The hydraulic jump in liquid helium

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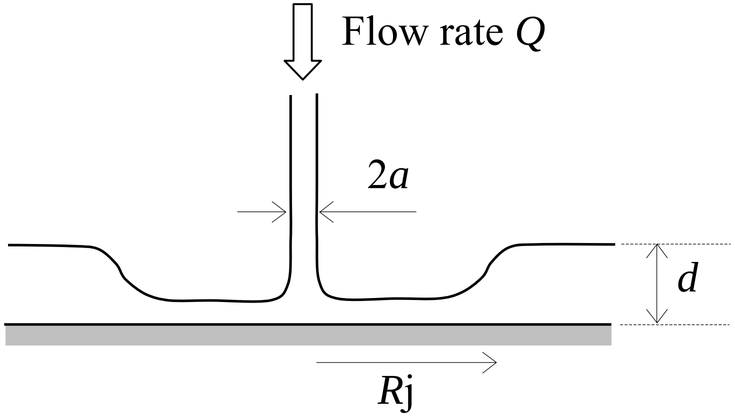
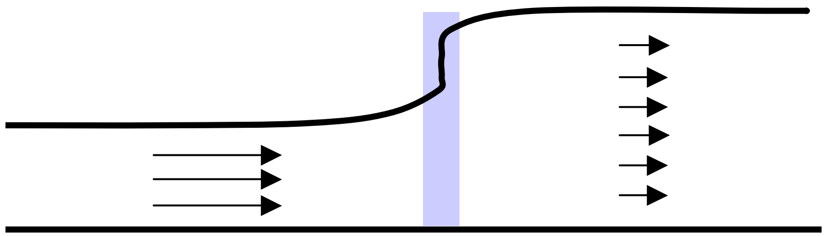
The hydraulic jump

- Easy to observe



- A theoretical challenge

- *First step* [Rayleigh 1914] :

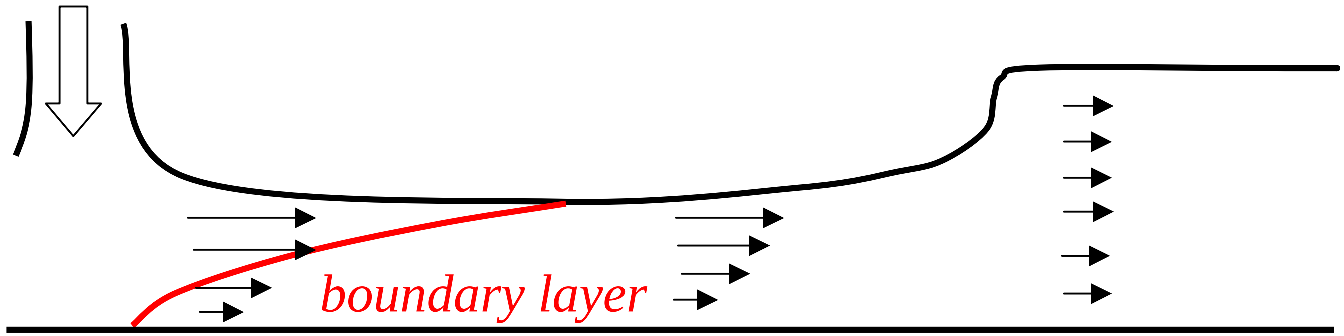


the jump is a shock
inviscid potential flow

$$\Rightarrow \quad \frac{R_j d^2 g a^2}{Q^2} + \frac{a^2}{2\pi^2 R_j d} = \frac{1}{\pi^2}$$

neglecting outside momentum $\Rightarrow \quad R_j \propto \dfrac{Q^2}{d^2 g a^2}$

- *Second step :*



the jump is still a shock

the velocity profile in the thin film is nearly parabolic

[Watson 1964]

with $Re \equiv Q/\nu a$

$$\Rightarrow \quad \frac{R_j d^2 g a^2}{Q^2} + \frac{a^2}{2\pi^2 R_j d} = 0.01676 \left\{ \left(R_j/a\right)^3 \mathrm{Re}^{-1} + 0.1826 \right\}^{-1}$$

- *Going further* :

  * Experiments : recirculating region at the jump

$\rightarrow$ need for a more accurate description of the flow

difficult since i) Re is not small

ii) free boundary

new approximation schemes [Bohr et al. 2000]

  * Effect of the surface tension ?

Changes the momentum balance at the jump

$$\frac{R_j d^2 g a^2}{Q^2}\left(1+\frac{2}{\mathrm{Bo}}\right)+\frac{a^2}{2\pi^2 R_j d}=0.01676\left\{\left(R_j/a\right)^3\mathrm{Re}^{-1}+0.1826\right\}^{-1}$$

[Bush et al. 2003]

$$\mathrm{Bo} \equiv \mathrm{R_j}\,\Delta \mathrm{h}/\mathrm{L_c}^2$$

- Using liquid helium ?

Extension of experiments with viscous fluids

- easy to reach high Re
- effect of surface tension

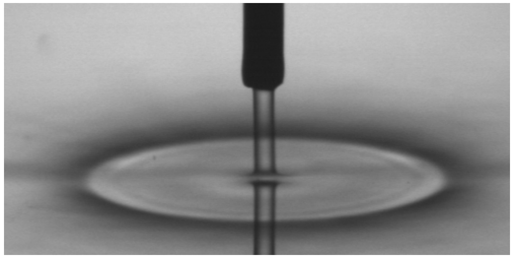
What happens at the lambda point ?

