Effective Horizons in Quantum Phase Transitions

Ralf Schützhold

Institute for Theoretical Physics

Dresden University of Technology

Motivation and Contents

- Equilibrium properties well understood for many systems, e.g., near a phase transition
- Dynamical (time-dependent) phase transitions
  $\longrightarrow$ response time typically diverges
  $\longrightarrow$ non-equilibrium properties
- Analogy to expanding/contracting universe
  $\longrightarrow$ effective horizon (universal behaviour)
  $\longrightarrow$ loss of causal contact
- Amplification of quantum fluctuations
  $\longrightarrow$ seeds for pattern formation etc.
- Physical examples and induced spectra
- Relation to real cosmic inflation?

Dynamical Phase Transition

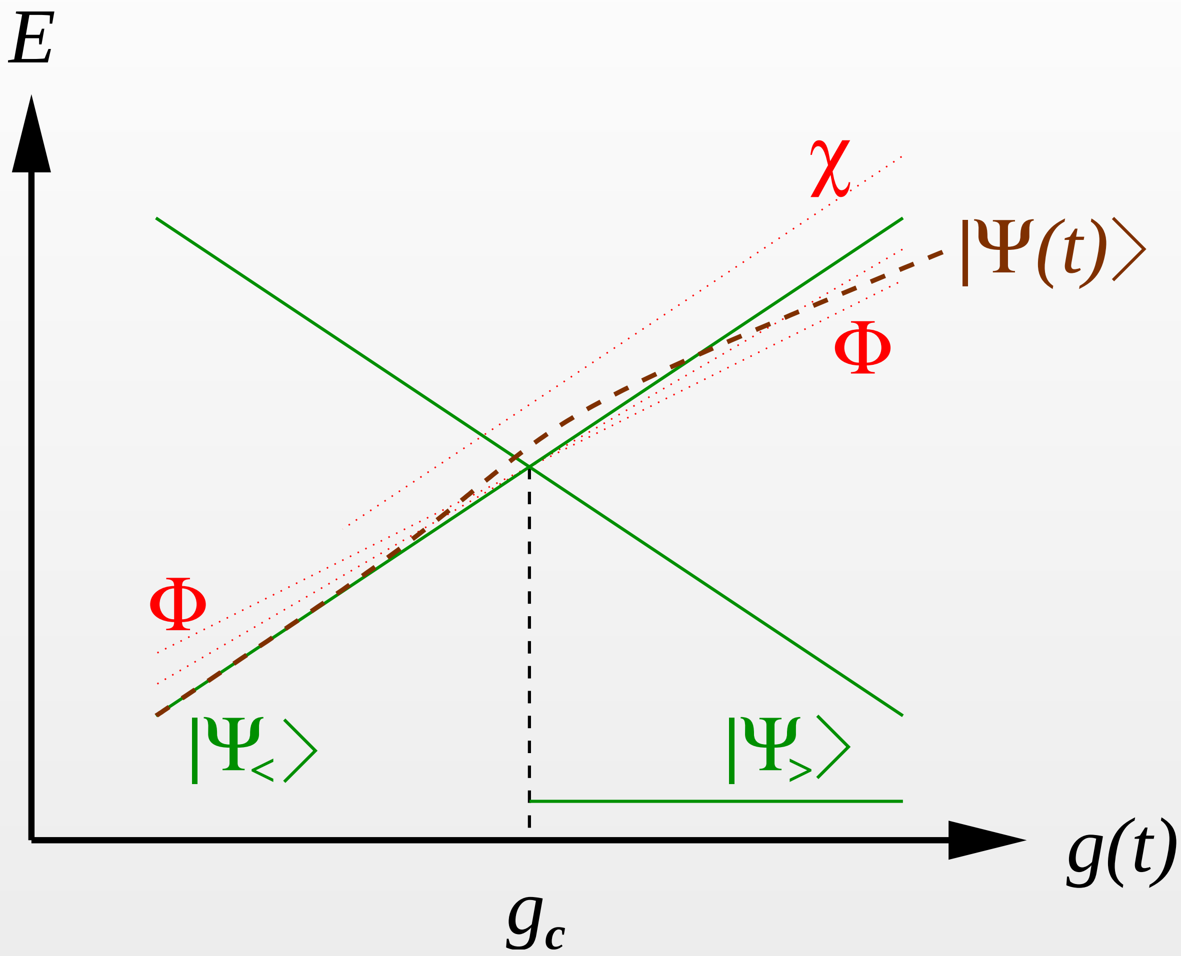
Zero temperature
$T = 0$

External parameter
$g = g(t)$

Two competing ground states
$|\Psi_<\rangle$ and $|\Psi_>\rangle$

Quasi-particle excitations
χ and Φ (unstable)

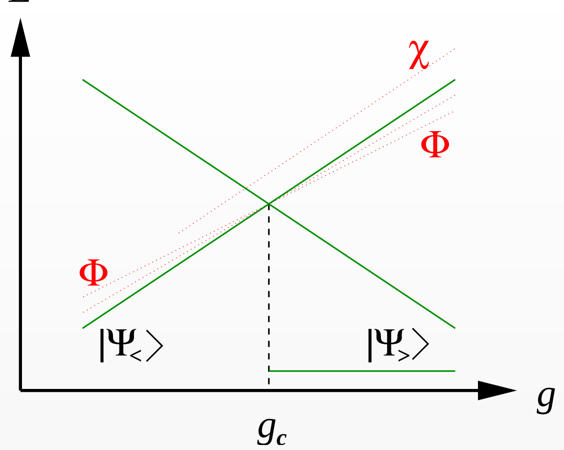
Actual quantum state $|\Psi(t)\rangle$



Sweeping through phase transition (critical point g_c)

$\rightarrow$ non-equilibrium dynamics $g(t)$

Universal Behaviour



- Homogeneity and isotropy (on large length scales)
- Linearity: small (quantum) fluctuations Φ
- Analyticity: dispersion relation $\omega(k)$ analytic
- Vanishing gap: $\omega(k=0)=0$ (e.g., Goldstone)
- Independence: one scalar mode Φ

