Josephson junctions as on-chip noise detectors

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Outline

Quantum noise

Josephson junctions

Decay of zero voltage state, MQT

Noise detection by Josephson junctions

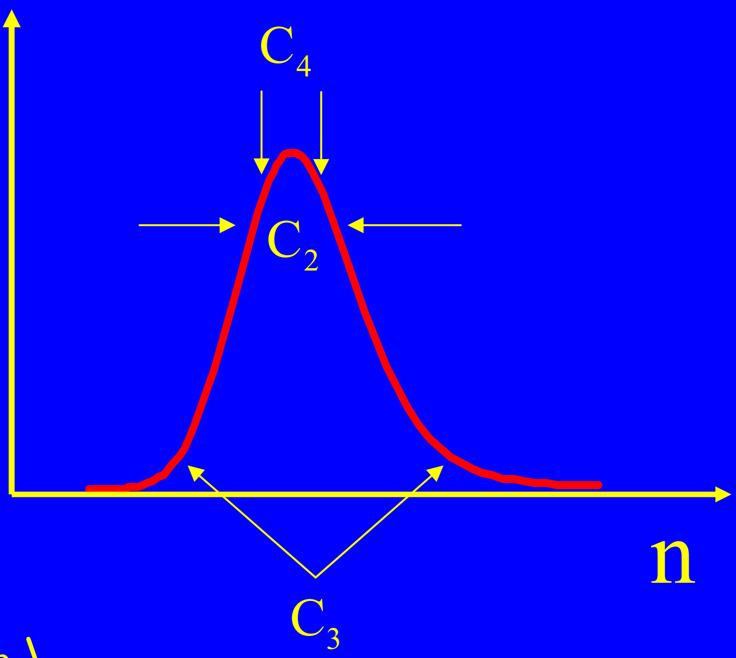
Third cumulant

Forth cumulant

Electrical noise of a mesoscopic conductor

$I = e\,n/\tau$

$P_\tau(n)$



$$C_1 = \langle n \rangle, \quad C_2 = \left\langle \left(n - \langle n \rangle \right)^2 \right\rangle$$

$$C_3 = \left\langle \left(n - \langle n \rangle \right)^3 \right\rangle$$ skewness

$$C_4 = \left\langle \left(n - \langle n \rangle \right)^4 \right\rangle - 3C_2^2$$ sharpness

Electrical noise of a mesoscopic conductor

complete information on I(t)

noise correlation functions

$$\langle I(t)I(0)\rangle \quad \Rightarrow \quad S_2(\omega)$$

$$\langle I(t)I(t')I(0)\rangle \quad \Rightarrow \quad S_3(\omega,\omega')$$

....

How to measure noise cumulants

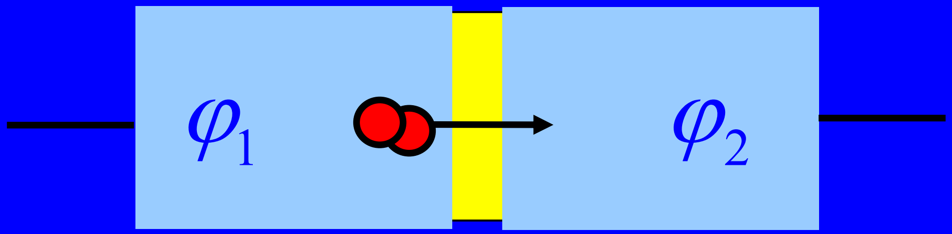
central limit theorem reduces higher cumulants for large sampling times
correlated signals at short times due to finite bandwidth

Reulet et al. use analog mixers to generate the third power of I
average by low pass filter
works for low impedance noise sources (50 Ω)
environmental effects

Reznikov et al. digitize signal form low temperature amplifier
works for very high impedance noise sources (MΩ)
amplifier nonlinearity

On-chip detectors Tobiska & Nazarov, Pekola, Lindell et al., Heikkilä et al.
coupling of two mesoscopic devices

Josephson Junction



superconducting electrodes

Josephson energy: $-E_J\cos(\theta)$ $\theta = \varphi_1 - \varphi_2$

$$E_J = \frac{\hbar I_c}{2e}$$

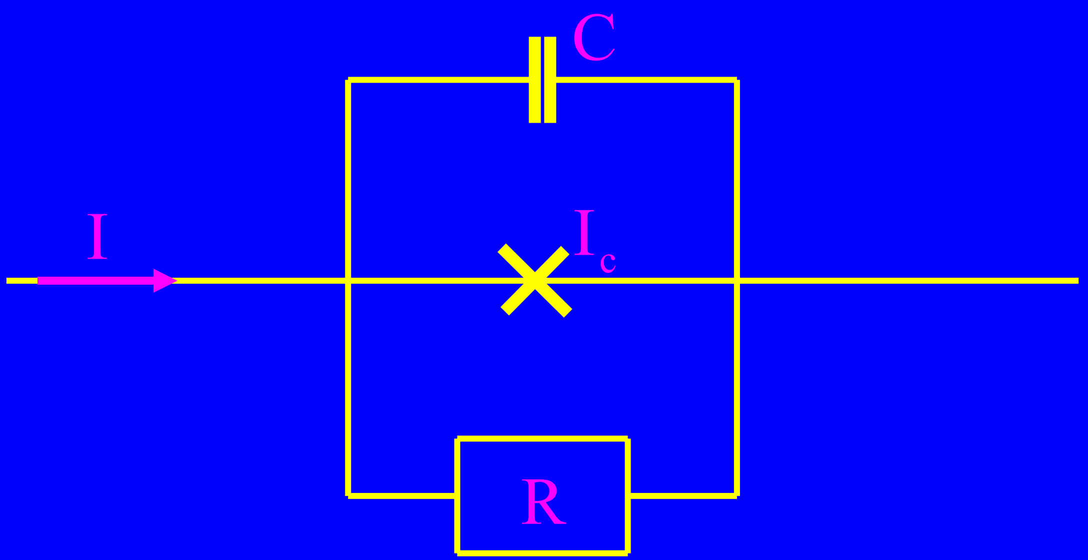
$$H = \frac{Q^2}{2C} - E_J\cos(\theta)$$