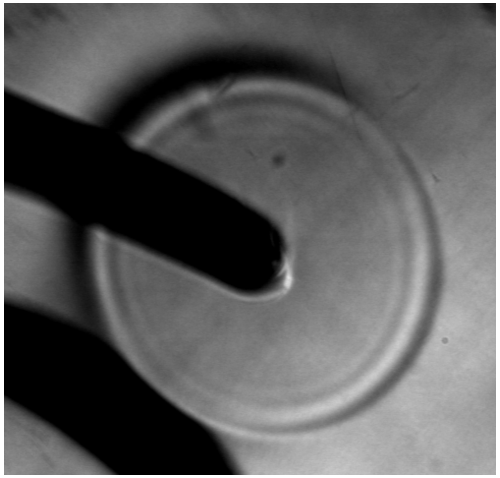
- is it possible to reach a true inertial regime ?
- white hole analog ?

the flow velocity in the interior region exceeds the speed of surface waves

[Volovik 2005]

Experimental Setup

- Optical He-4 cryostat (T > 1.5K)
- Side view :  jet diameter : 0.2 mm
- Top view :

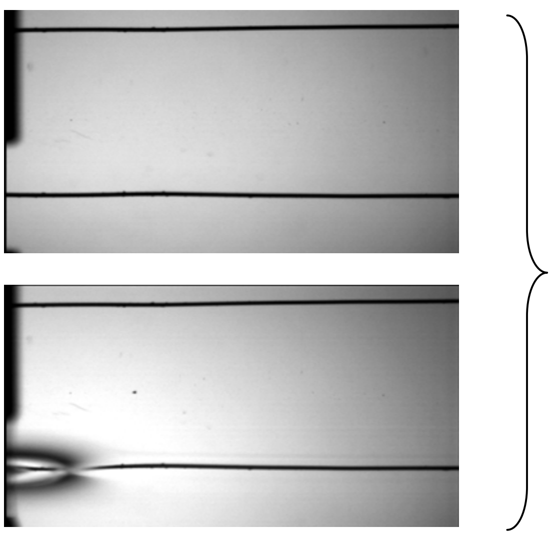


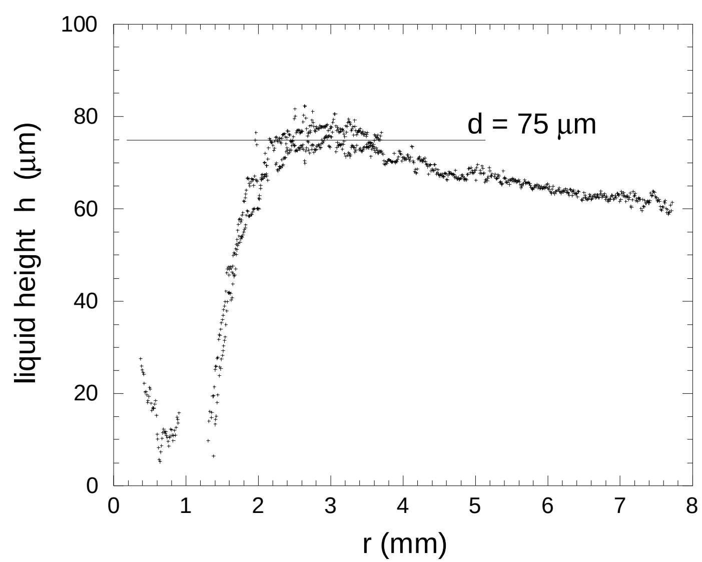
- He-4 flow
  - gas flow measured at room temperature
  - condensation + thermalization in the He bath