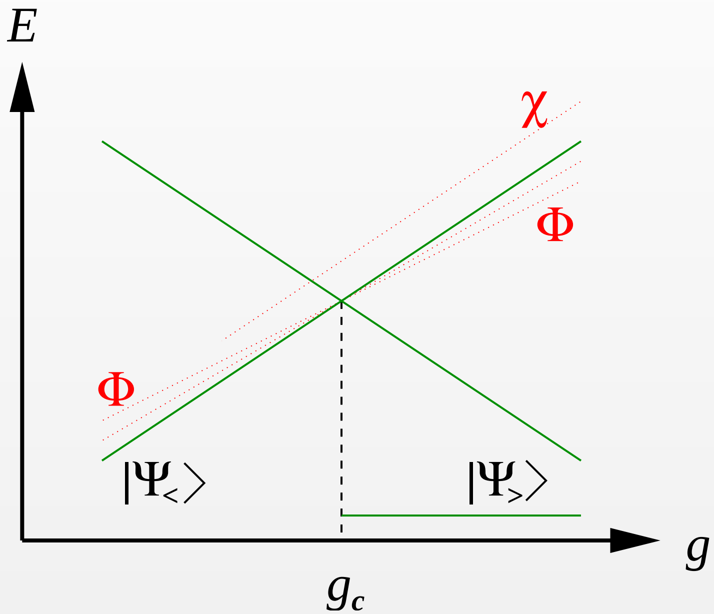
Low-energy effective action

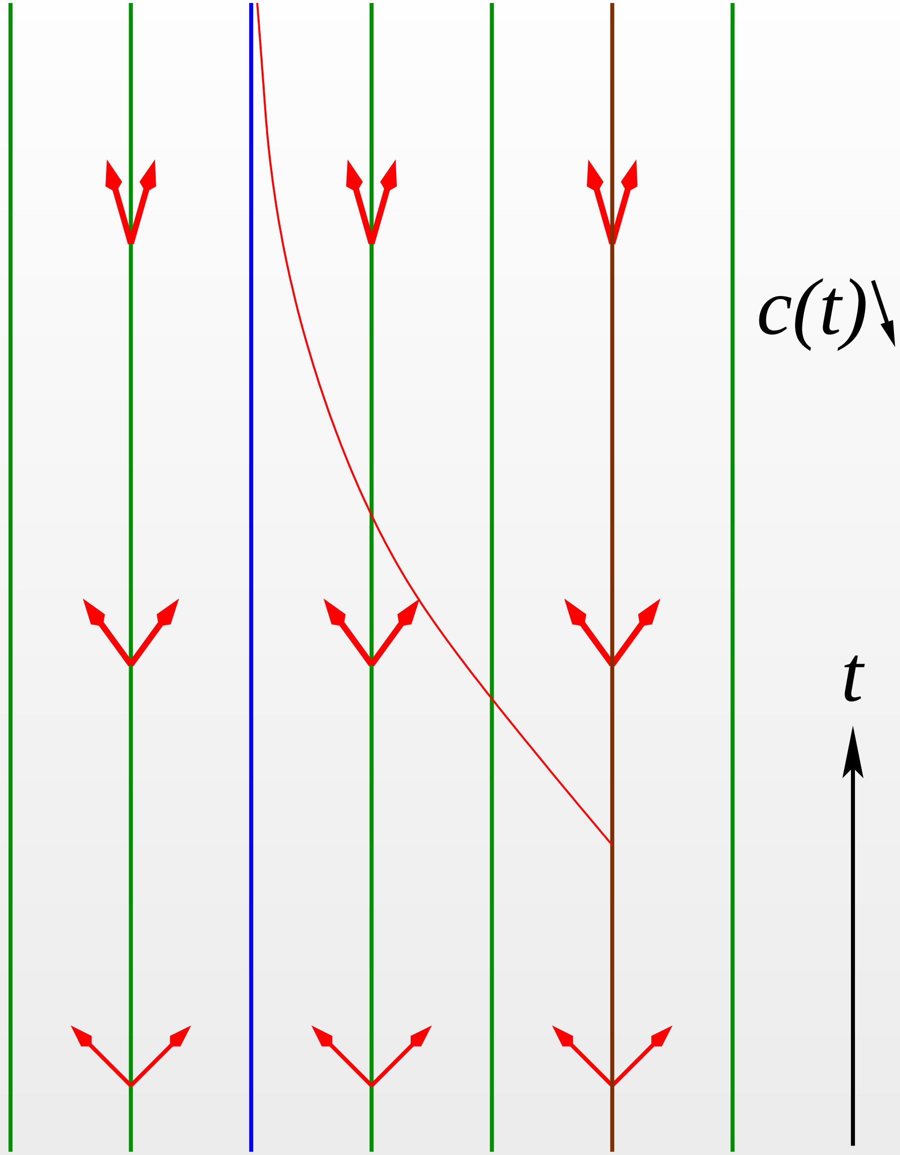
$$\mathcal{L}_{\rm eff} = \frac{1}{2}\left(\frac{1}{\alpha[g(t)]}\,\dot{\Phi}^2 - \beta[g(t)]\,(\boldsymbol{\nabla}\Phi)^2\right)$$

Effective metric (expanding/contracting universe)

$$ds^2_{\rm eff} = \sqrt{\alpha\beta^3}\,dt^2 - \sqrt{\beta/\alpha}\,d\boldsymbol{r}^2$$