no quasiparticles

$$[Q, \theta] = -2ie$$

$$U = \frac{Q}{C} = \frac{\hbar}{2e}\dot{\theta}$$

Current Biased Josephson Junction

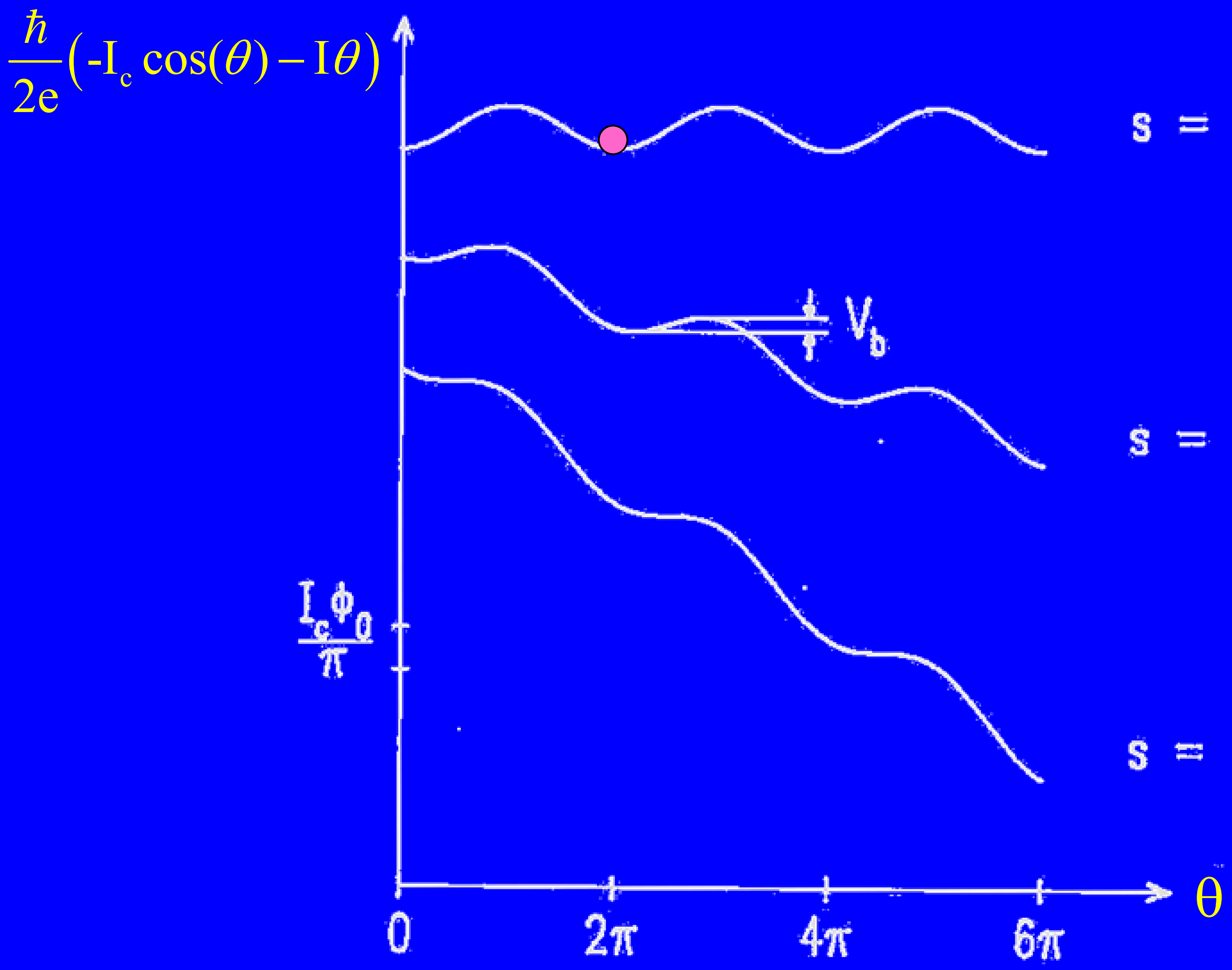


$$I = C\dot{U} + \frac{1}{R}U + I_c \sin\theta$$

$$C\left(\frac{\hbar}{2e}\right)^2 \ddot{\theta} + \frac{1}{R}\left(\frac{\hbar}{2e}\right)^2 \dot{\theta} + \frac{\partial}{\partial\theta}\left\{\frac{\hbar}{2e}\left(-I_c\cos(\theta) - I\theta\right)\right\} = 0$$