    $Q = 2 - 50\ mm^3/s$

  $\rightarrow R_j = 0.4 - 4$ mm

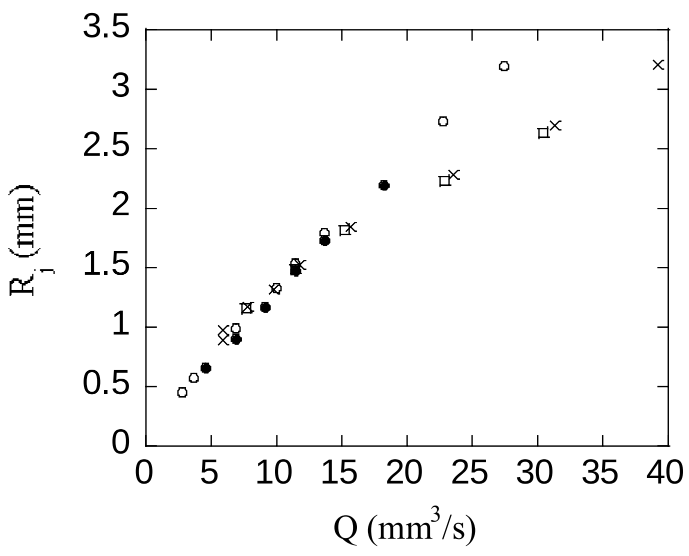
- Thickness of the liquid flow

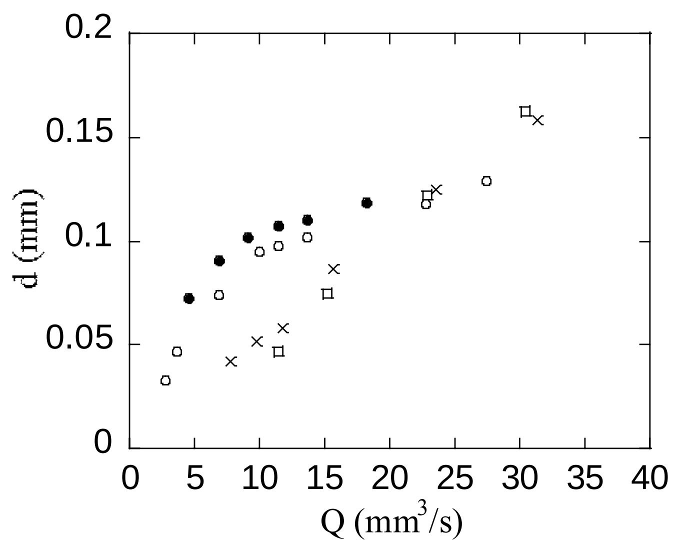




Normal liquid He-4

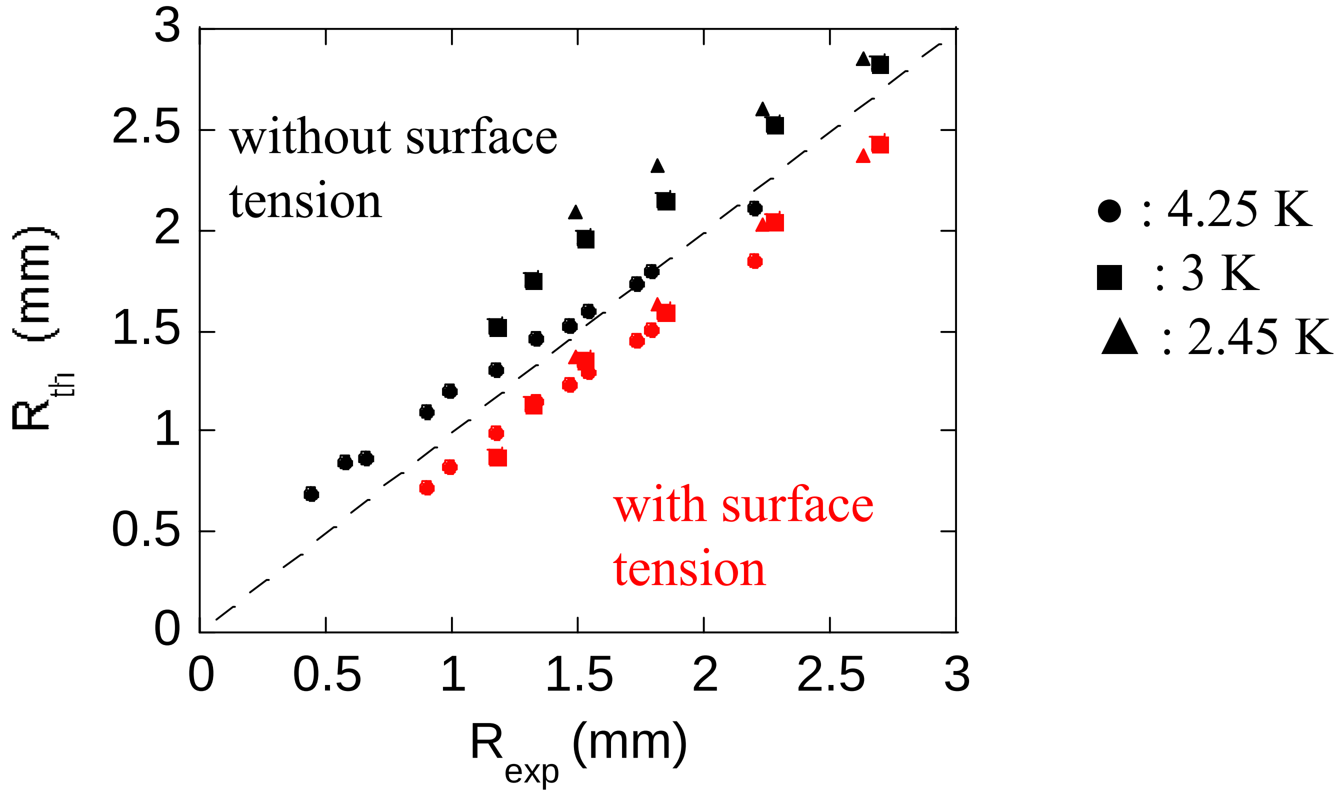
- Jump radius R_j and outside depth d





temperature : 4.25 – 2.4 K
inside depth ~ 10 μm

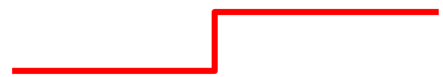
- Comparison with « shock » models

