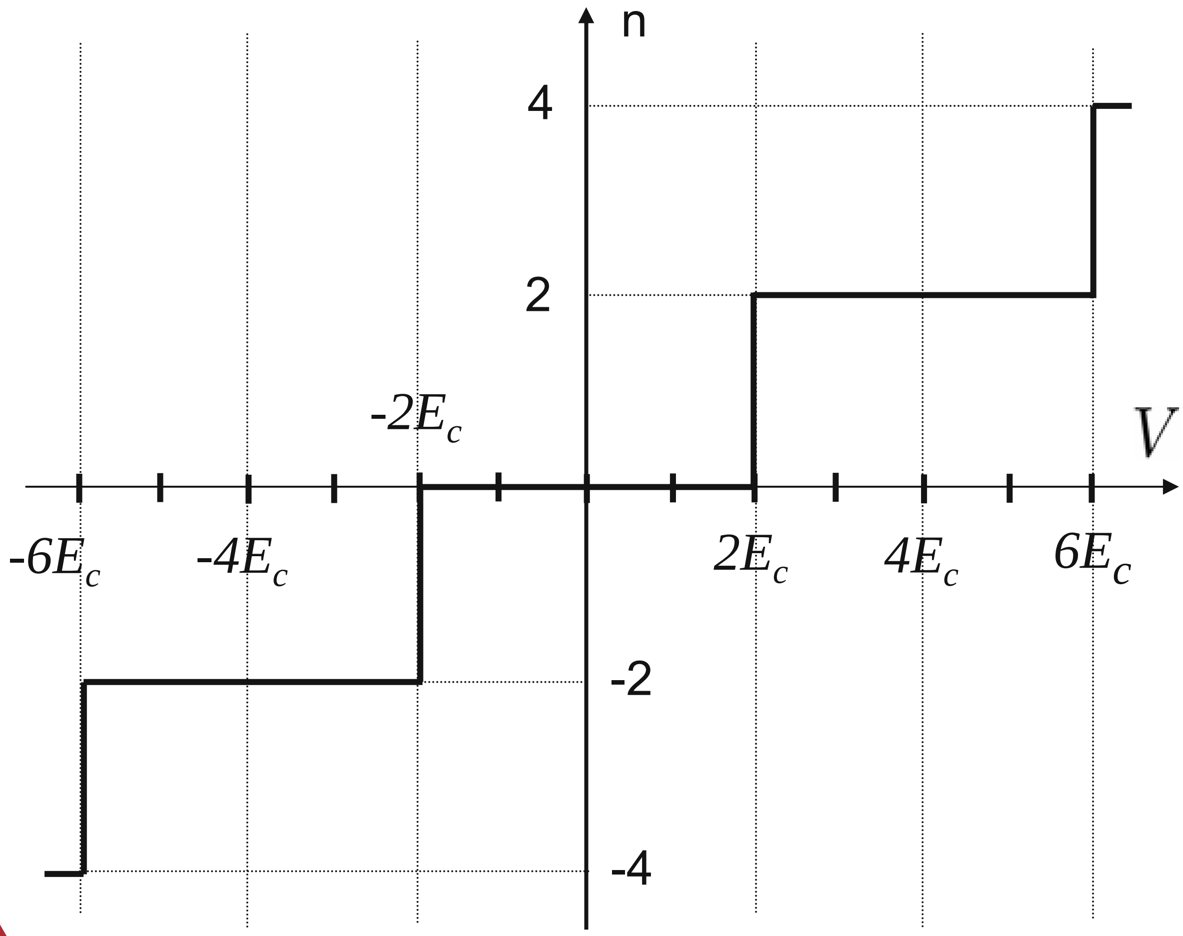
Single - and two particle excitation energies