"Cosmic" Horizon





Energy of excitations

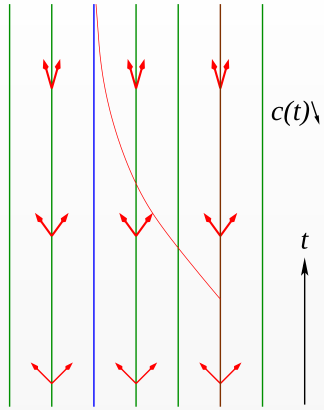
$\mathcal{H} = \frac{1}{2}[\alpha\Pi^2 + \beta(\boldsymbol{\nabla}\Phi)^2]$

$\rightsquigarrow \alpha(g_c) = 0$ or $\beta(g_c) = 0$

Effective metric: $ds^2_{\text{eff}} = \sqrt{\alpha\beta^3}\,dt^2 - \sqrt{\beta/\alpha}\,d\boldsymbol{r}^2$

$\rightsquigarrow c_s^2 = \alpha\beta \downarrow 0 \rightsquigarrow$ horizon $\rightsquigarrow$ amplification

Quantum Fluctuations

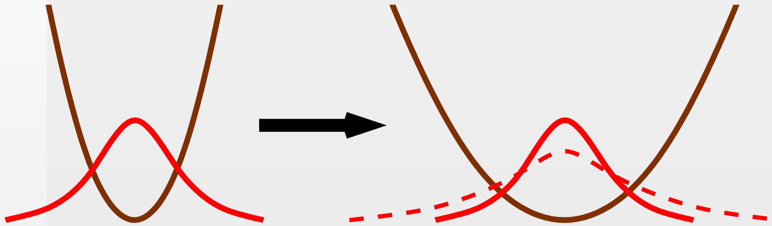


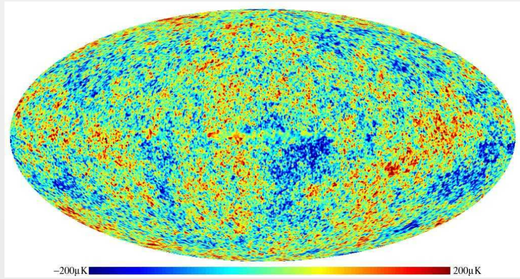
Size of "cosmic" horizon always decreases

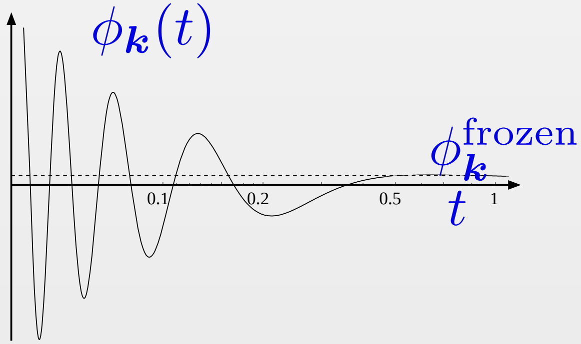
$$\frac{d}{dt}\Delta r(t) = \frac{d}{dt}\int_t^\infty dt'\, c(t') = -c(t) < 0$$

Oscillation $\lambda \ll \Delta r(t)$ $\rightarrow$ horizon crossing $\rightarrow$

$\rightarrow$ freezing $\lambda \gg \Delta r(t)$ and squeezing



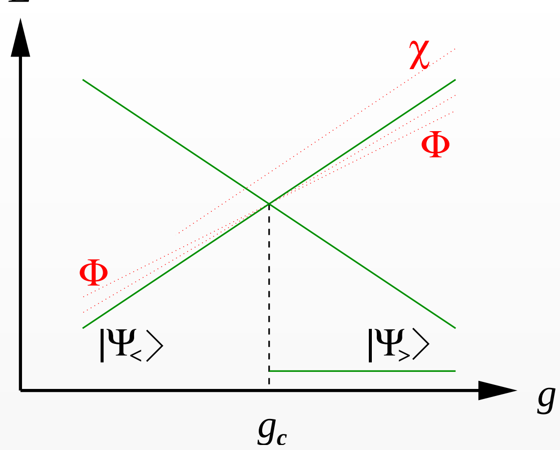




Amplification of quantum fluctuations $\hbar\omega/2$

Analogous to early universe (WMAP)

