Josephson ratchets:

Experimental observation of directed motion induced by deterministic drives and by fluctuations

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Ratchet effect

directed motion -- no net driving force !
in periodic & asymmetric potential = **ratchet potential**

motivation: **„molecular motors“ in biology**

- muscle contraction
- intra-cellular transport

applications: **- separation devices for macro-molecules,**
- synthetic molecular motors, ...

theory: extensively treated (mostly 1-dim.)

- rocking ratchets
- flashing ratchets
- …

- P. Reimann, Phys. Rep. **361** (2002)
- *special issue on Ratchets and Brownian Motors* Appl. Phys. A **75** (2002)

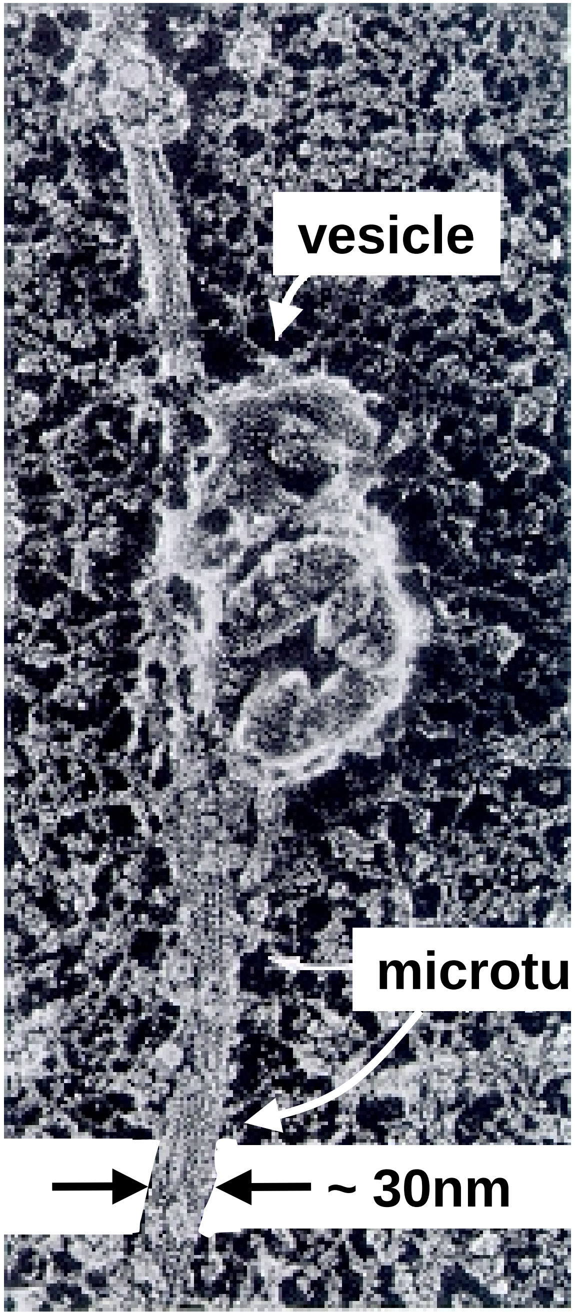
experiments: few demonstrations !

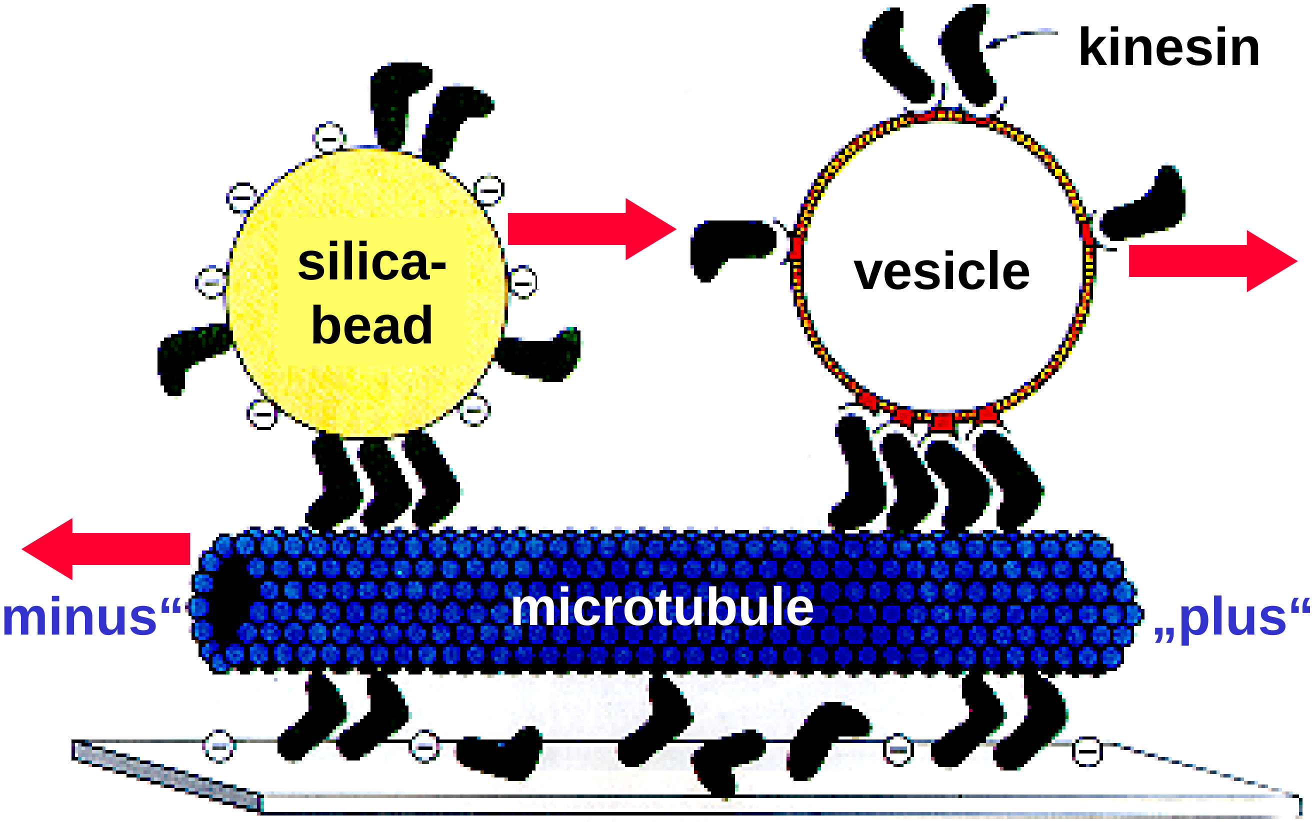
Josephson ratchets:

- Josephson fluxons M. Beck *et al*., PRL **95** (2005)

- SQUIDs A. Sterck *et al*., PRL **95** (2005)

Intra-cellular transport: Kinesin on microtubule





transport with discrete steps
length ≅ period of microtubule

Outline

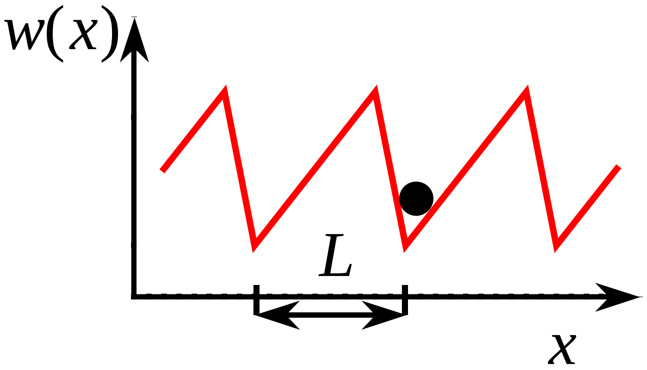
- **Introduction:**
  - ↷ 1D rocking ratchet
  - ↷ Josephson junction (JJ)
  - ↷ 3-junction SQUID ratchet

- **Investigation of 3-JJ SQUID ratchets**

  (experiment and numerical simulation)
  - ↷ harmonic drive (adiabatic & non-adiabatic)
  - ↷ stochastic drive

1D rocking ratchet

ratchet potential:



$\Rightarrow$ periodic
$\Rightarrow$ broken reflection-symmetry

$$m\ddot{x} + \xi\dot{x} = -\partial_x w(x) + F_d(t) + F_{th}(t)$$

external force

$$< F_d(t) >= 0$$

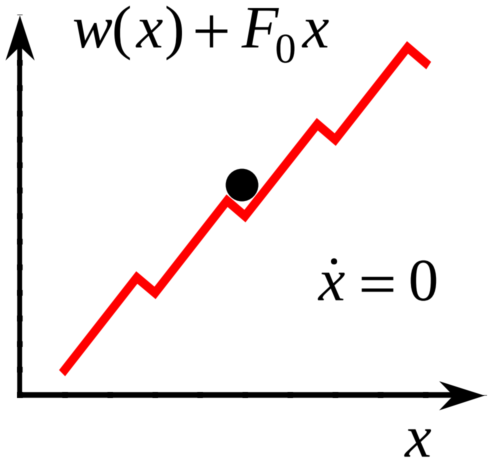
stochastic Langevin-force

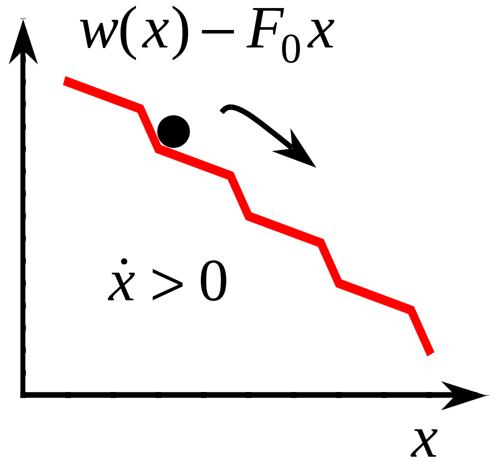
$$< F_{th}(t) >= 0$$

$$< F_{th}(0) F_{th}(t) >= 2 k_B T \xi \delta(t)$$

(equilibrium – fluctuations, Gaussian distributed)

harmonic drive: $F_d(t) = F_0 \sin\omega\, t$





transport → shallow slope

$$< \dot{x} > (F_d(t), \xi, T, m) = ?$$

Josephson junction

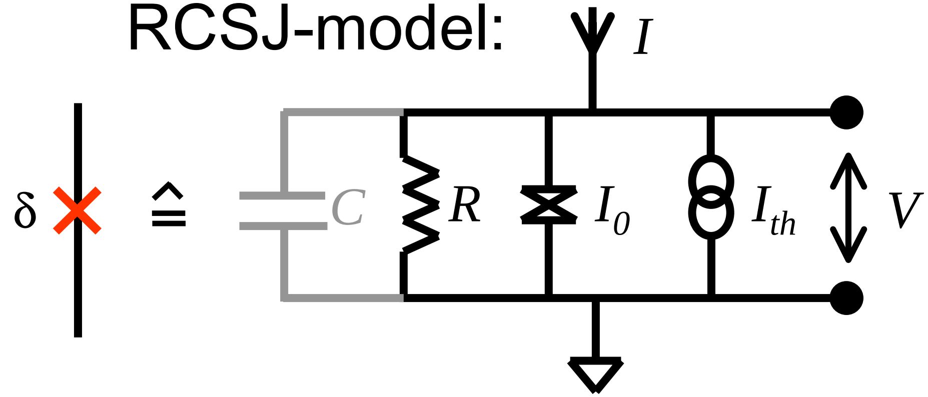
Josephson (1962):