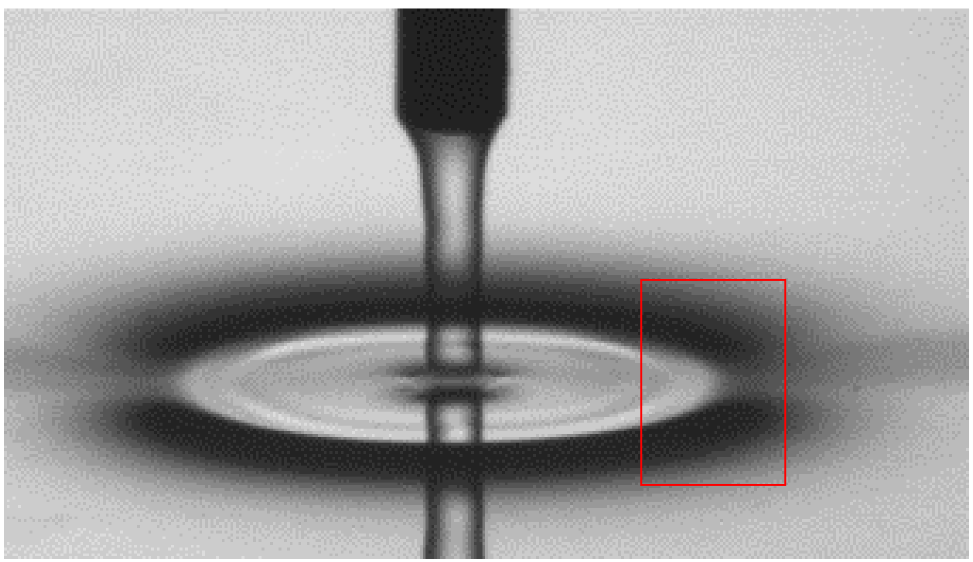


- Agreement between model and experiment
- Better agreement *with* the surface tension correction, but this correction is *too large*

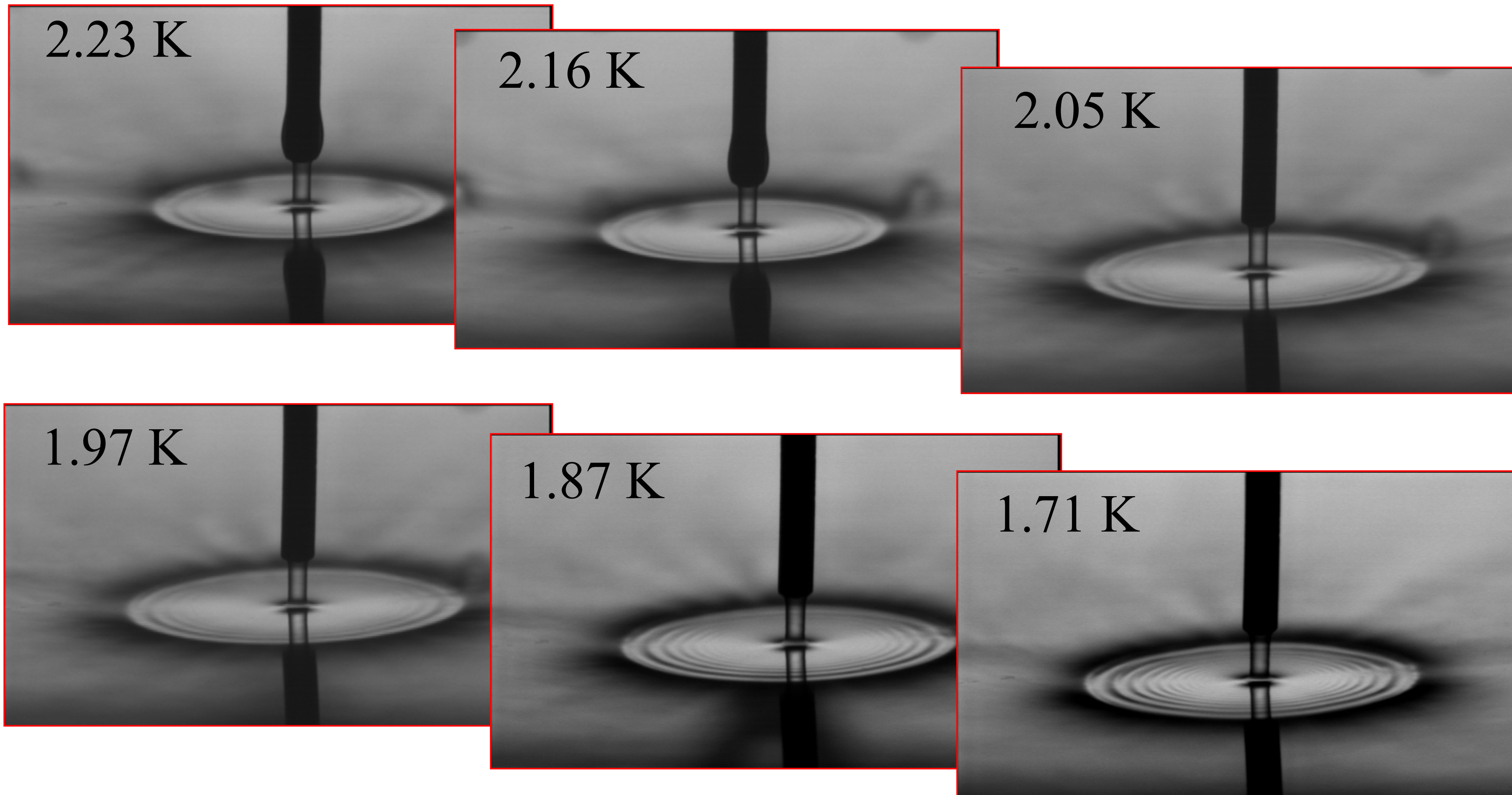
- Comparison with « shock » models

The surface tension correction is calculated assuming

a *sharp* jump : 



Going through the lambda point



No (large) change in the jump radius R_j at T_λ ?