Simple Example



Bose-Einstein condensate with varying

Coupling $g(t)$ (e.g., Feshbach)

$$\mathcal{L}_{\rm eff} = \frac{1}{2}\left(\frac{1}{g(t)}\,\dot{\Phi}^2 - \frac{\varrho_0}{m}\,(\boldsymbol{\nabla}\Phi)^2\right)$$

Critical point at $g_c = 0$ (repulsive $\to$ attractive)

- Sweep $g(t) \propto t \to$ power-law expanding universe
  Spectra: $\sigma(\Phi) = k^{-4/3}$, $\sigma(\delta\varrho) = k^{4/3}$
- Sweep $g(t) \propto 1/t^4 \to$ de Sitter inflation
  Scale-invariant spectrum: $\sigma(\Phi) = k^{-3}$
  (density fluctuations do not freeze $\delta\varrho \propto t$)

Bose-Hubbard Model

$$\hat{H} = J(t) \sum_{\alpha\beta} M_{\alpha\beta} \hat{a}^\dagger_\alpha \hat{a}_\beta + \frac{U}{2} \sum_\alpha (\hat{a}^\dagger_\alpha)^2 \hat{a}^2_\alpha$$

Interaction U, tunneling rate $J(t)$, lattice matrix $M_{\alpha\beta}$

Large integer filling $n = \langle \hat{n}_\alpha \rangle = \langle \hat{a}^\dagger_\alpha \hat{a}_\alpha \rangle \gg 1$

Superfluid→Mott phase transition at $J_c = \mathcal{O}(U/n)$

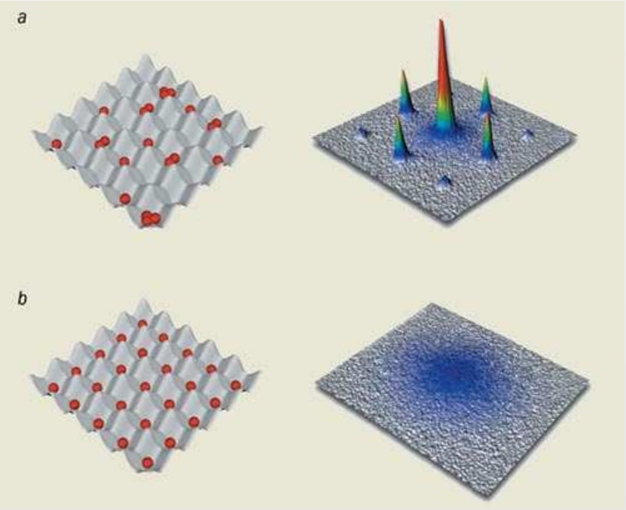
Exponential sweep of tunneling rate (→ experiments)