$$I_s = I_0 \sin\delta$$

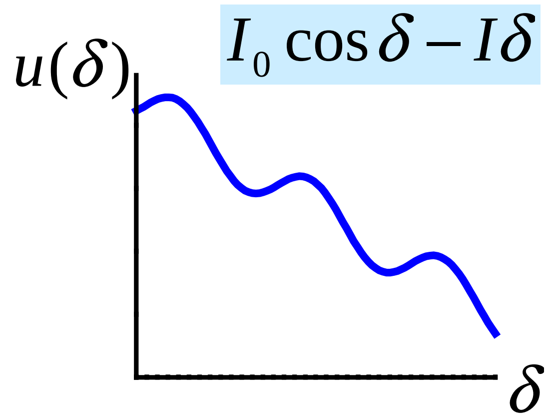
$$V = \frac{\Phi_0}{2\pi}\dot{\delta}$$

δ: Phase difference of superconducting wave function

RCSJ-model:



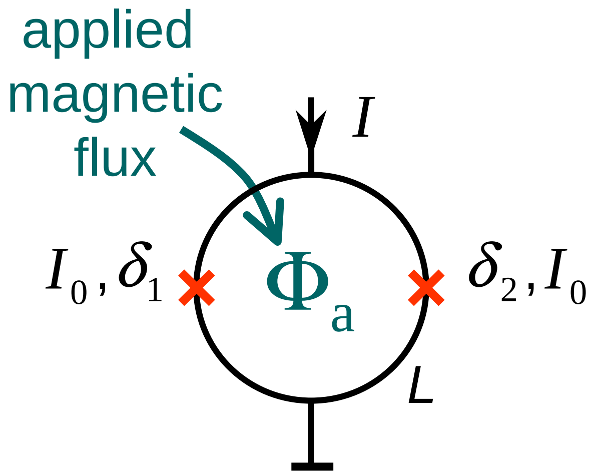
tilted washboard potential:



equivalence:

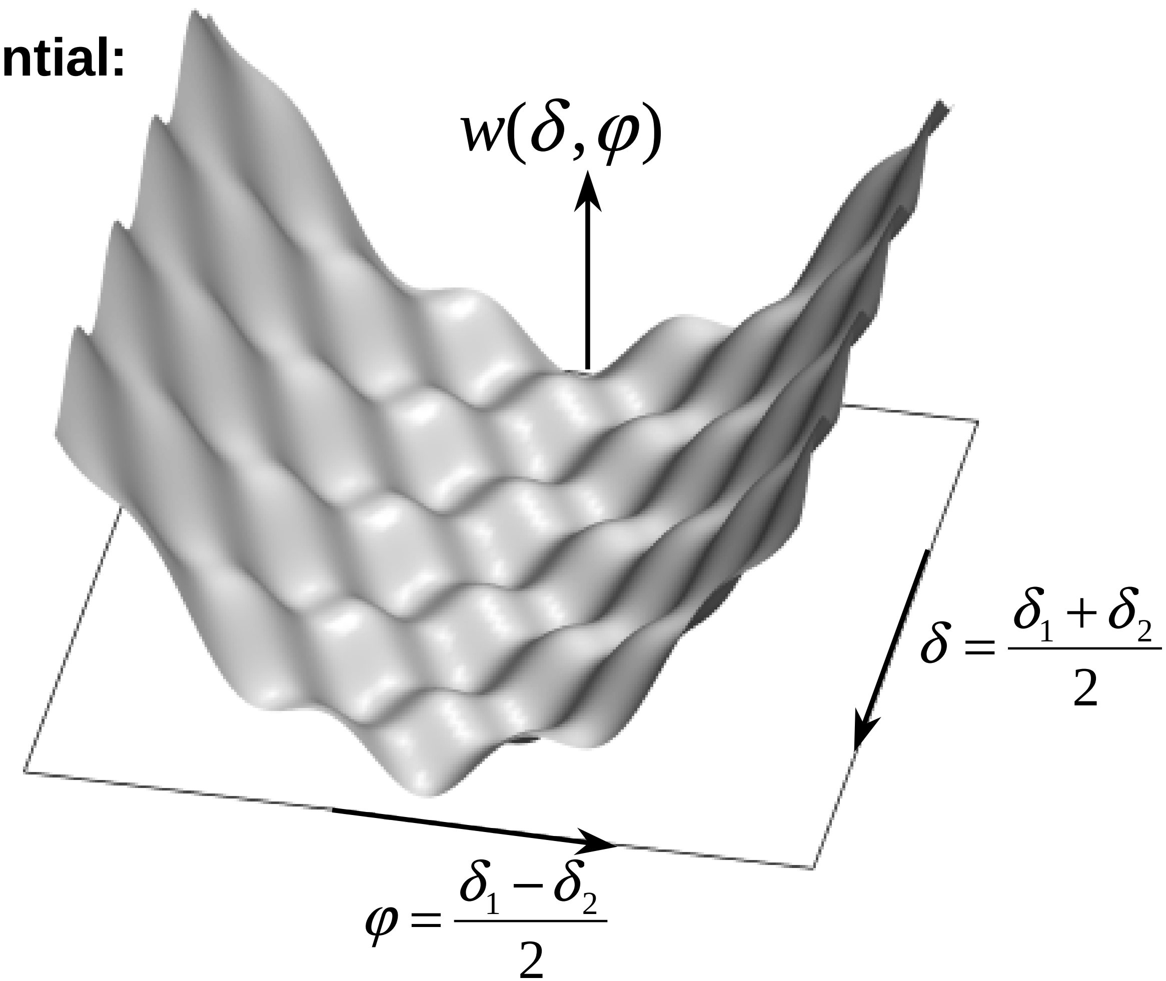
$$m\ddot{x} + \xi\dot{x} = -\nabla w(x) + F_d(t) + F_{th}(t) \quad \Leftrightarrow \quad \frac{\Phi_0 C}{2\pi}\ddot{\delta} + \frac{\Phi_0}{2\pi R}\dot{\delta} = -I_0\sin\delta + I(t) + I_{th}(t)$$