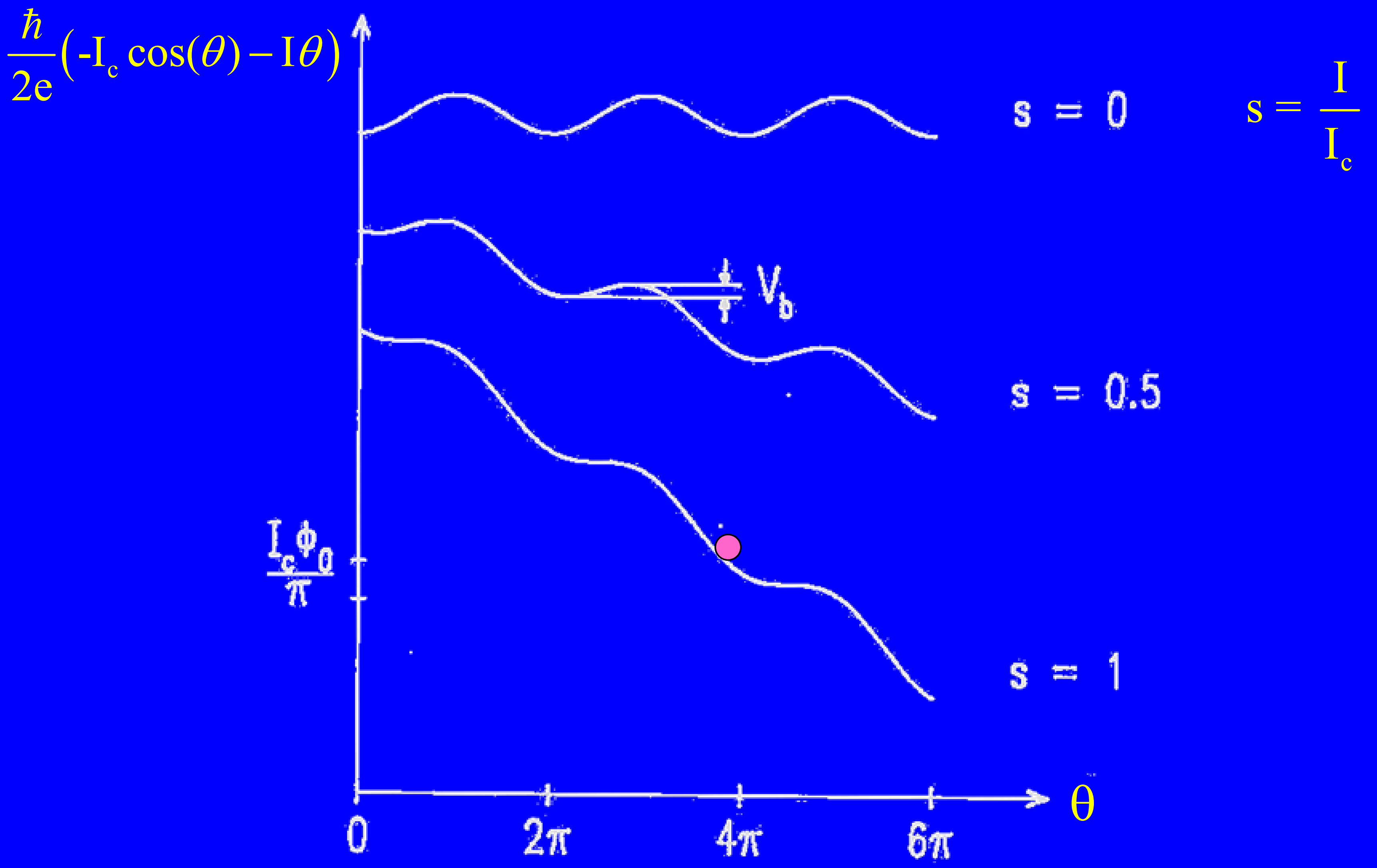
kinetic dissipative potential

"Particle" in Tilted Washboard Potential



$s = \frac{I}{I_c}$

"Particle" in Tilted Washboard Potential



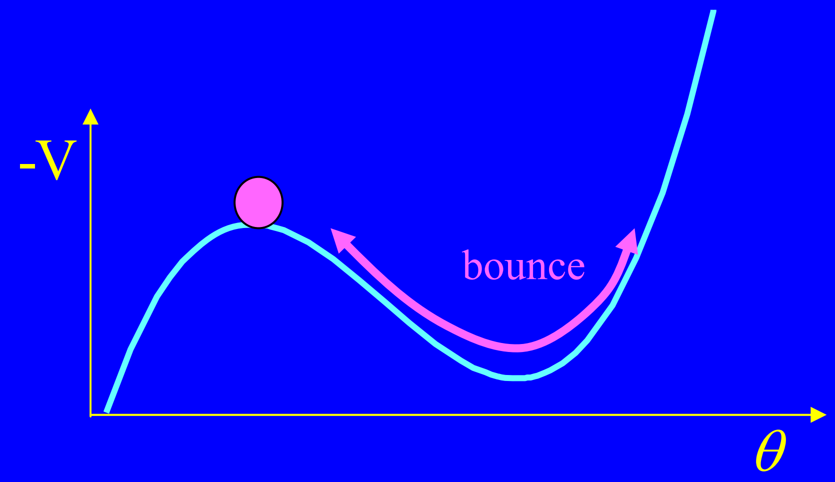
Transition U=0 → U ≠ 0

MQT Rate

bounce technique

$$Z = \mathrm{tr}\, e^{-\beta H} = \int \mathcal{D}[\theta]\, e^{-S[\theta]}$$

$$Z = Z_{well} + Z_{bounce}$$



$$\Gamma = \frac{2}{\beta}\, \mathcal{Im} \ln Z = \frac{2}{\beta} \frac{\mathcal{Im}\, Z_{bounce}}{Z_{well}} = f\, e^{-S_{bounce}}$$