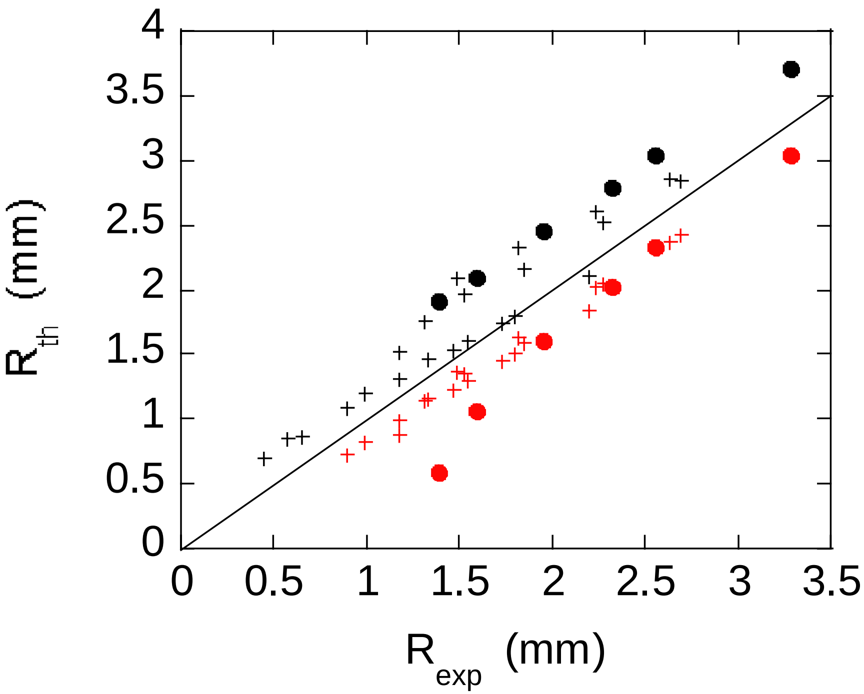
Inner region :

thickness $e \sim 10\ \mu m$

$Q > 10\ mm^3/s$

➡ $u > u_c \sim 50$ mm/s for r < 3 mm

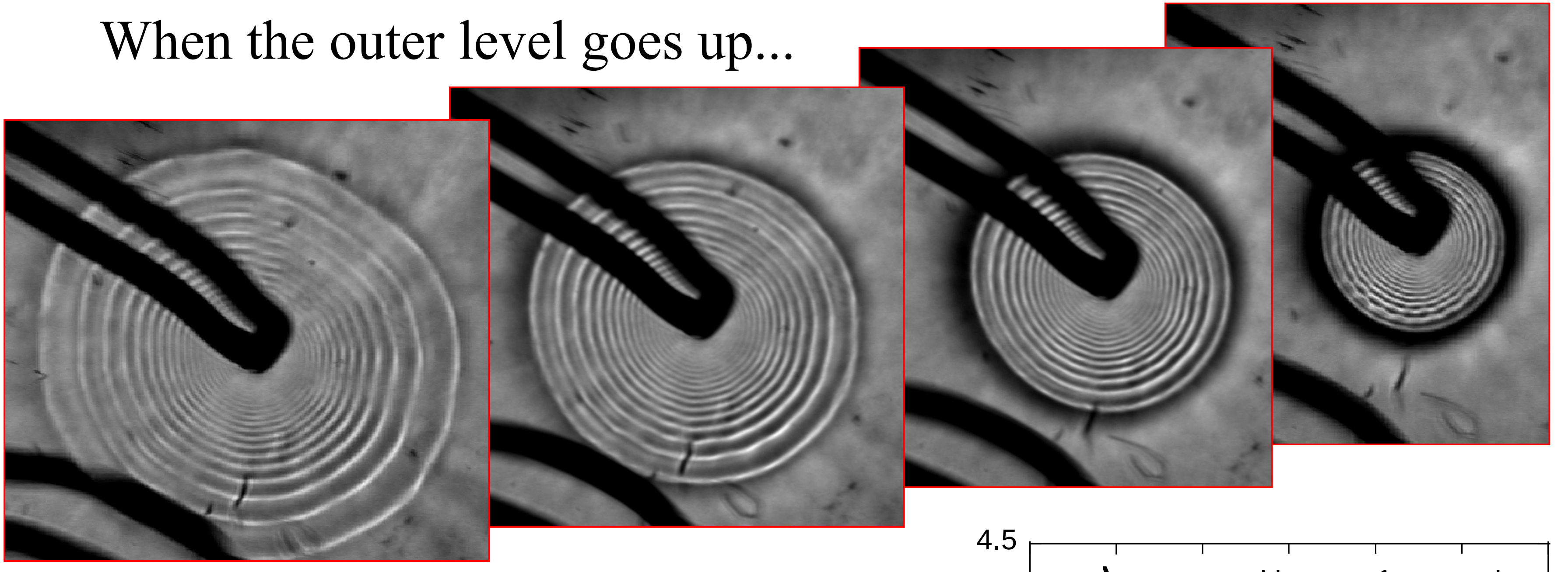
→ superfluid similar to a normal fluid with $\eta \approx \eta_n$