$$J(t) = J_0 \exp\{-\gamma t\}$$

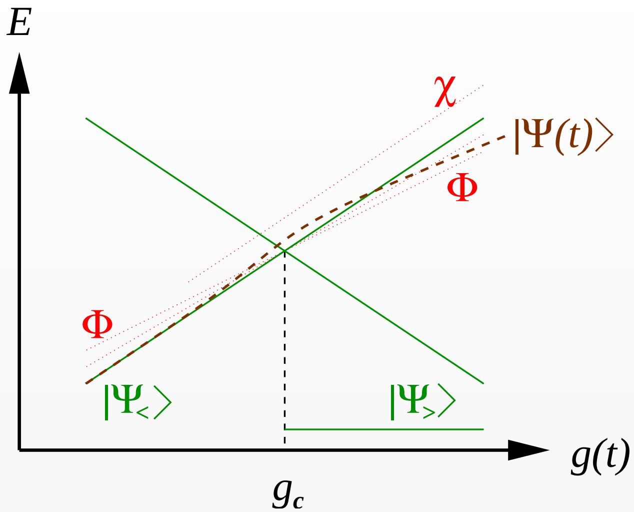
Adiabaticity parameter

$$\nu = \frac{Un}{\gamma}$$

Fast $\nu \ll 1$ vs slow $\nu \gg 1$



Decay of Superfluid



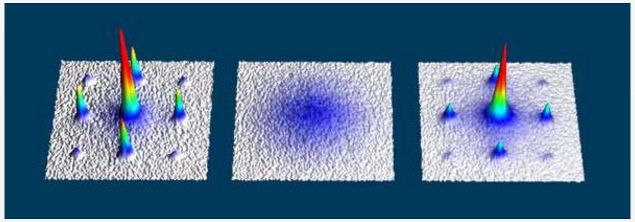
Off-diagonal long-range order

$$\langle \hat{a}_\alpha^\dagger \hat{a}_\beta \rangle = n - \gamma^2 t^2 \frac{\nu}{\pi} \left[1 - e^{-2\pi\nu}\right]$$

$\rightarrow$ independent of α, β

$\rightarrow$ peak at $k = 0$ decreases

($\rightarrow$ M. Greiner *et al*)



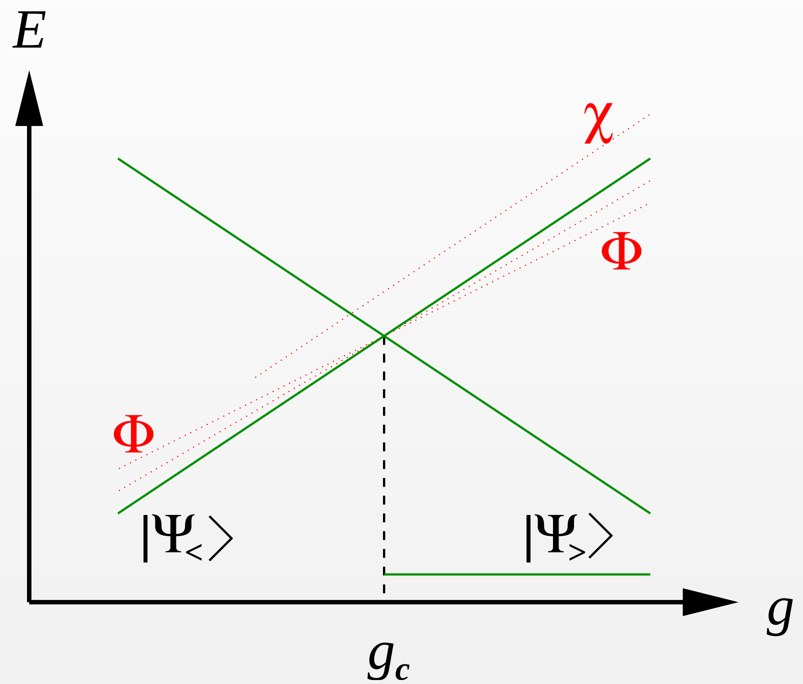
Superfluid fraction defined via $\boldsymbol{j} = \varrho_{\rm sf} \boldsymbol{\nabla} \Phi$ decreases

$$n_{\rm sf} = n - \gamma^2 t^2 \frac{\nu}{\pi} \left[1 - e^{-2\pi\nu}\right]$$

Rapid sweep $\nu \ll 1 \rightarrow$ decay with $n - 2(Unt)^2$ (independent of γ)

Adiabatic sweep $\nu \gg 1 \rightarrow$ decay $n - \gamma U n t^2$ of superfluid fraction much slower