velocity	$\dot{x}$	↔	$\dot{\delta}\,(\Phi_0/2\pi) = V$	voltage
force	$F_d\,,\,F_{th}$	↔	$I\,,\,I_{th}$	current
friction	ξ	↔	$1/R$	conductance
mass	m	↔	C	capacitance



dc SQUID

2D potential:



bias current ***I***

tilts potential along δ

magnetic flux Φ_a

shifts minimum along φ

$$w(\delta,\varphi,t) = 1 - \cos\delta \cdot \cos\varphi + \frac{1}{\pi\beta_L}\left(\varphi - \pi\frac{\Phi_a}{\Phi_0}\right)^2 - \frac{I(t)}{2I_0}\delta$$

screening parameter: $\beta_L \equiv 2LI_0/\Phi_0$

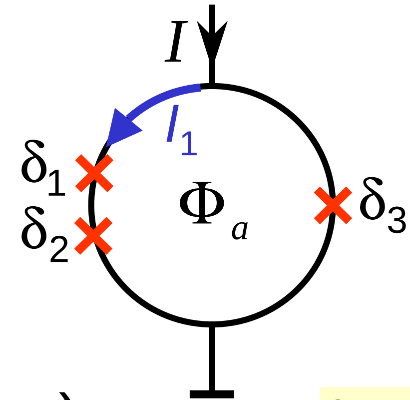
3-junction SQUID: A Josephson ratchet

$$\beta_C \equiv \frac{2\pi I_0 R^2 C}{\Phi_0}$$

- overdamped JJs: $\beta_C << 1$ intertial term negligible

- 2 identical JJs: $\delta_1 = \delta_2$ ↷ $\delta_\ell = \delta_1 + \delta_2$
  (in series)

$I_1 = I_{0,1} \sin(\delta_\ell / 2)$



- small inductance $\beta_L << 1$
  phases (left & right) rigidly coupled $\delta_3 - \delta_\ell \approx 2\pi(\phi_a + n)$