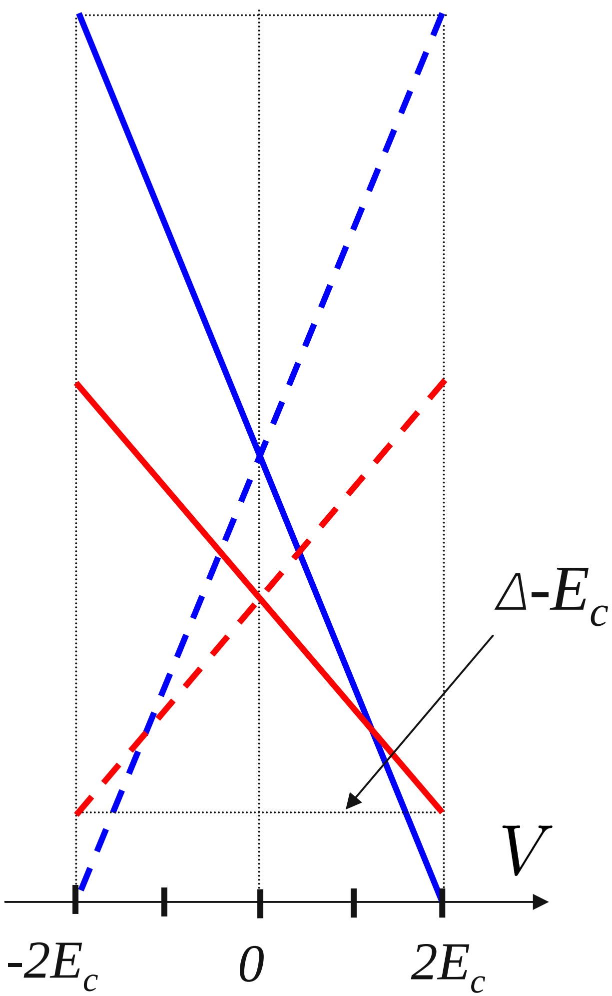
solid lines – creation
dashed lines - annihilation

Electron number and excitation energies as functions of V, $\Delta>$

Electron occupation number: n(V)



Single - and two particle excitation energies



solid lines – creation
dashed lines – annihilation

Density of states

Long range Coulomb interaction

$$H_c = \sum_{ij} n_i E_c^{ij} n_j - V_i n_i + \Delta P(n_i + p_i) \qquad E_c \sim e^2/2\,\tilde{\kappa}\,r \quad r \to \infty$$

Main conclusions of the single grain model stay the same!

1. $E_C > \Delta$: Gapless electrons.

   Pair gap: $2(E_C - \Delta)$

2. $E_C < \Delta$: Gapless pairs.

   Electron gap: $\Delta - E_C$

DOS: Efros-Shklovskii approach:

Energy to replace electron form i to j

$$\mathcal{E}^{ij}_{-+} = \mathcal{E}^{i}_{-} + \mathcal{E}^{j}_{+} - 2E_c^{ij} > 0$$

$$\nu_1(\varepsilon) = \alpha_{d1}\,(\tilde{\kappa}/e^2)^d \varepsilon^{d-1}$$

Energy to replace a pair from i to j

$$\mathcal{E}^{i\ \ j}_{2-2+} = \mathcal{E}^{i}_{2-} + \mathcal{E}^{j}_{2+} - 8E_c^{ij} > 0$$

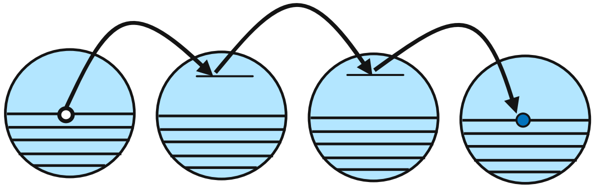
$$\nu_2(\varepsilon) = \alpha_{d2}\,(\tilde{\kappa}/(2e)^2)^d \varepsilon^{d-1}$$

The difference is due to the Cooper pair doubled charge only, therefore $\alpha_{d1} \approx \alpha_{d2}$.

Multiple cotunneling in granular superconductors. Hopping Conductivity

Electron hopping regime: Ec >>Δ.

Elastic regime



Hopping conductivity

$$\sigma \sim e^{-(T_0/T)^{1/2}} \qquad T_0 \sim e^2/\tilde{\kappa}\xi_{el}$$

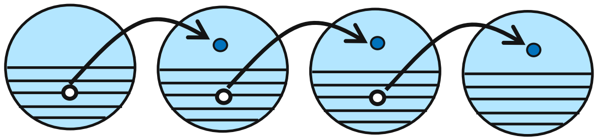
The presence of the gap results in a small correction to the localization length:

$$\xi_{el} = \frac{2\,a}{\ln(\,\bar{E}(\Delta)\,\pi/\bar{g}\,\bar{\delta})}$$

$$\bar{E}(\Delta) = \bar{E}(0) + c\Delta$$

Noticeable negative magneto-resistance.