prefactor f: fluctuations about bounce, zero mode, unstable mode

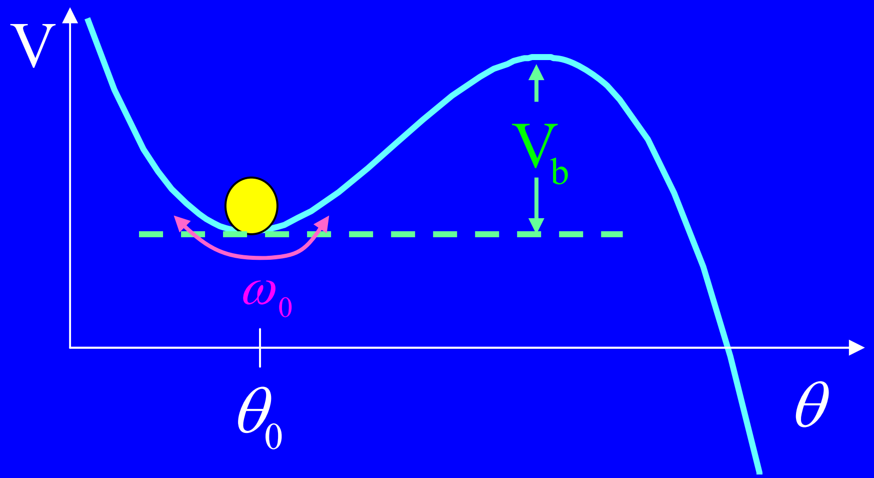
Bounce Technique

Decay rate form imaginary part of free energy

Langer (1967)
Miller (1975)
Callan and Coleman (1977), Stone (1977)

Caldeira and Leggett (1981)