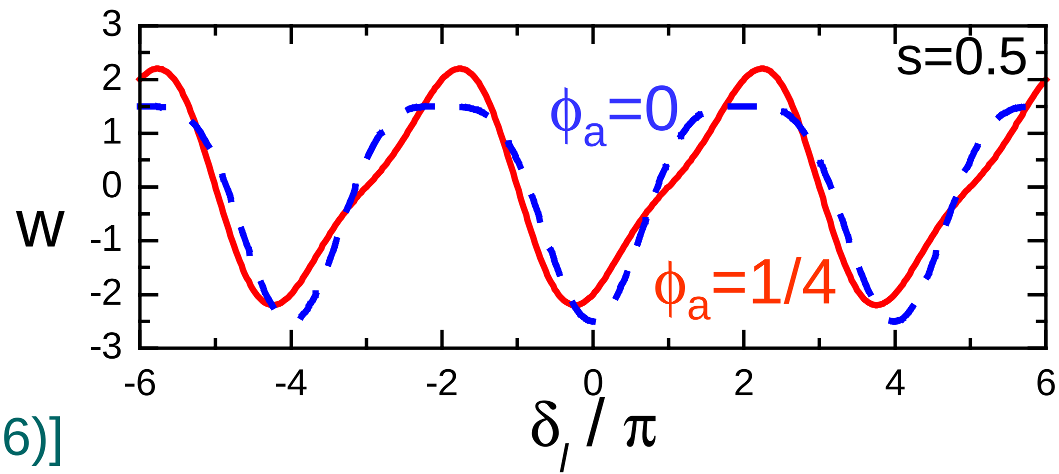
$n = 0,1,2\ldots$

$\beta_L \equiv 2 L I_0 / \Phi_0$

$\phi_a \equiv \Phi_a / \Phi_0$

↷ **effective 1D potential:**

$$w(\delta_\ell) = -s\cos(\delta_\ell + 2\pi\phi_a) - 2\cos(\delta_\ell / 2)$$



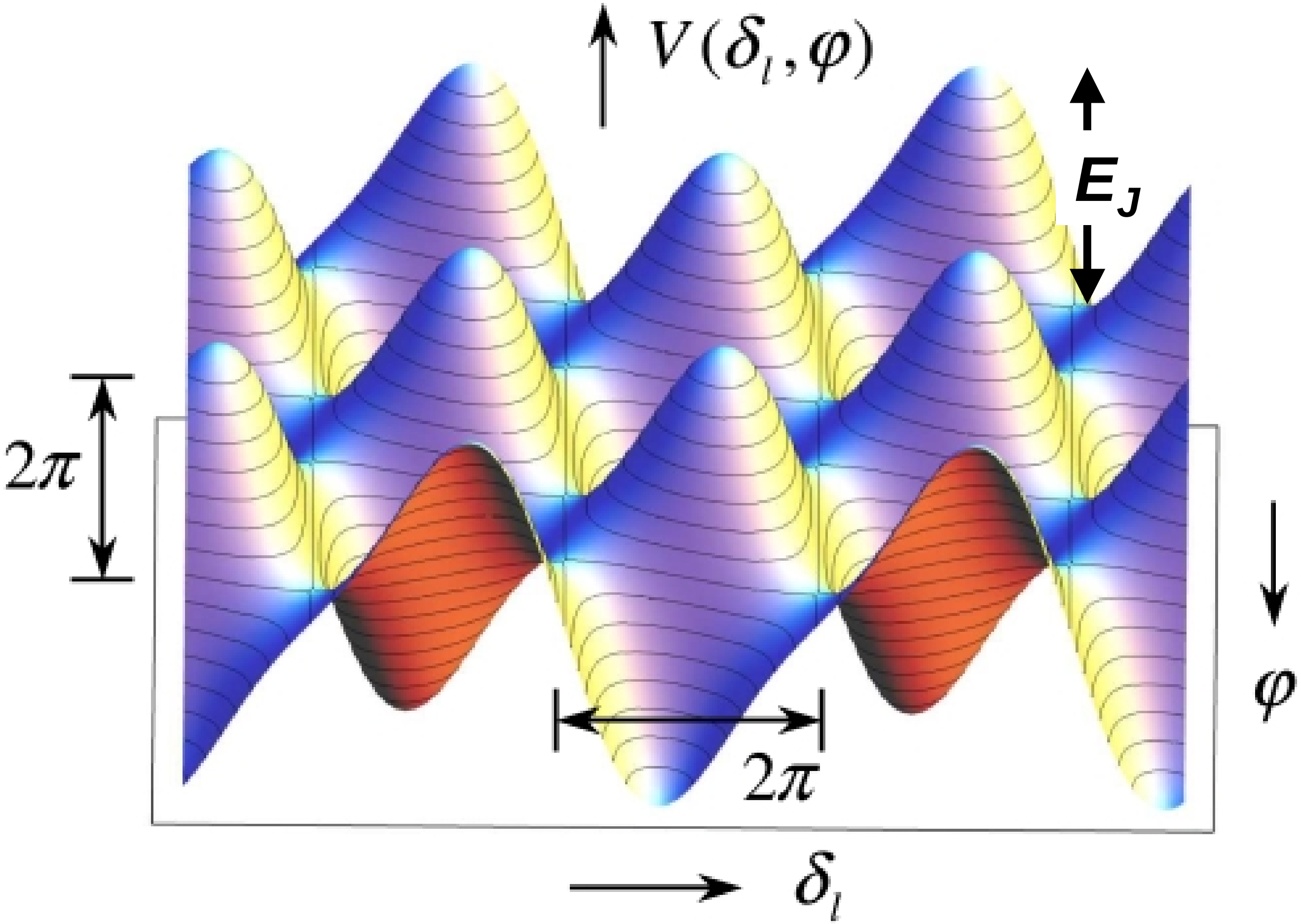
$$s \equiv \frac{I_{0,3}}{I_{0,1}}$$

[Zapata, *et al.*, PRL **77** (1996)]

Generalization

allow for $\delta_1 \neq \delta_2$