Inelastic regime



1. **T>>Δ:** ES law with essentially unaffected localization length:

$$\xi_{in}(T) = \frac{2\,a}{\ln[\,\bar{E}^2/16\pi T^2\bar{g}\,]}$$

2. **T<< Δ:** **Strong suppression of the inelastic cotunneling !**

$$\sigma \sim \exp\left[-N\left(\ln(\bar{E}^2/4\bar{g}T\Delta) + 2\Delta/T\right)\right]$$

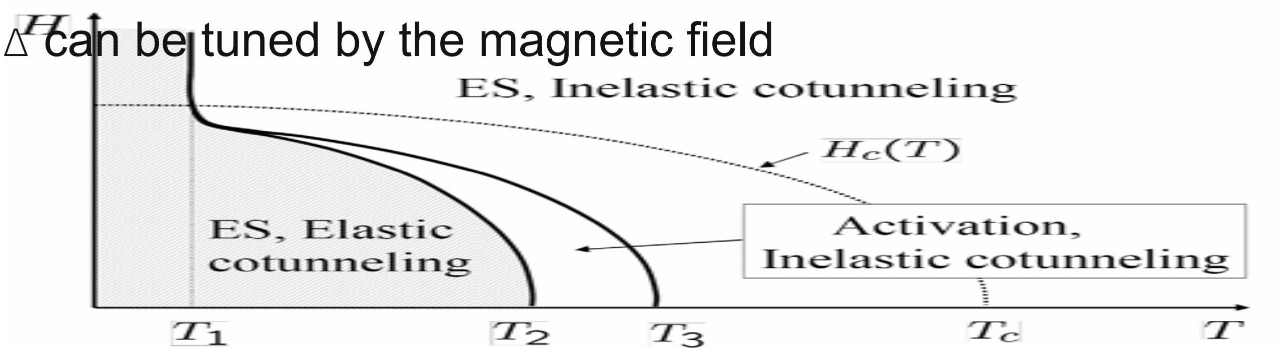
N is the typical tunneling order:

$$N = \sqrt{b\,e^2/16\,a\tilde{\kappa}\Delta} \sim \sqrt{E_c/\Delta}$$

Giant negative magneto-resistance!

Transport phase diagram in the magnetic field in the EH regime

The gap Δ can be tuned by the magnetic field



$T_1 \approx 0.1\sqrt{E_c\delta}$ - Crossover between elastic and inelastic regimes at Δ=0

$T_2 \approx \xi_{el}\Delta/a$ - Crossover between the elastic and inelastic activation behavior at H=0

$T_3 \approx \xi_{in}\Delta/a$ -Crossover between ES and activation inelastic regimes

Cooper pair hopping (CPH) regime

Hopping of Cooper pairs can be described in terms of the effective Hamiltonian

$$H = 4\sum_{ij} \hat{n}_i E_c^{ij} \hat{n}_j - 2\sum_i \hat{n}_i V_i + \frac{1}{2}\sum_{<ij>} J_{ij}\, e^{i\varphi_i - i\varphi_j}$$

φ - Cooper pair phase

$\hat{n} = -i\partial/\partial\varphi$ - Cooper pair number operator

$J_{ij} = g_{ij}\pi\Delta/2$ - Josephson couplings

$\sigma \sim e^{-(T_0/T)^{1/2}}$

Tunneling amplitude via perturbation theory in J:

$$A \sim \prod_{i=1}^{N} J_{i,i+1}/\tilde{\mathcal{E}}_2^i, \qquad \tilde{\mathcal{E}}_2^i = 2/[\,1/\mathcal{E}_{2+}^i + 1/\mathcal{E}_{2-}^i]$$

$\mathcal{E}_{2+}^i,\ \ \mathcal{E}_{2-}^i$ - Cooper pair creation and annihilation energies

Tunneling probability $P = A^* A \sim e^{-2r/\xi_{CPH}}$

$$\xi_{CPH} = \frac{a}{\ln(8\bar{E}/\pi\bar{g}\Delta)}$$

Conductivity:

$$T_0 \sim e^2/\tilde{\kappa}\,\xi_{CPH}$$

Positive magnetoresistance !

Experimental data

Granular aluminum samples.

A. Gerber, A. Milner, G. Deutscher, M. Karpovsky, A. Gladkikh PRL 1997.

Weak coupling insulating regime.

Grain size ~ 120A

Ec>>Δ → Electron hopping !

Theory predicts the giant negative magneto-resistance at T< Δ.

Explanation: suppression of the inelastic cotunneling by the superconducting gap.