Decay rate in cubic potential

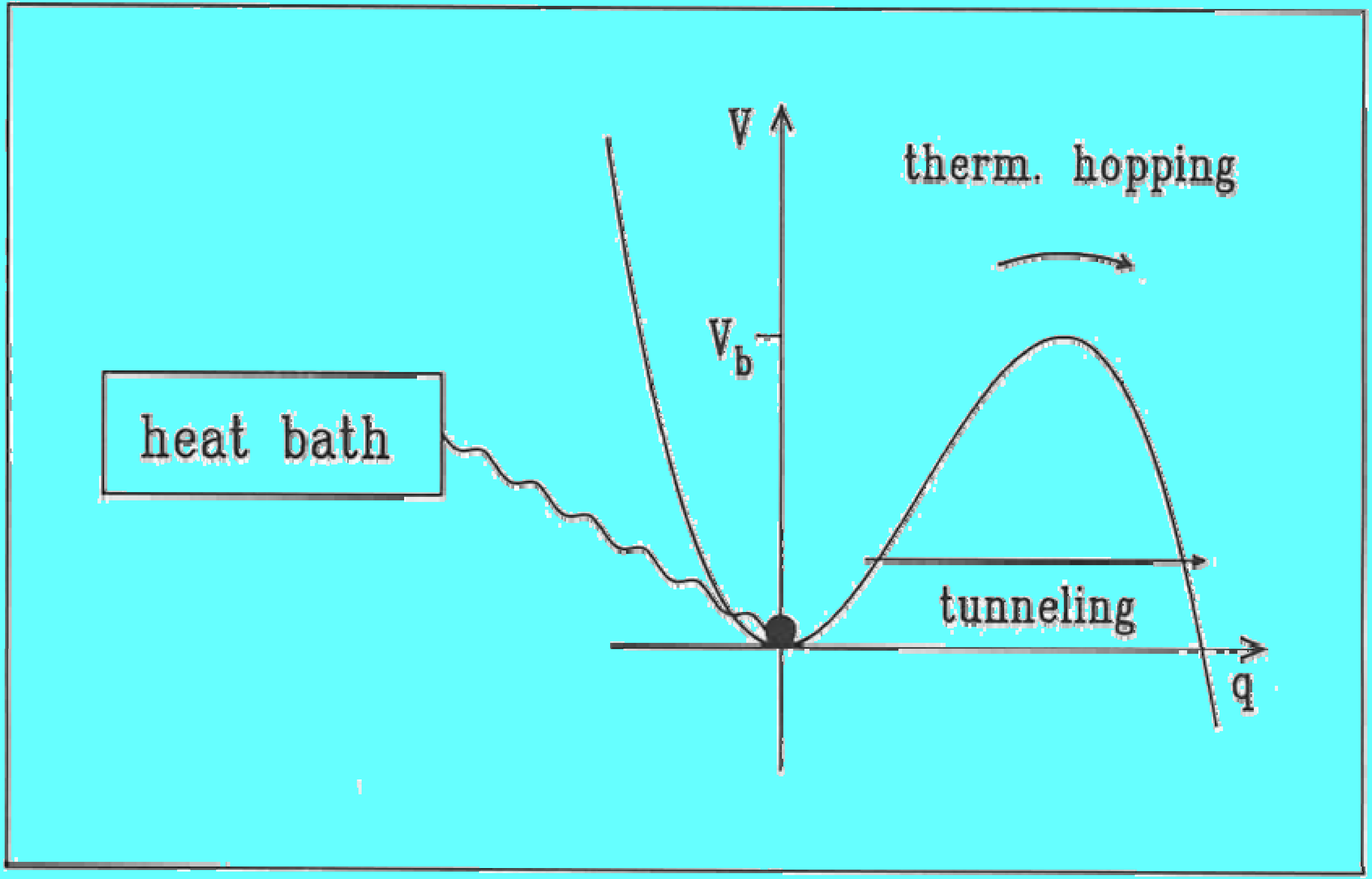


$V(\theta)$: cubic polynominal

$\omega_0^2 = V''(\theta_0)/m$, V_b

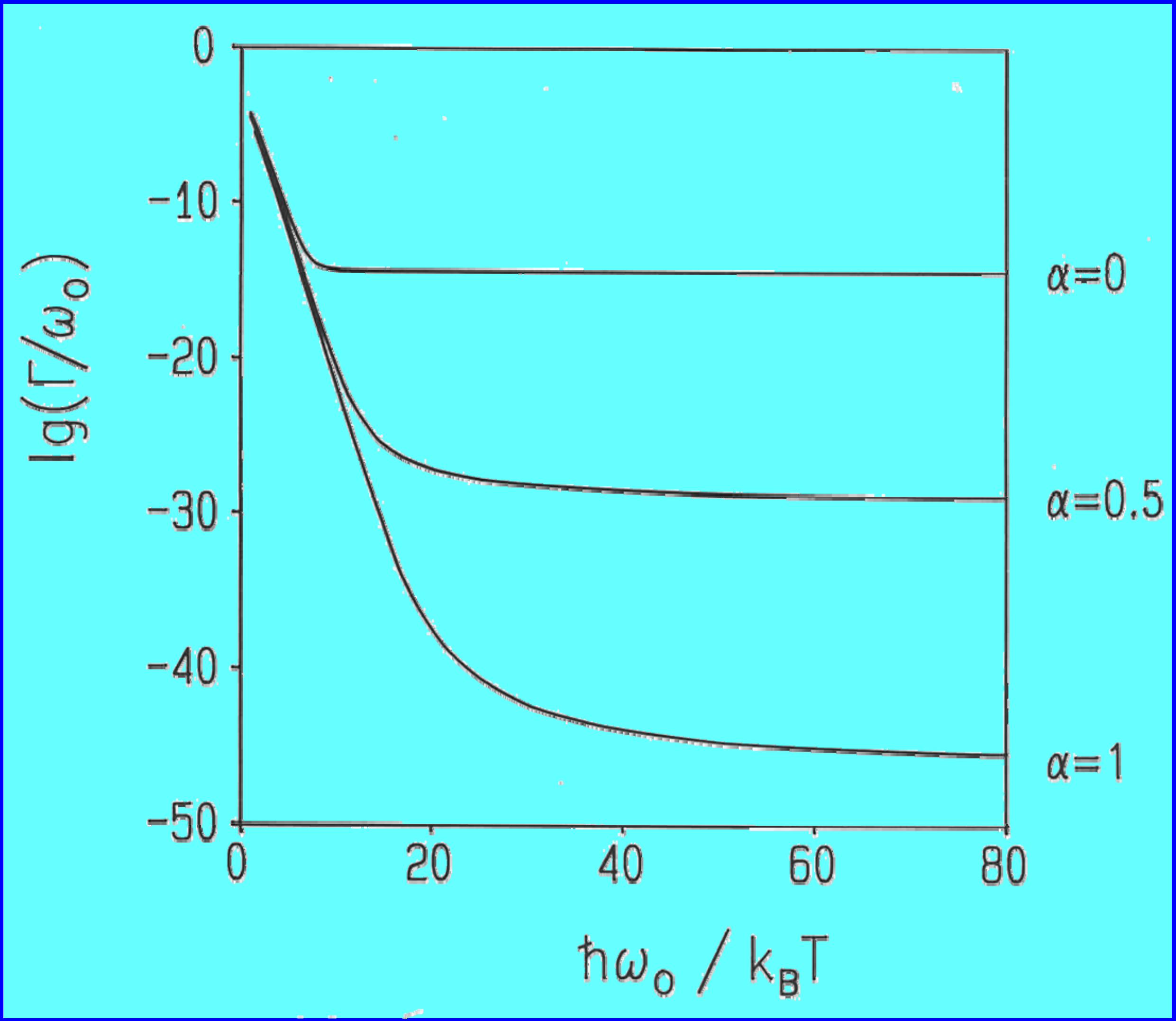
$$\Gamma_0 = 6\sqrt{6\omega_0 V_b / \pi}\ \exp\left(-\frac{36}{5}\frac{V_b}{\omega_0}\right)$$