for junctions in the left arm

↷ additional variable: $\varphi \equiv \delta_1 - \delta_2$



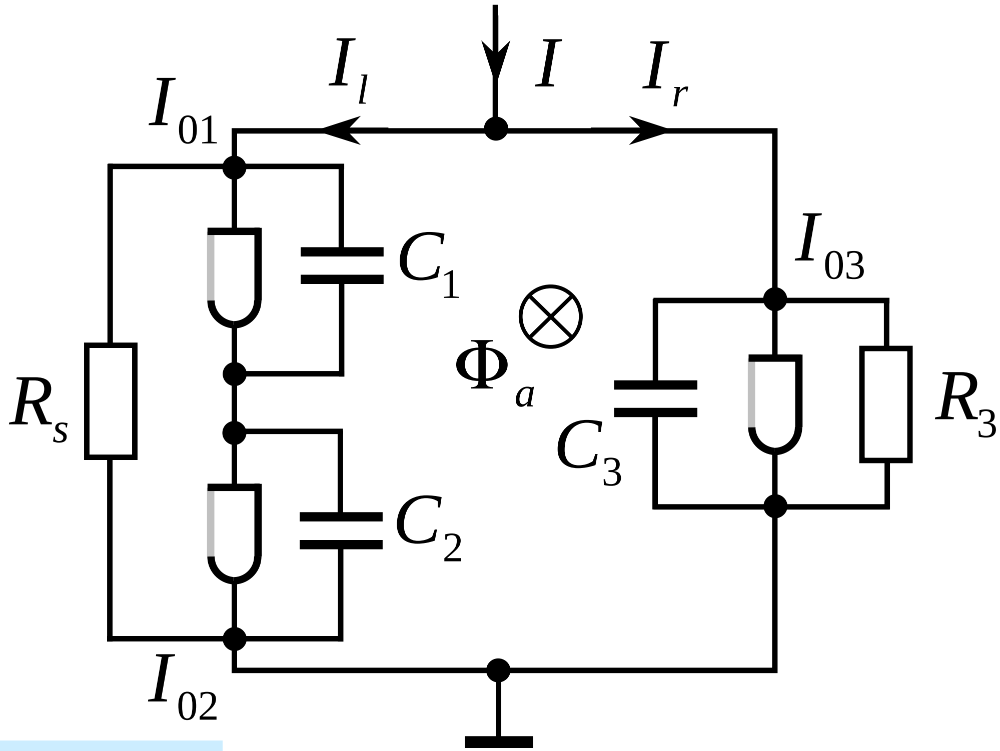
noise parameter:

$$\Gamma \equiv \frac{k_B T}{E_J} = \frac{2\pi k_B T}{I_0 \Phi_0}$$

Modification

[A. Sterck, et al., PRL **95** (2005)]

common shunt → for junctions in the left arm



modifies equations of motion in $\varphi = \delta_1 - \delta_2$:

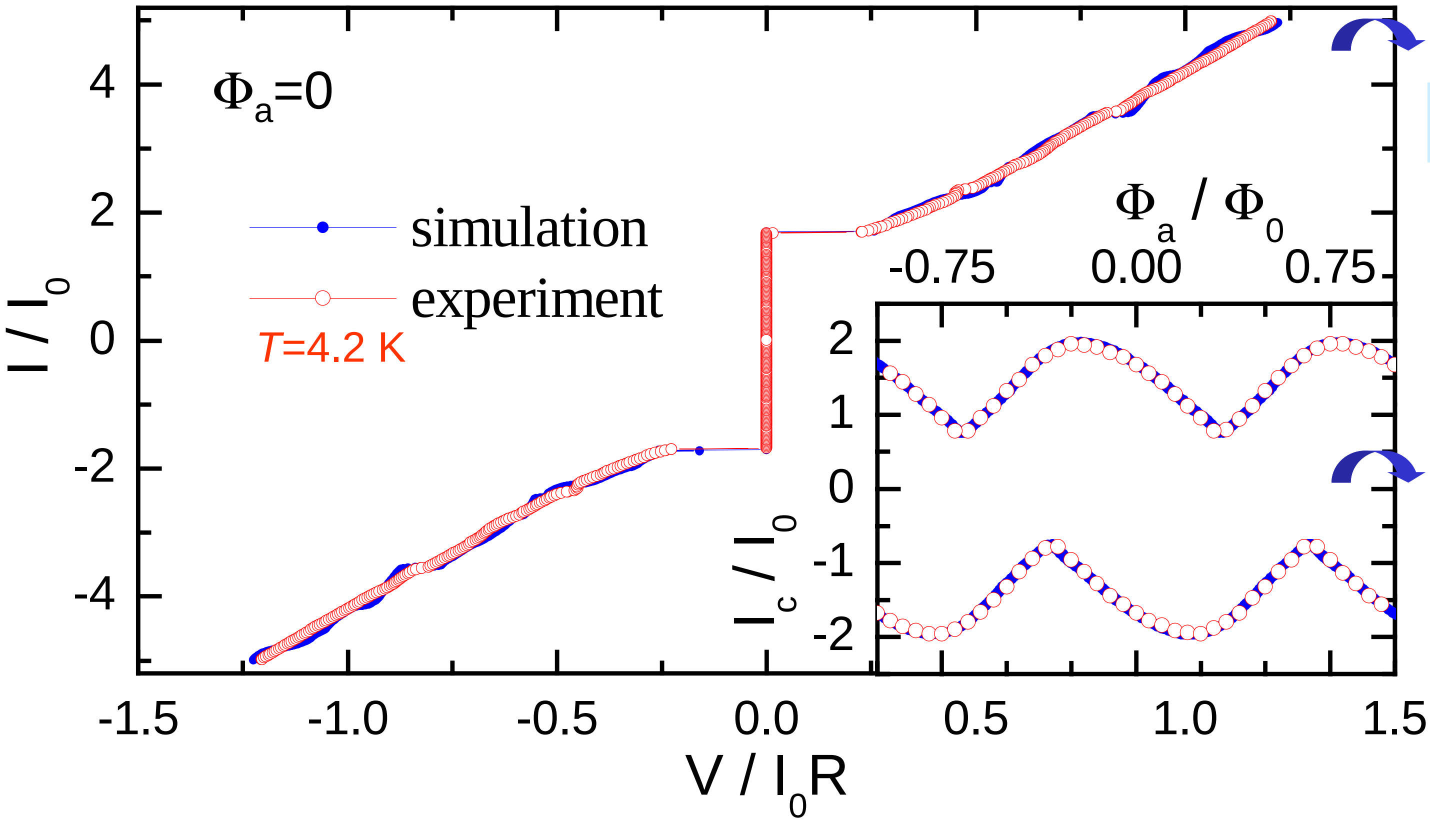
common shunt → underdamped

individual shunts → overdamped

fabrication:
Nb/AlO_x/Nb-technology

j_0=1 kA/cm^2

Experiment: dc properties



$I_0 = 98\ \mu A$

$\Gamma \equiv 2\pi k_B T / (I_0 \Phi_0) = 2 \cdot 10^{-3}$

$R_s \approx R_3 \equiv R = 1.4\ \Omega$

$V_c \equiv I_0 R = 130\ \mu V$

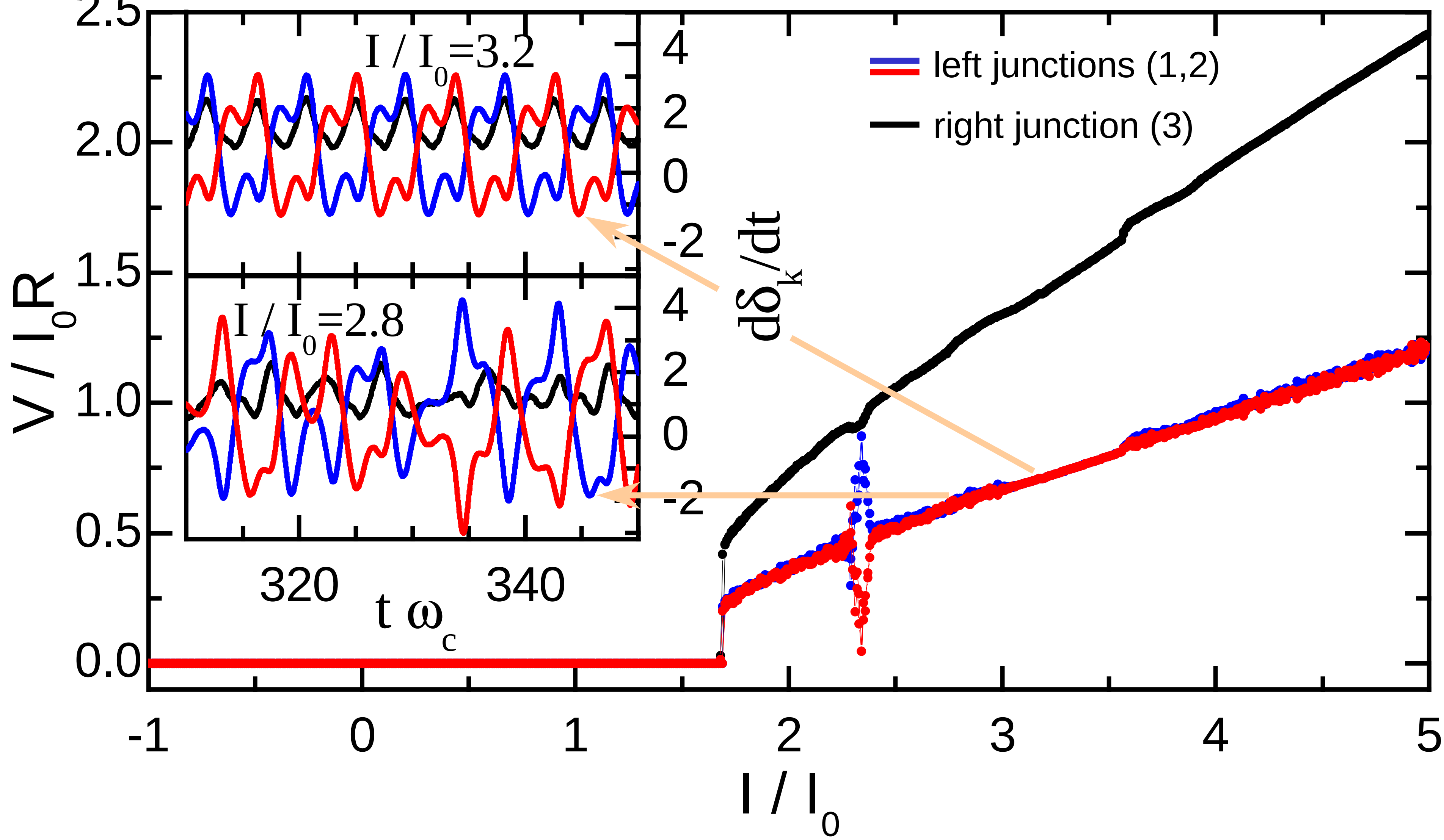
$\omega_c / 2\pi = f_c \equiv V_c / \Phi_0 = 63\ GHz$