Similarities to Cosmic Inflation

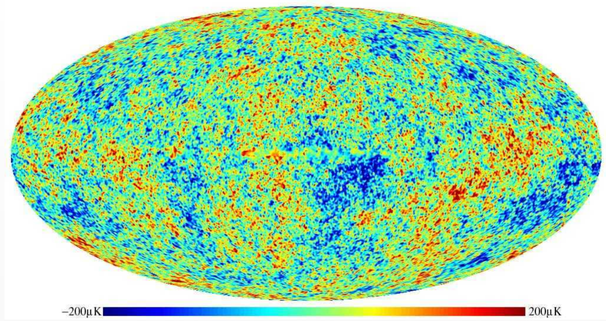


- Release of energy
  $\to$ (p)re-heating
- Robust against initial
  (small-scale) perturbations
- Universality
  (no fine-tuning)
- Amplification of quantum fluctuations

But: different spectrum in general (e.g., $k^{-4/3}$)

- Preferred frame (rest frame of medium)
- No unique/constant propagation speed
- Neglect of (quantum) back-reaction

Speculations...



Postulate:

- No (locally) preferred frame
- Unique/constant propagation speed

$$\mathcal{A} = \frac{1}{2} \int dt\, d^3r\, \frac{\dot{\Phi}^2 - (\boldsymbol{\nabla}\Phi)^2}{t^2}$$

- $\leftrightarrow$ scale-invariance $\mathcal{A}[\lambda t, \lambda \boldsymbol{r}] = \mathcal{A}[t, \boldsymbol{r}]$
- Dominated by (quantum) back-reaction?

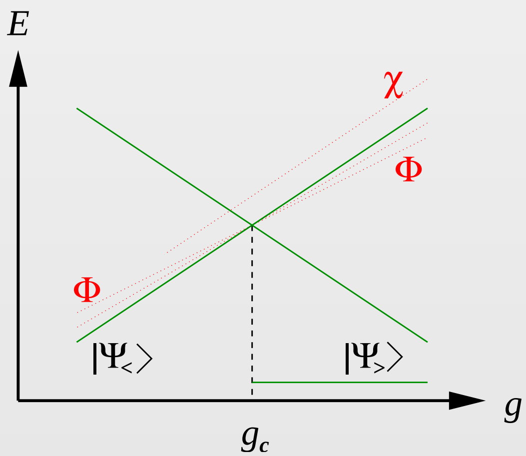
$\rightarrow$ correct $1/k^3$-spectrum (conformal de Sitter metric)

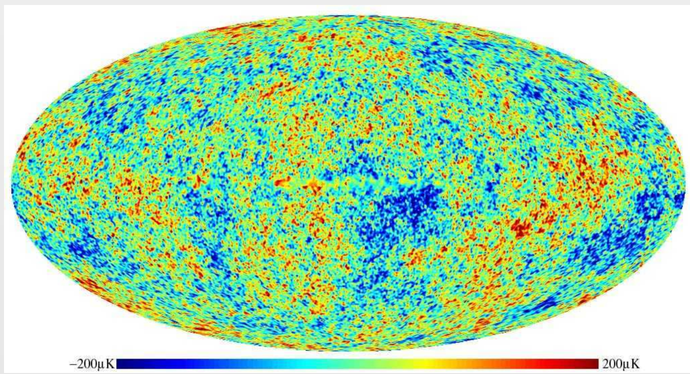
Was cosmic inflation just a phase transition?

Summary

- Analogy between cosmic inflation and quantum phase transitions in the laboratory
- Horizons → loss of causal contact → non-adiabatic behaviour → amplification of quantum fluctuations
- Interdisciplinary know-how transfer
- Relation to real cosmic inflation?

R. S., Phys. Rev. Lett. **95**, 135703 (2005)





Acknowledgements

- German Research Foundation (DFG): Emmy-Noether Programme
- Alexander von Humboldt foundation
- ESF-Programme Cosmology in the Laboratory
- Pacific Institute of Theoretical Physics
- EU-IHP ULTI, CIAR, NSERC
- many interesting discussions…