zero temperature, no dissipation



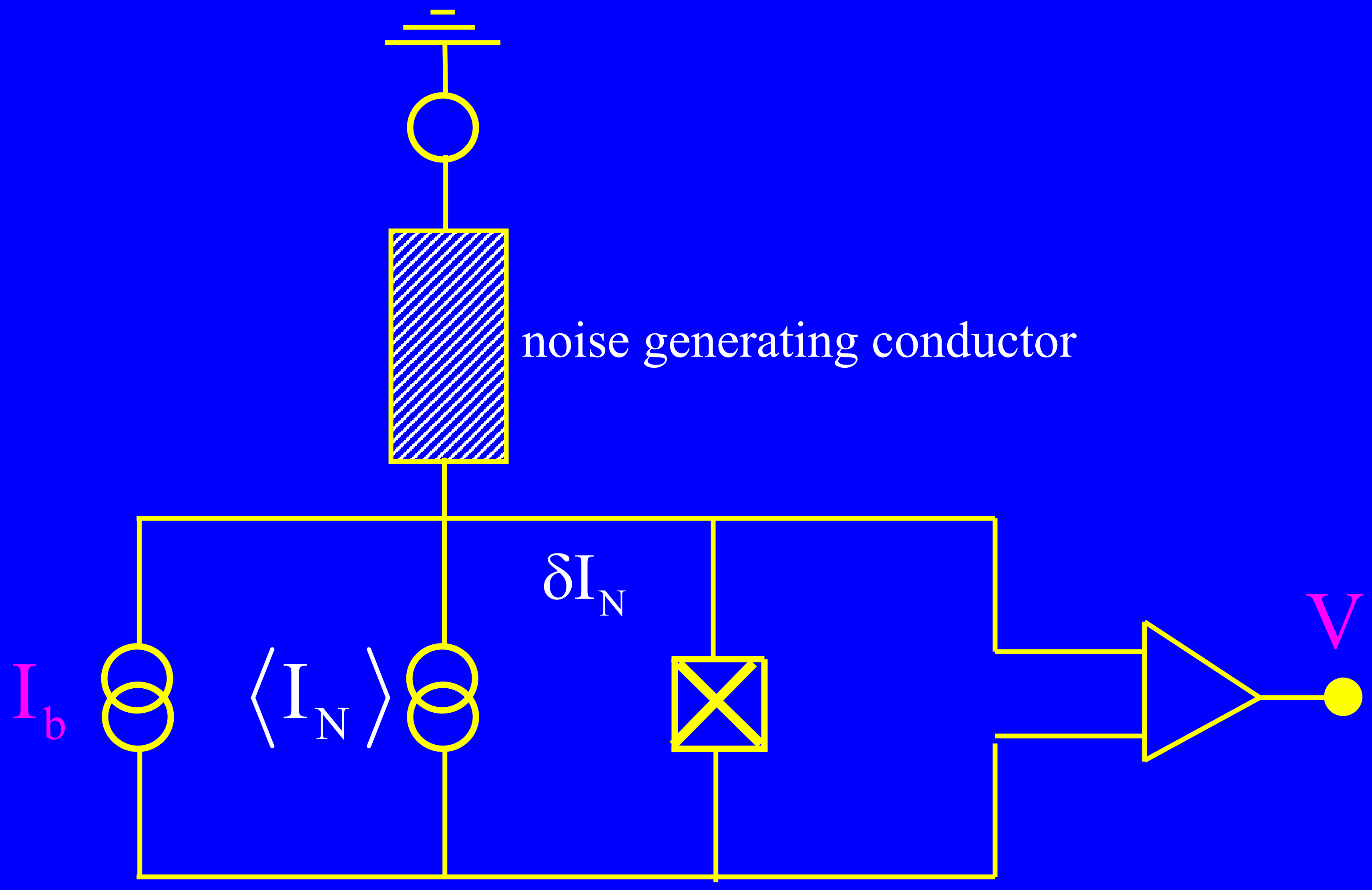
V
therm. hopping
V_b
heat bath
tunneling
q

MQT Rate at Finite T with Dissipation

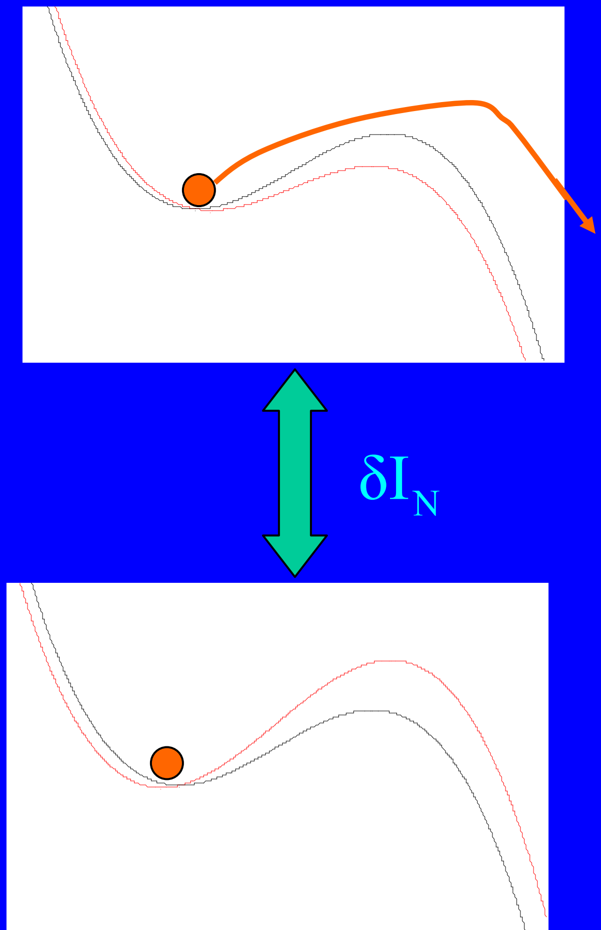


HG, Olschowski & Weiss, PRB 36, 1931 (1987)