$\beta_L = 0.1$

$I_{0,3} / I_{0,1} \equiv s = 0.5$

$I_{0,1} / I_{0,2} \equiv q = 0.99$

Simulation: dc I-V-characteristic

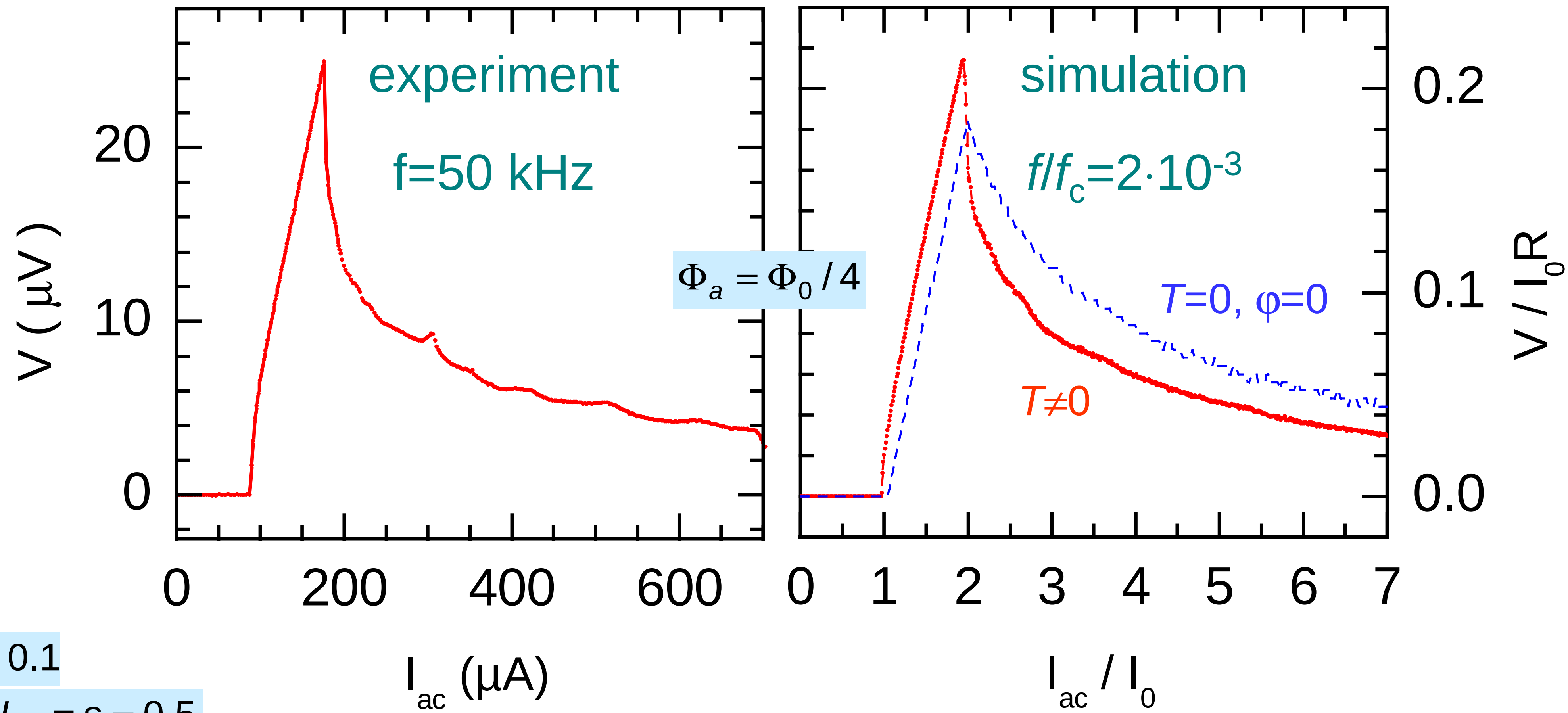


incoherent oscillations ⟺ „noisy“ IVC

j_0=1 kA/cm²

Harmonic drive: Adiabatic regime

$\Gamma = 2 \cdot 10^{-3}$ $V_c \equiv I_0 R = 130\,\mu V$ $f_c \equiv V_c / \Phi_0 = 63\ GHz$



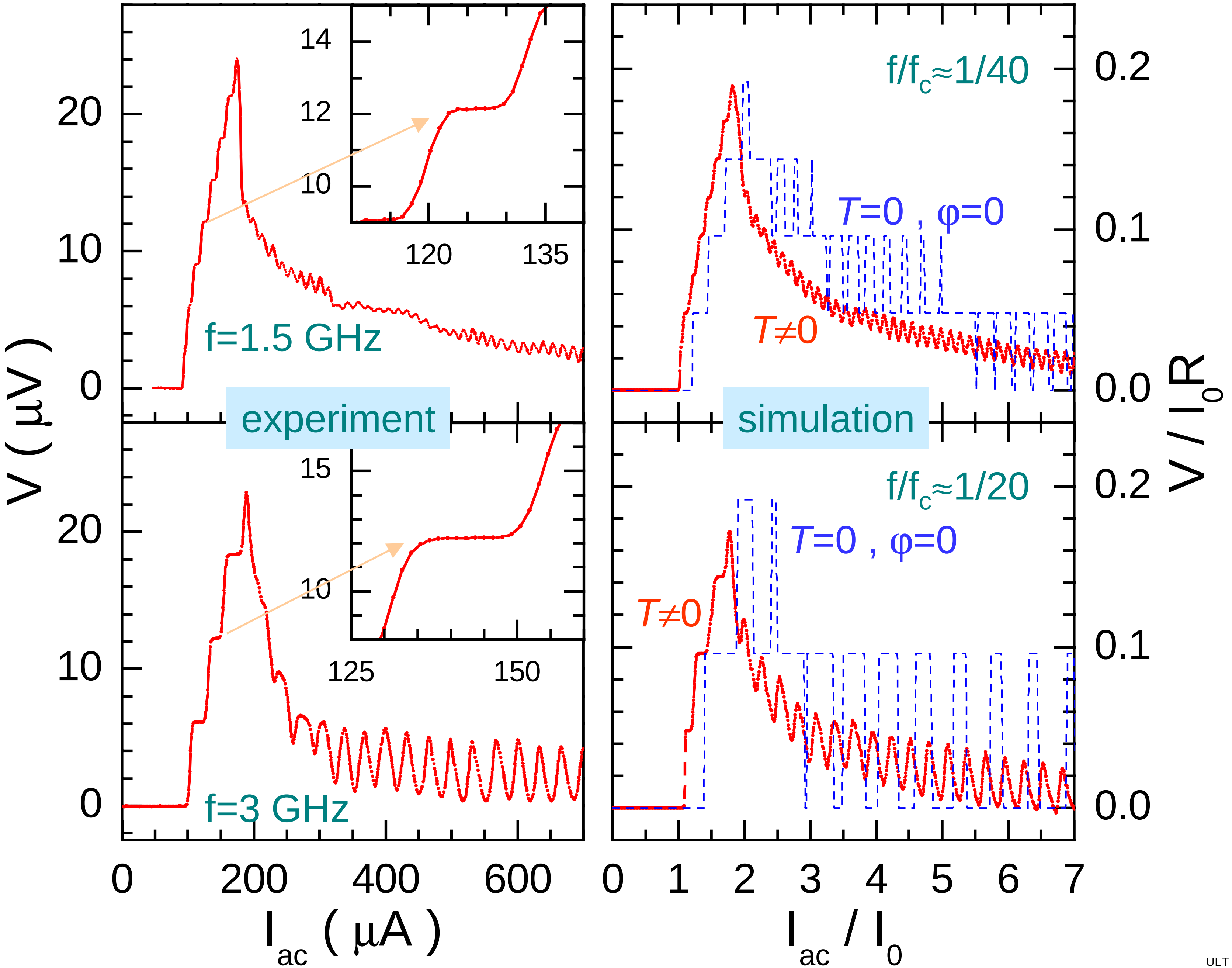
$\beta_L = 0.1$

$I_{0,3} / I_{0,1} \equiv s = 0.5$

$I_{0,1} / I_{0,2} \equiv q = 0.99$

j_0=1 kA/cm^2

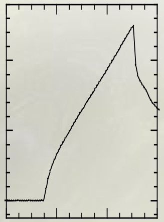
Harmonic drive: Non-adiabatic regime



Conclusions

- **SQUID ratchets → experiments !**
  - **detection of directed transport = measure voltage !**
  - **control over important parameters:**
    - **asymmetry** [design & applied flux]
    - **noise parameter** [design & temperature]
    - **drive frequency & amplitude & spectral distribution** [bias current]

- **experimental demonstration of rectification:**



  - **harmonic drive:**
    - **„low“ frequency → underdamped ratchet**
    - **„high“ frequency → quantization of the ratchet effect (Shapiro steps)**