Good data collapse for T = 4.25 – 1.5 K

Some more data at T = 1.5 K

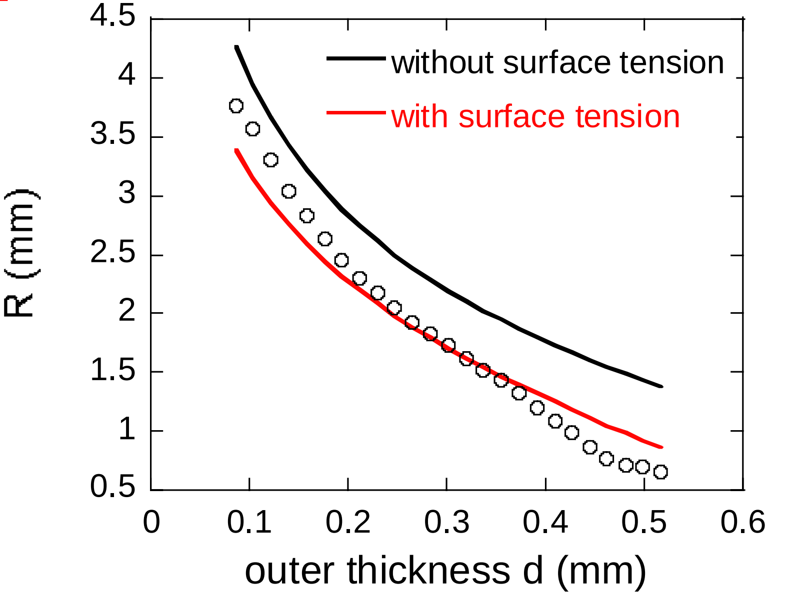
When the outer level goes up...



…the jump shrinks.

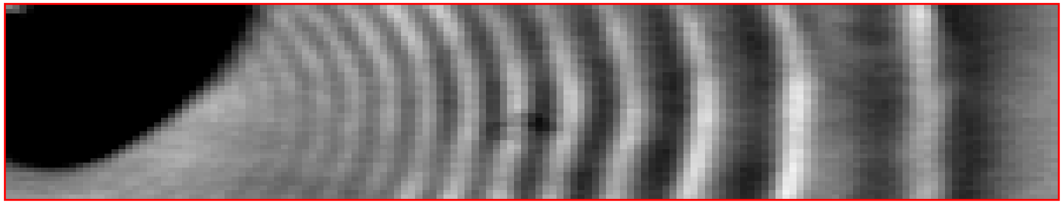
→ $R_j(d)$ at constant flow rate

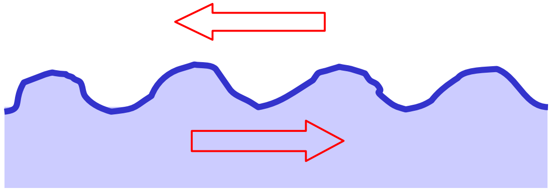
Agreement with "shock" models
+ viscous flow with $\eta = \eta_n$



Capillary waves in the inner region

- *Stationary* pattern in the lab frame





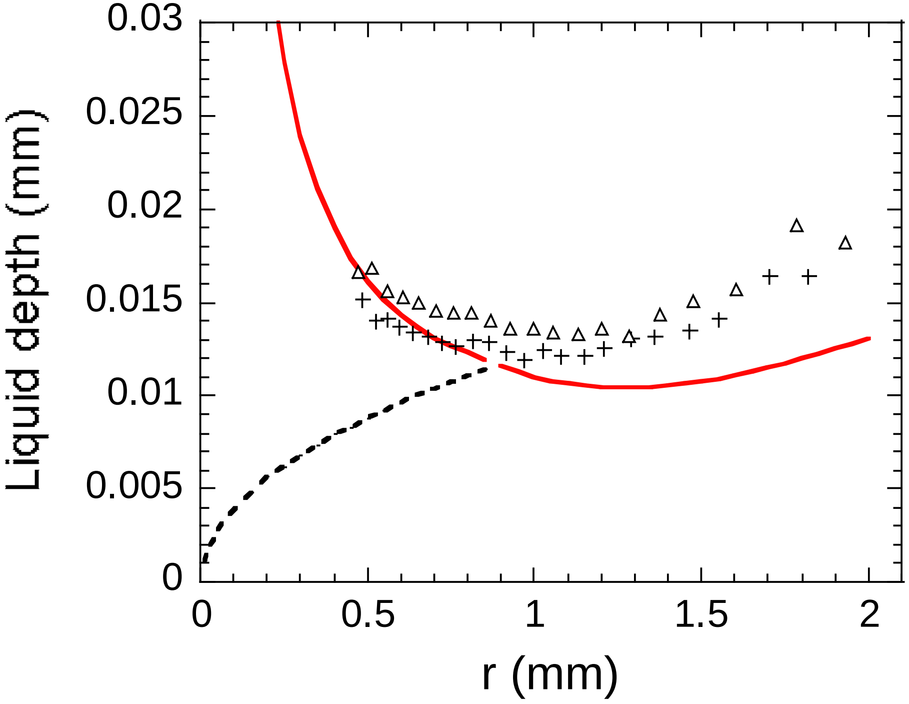
Wave velocity *opposite* to the fluid velocity

Oscillation - originates at the jump
and decreases as it propagates
- *due to an instability of the flow* ?