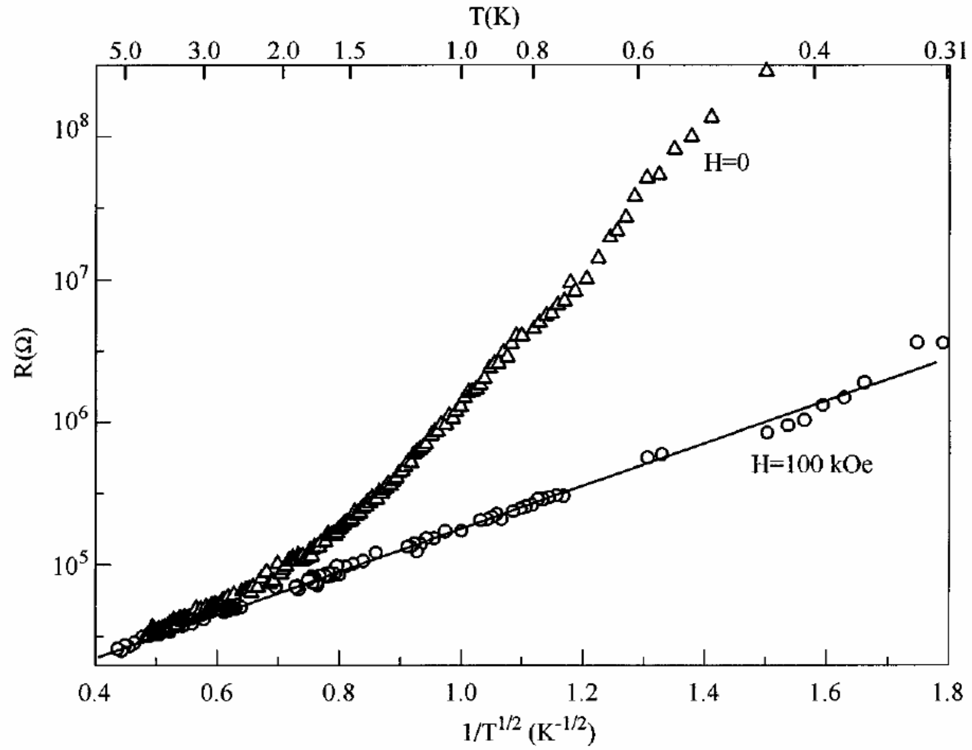
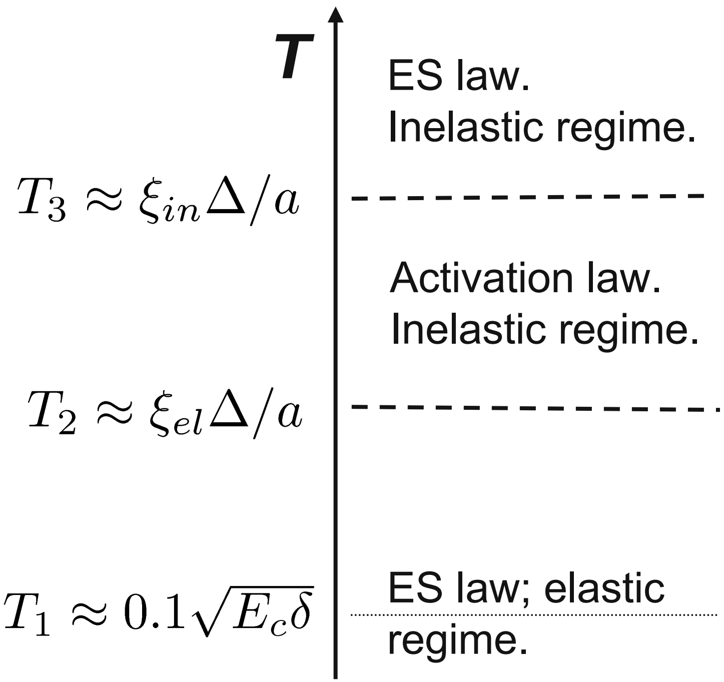


FIG. 1. Resistance of sample 1 measured at zero (triangles) and 100 kOe field (circles) as a function of the inverse square root of the temperature. Sample 1 room temperature resistance is 2×10^3 Ω.

Hopping conductivity in superconductors: Results

Hopping law

Ec > Δ. Electron hopping.



Negative magnetoresistance

Ec < Δ. Cooper pair hopping.

1. **ES law for Cooper pair transport.**
2. **Positive magnetoresiatnce.**

3. Possible scenario at g ~ 1: Renormalization of the charging energy due to tunneling coupling.