The local wavelength $\lambda(r)$ depends on the local { fluid velocity, fluid depth }

- Analysis of the wavelength
  - $\lambda << r \rightarrow$ 1D wave
  - $\lambda << H \rightarrow$ surface waves in shallow "water"

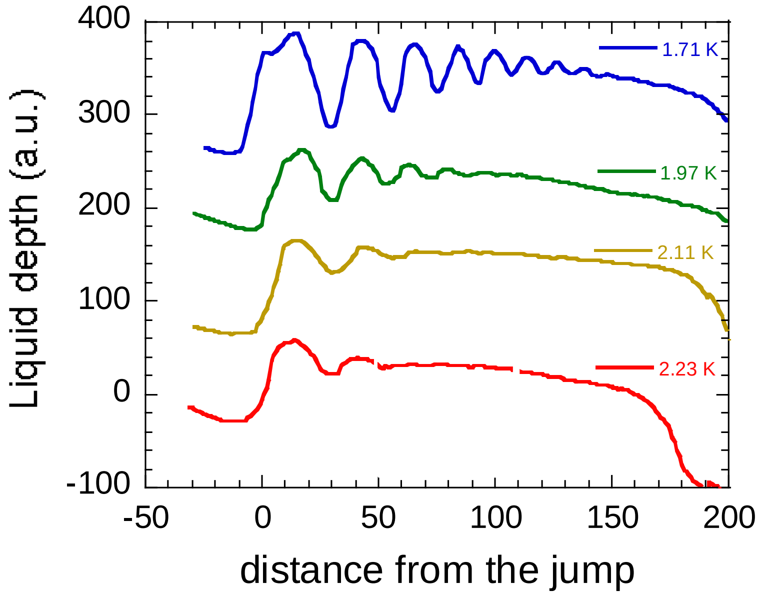
$$\omega^2 = \left(gk + (\alpha / \rho)k^3\right) \mathrm{th}\, kH$$

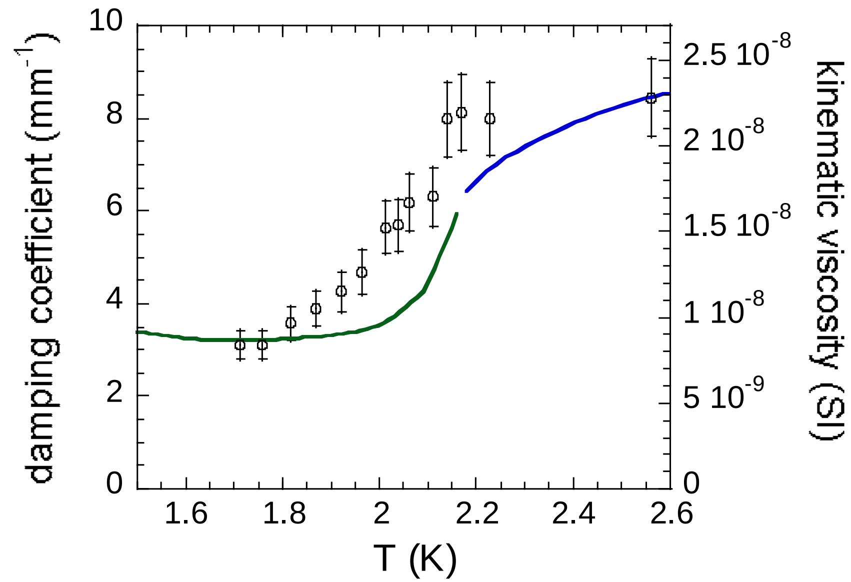


Agreement with
- Watson's calculation for a viscous liquid
- direct measurement (~ 10 μm)

- # Damping

The damping appears much weaker below T_λ :





→ the imaginary part κ of the wave vector *k* decreases
by the same amount as the viscosity

Seems consistent with the idea that superfluid = viscous fluid

- Calculation (estimation) of the damping

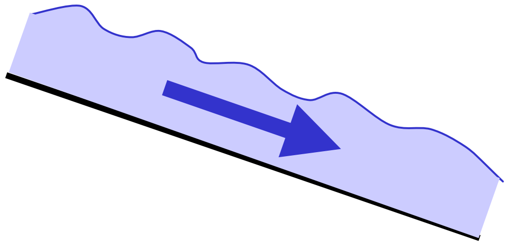
Velocity profile : close to a parabolic profile

Real flow inside the jump

➡ simpler 1D geometry : flow down a tilted plane

parabolic background flow

+ perturbation $\delta h\, e^{\,i(kx-\omega t)}$



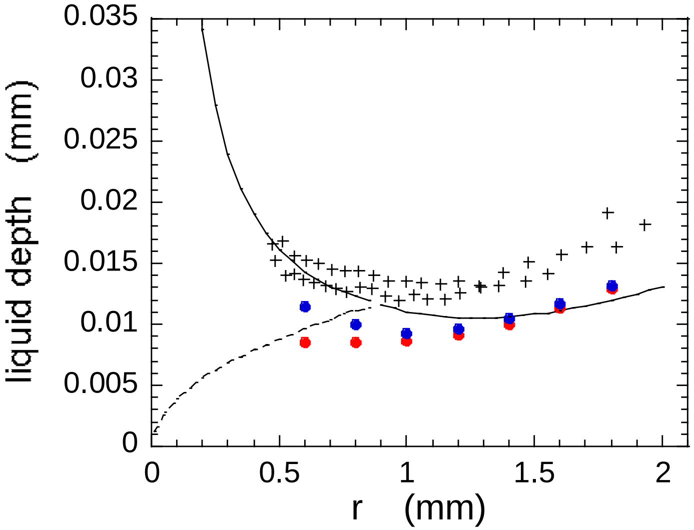
Large literature on the flow stability…

…here, we look for

a *particular* solution for *k*

corresponding to $\omega = 0$

Real part of the wave vector →



- Calculation (estimation) of the damping

  – Damping for the normal fluid

Imaginary part of the wave vector → $\kappa = 10.6\ \mathrm{mm}^{-1}$

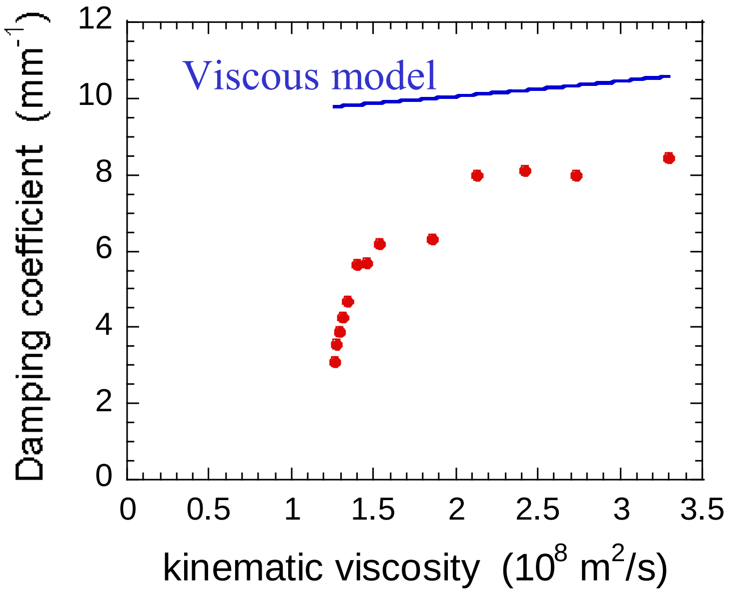
experiment → $\kappa = 8.4\ \mathrm{mm}^{-1}$

} at T = 2.5 K

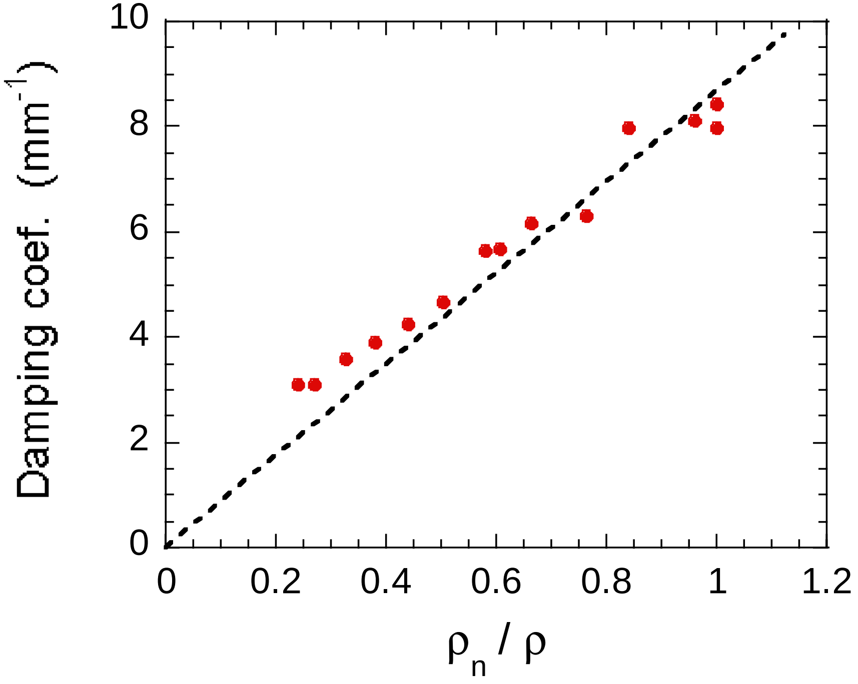
Correct estimate for the normal fluid

  – Temperature dependence

Too small



→ something special happens below T_λ



Conclusion

- Above T_λ : good agreement with the usual (viscous) model

- The main features of the jump do not change at T_λ:
  the value of R_j and the liquid depth inside the jump are consistent with the hypothesis that the fluid behaves as a viscous fluid with viscosity η_n

- However, the damping of capillary waves inside the jump is not consistent with this hypothesis.

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- Is it possible to reach a true superfluid regime ?