Detection of Noise Cumulants



Josephson Junction Driven by Electrical Noise



Escape rate: exponentially sensitive to potential variations

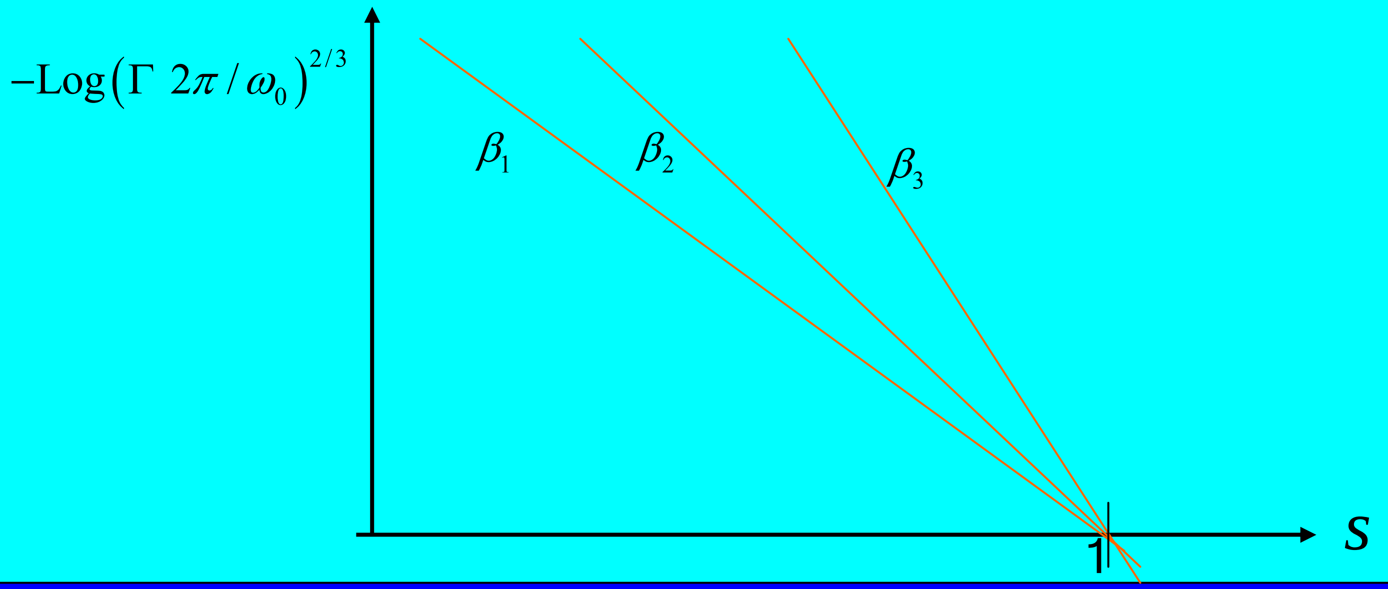
Classical switching rate of a Josephson junction

$E_J \gg kT$,
$s = I_b/I_0 < 1$:

$$\Gamma = \frac{\omega_0}{2\pi}\, a_{th}\, \exp\left[-\frac{4\sqrt{2}}{3}\beta E_J (1-s)^{3/2}\right]$$

$\omega_0 = \sqrt{2E_C E_J/h^2}\,(1-s)^{3/2}$ $\qquad a_{th} = O(1)$

Scaling: $-\mathrm{Log}\left(\Gamma\ 2\pi/\omega_0\right)^{2/3} \propto \beta^{2/3}(1-s)$



Thermal escape in presence of non-Gaussian noise

Noise generated by a mesoscopic conductor

$$\eta(\mathrm{t}) = \frac{\hbar}{2e}\delta \mathrm{I_N}(\mathrm{t})$$

Equation of motion

$$\mathrm{M}\ddot{\varphi} + \mathrm{M}\gamma\dot{\varphi} + \underbrace{\mathrm{V}'(\varphi) - \eta(\mathrm{t})}_{\mathrm{V}'_{\mathrm{eff}}(\varphi,\mathrm{t})} = \xi(\mathrm{t})$$

Thermal escape driven by non-Markovian, non-Gaussian noise

Generalized Fokker-Planck equation

$$\frac{\partial W_\eta(q,p,t)}{\partial t} = \left[-\frac{p}{M}\frac{\partial}{\partial q} + \frac{\partial}{\partial p}\left(V'(q) + \gamma p \right) + M\gamma k_B T \frac{\partial^2}{\partial p^2} \right] W_\eta(q,p,t) - \eta(t)\frac{\partial}{\partial p} W_\eta(q,p,t)$$

$$\bar{W}(p,q,t) = \left\langle W_\eta(q,p,t) \right\rangle_\eta$$

$$\frac{\partial \bar{W}}{\partial t} = \mathbf{L_0}\bar{W} - \left(C_2 \frac{\partial^2}{\partial p^2} + C_3 \frac{\partial^3}{\partial p^3} \right) \bar{W}$$

$$C_2 = \int_0^\infty dt \left\langle \eta(t)\eta(0) \right\rangle \qquad C_3 = \int_0^\infty dt \int_0^\infty dt' \left\langle \eta(t+t')\eta(t')\eta(0) \right\rangle$$

$$\left\langle I \right\rangle \gg e\omega_0$$

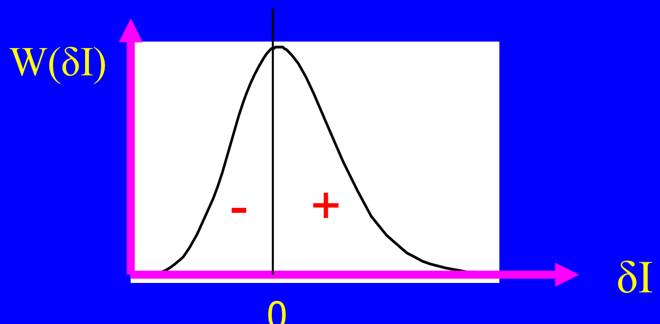
Effective temperature

$$T_{eff} = T\left(1 + \frac{C_2}{M\gamma kT}\right)$$

Rate expression:

$$\Gamma \propto e^{-\beta_{eff} V_b (1-g)} \qquad g = \frac{9V_b}{5q_0}\frac{C_3}{\left(M\gamma kT_{eff}\right)^2}$$

describes rate enhancement/suppression



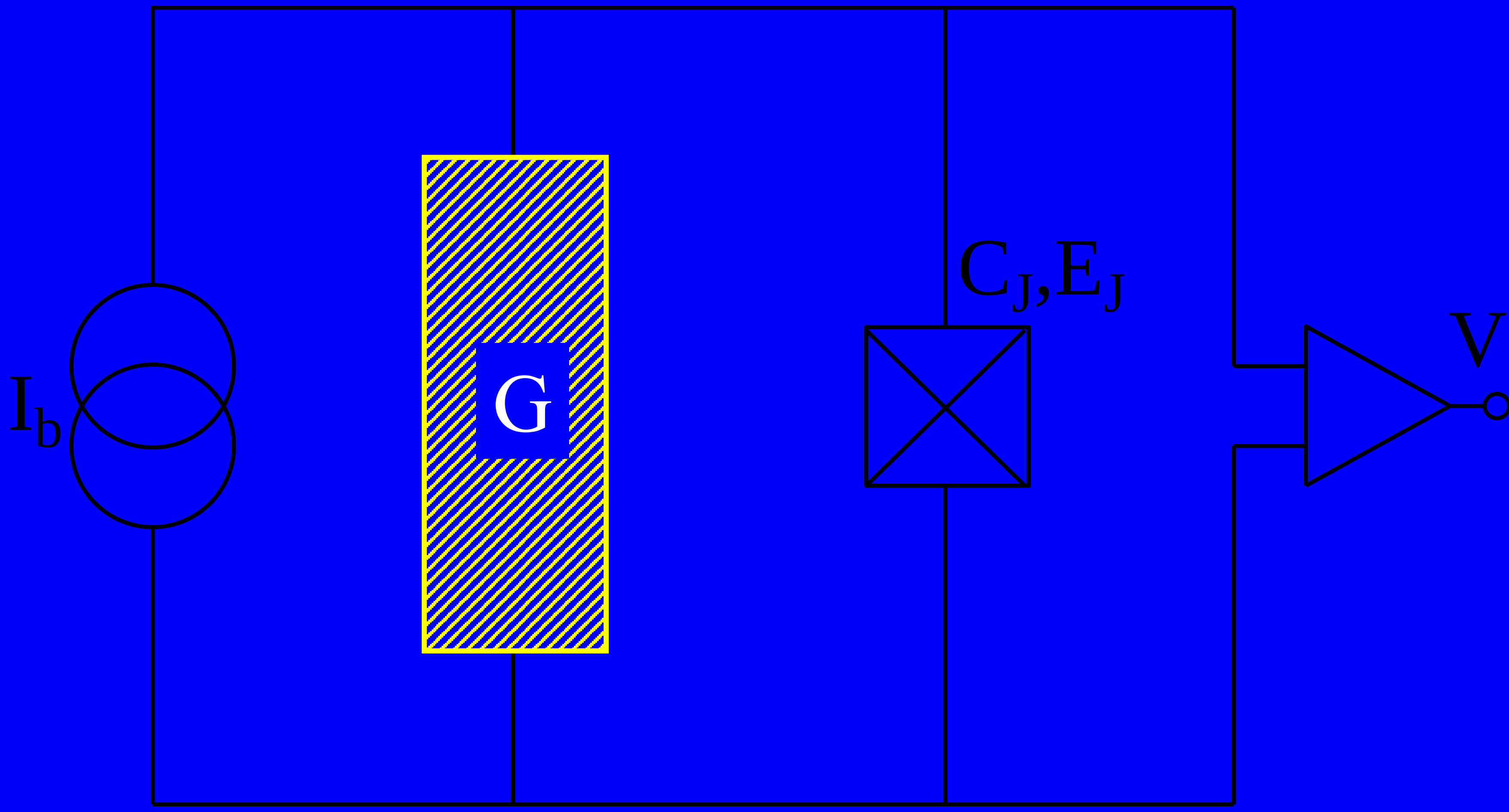
Rate asymmetry

$$R = \frac{\Gamma(I_b)}{\Gamma(-I_b)} = \exp\left\{ \frac{8\sqrt{2E_J^3}}{15kT_{eff}} (1-s)^{3/2} g(s_N) \right\}$$

$$s = \frac{I_b}{I_c}, \quad s_N = \frac{I_N}{I_c}$$

J. Ankerhold *to be published*

→ Exp. Results: H. Pothier



No current through conductor G before switching

Noise Generating Functional

$$G[\phi] = e^{-S_G[\phi]} = \left\langle \mathcal{T} \exp\left[\frac{\mathrm{i}}{e} \int_C dt\; I(t)\phi(t) \right] \right\rangle$$

$$C_n(t_1,\ldots,t_n) = -\left(-\mathrm{i}e\right)^n \partial^n S_G[\phi] / \partial\phi(t_1)\cdots\partial\phi(t_n)\,|_{\phi=0}$$

Noise cumulants

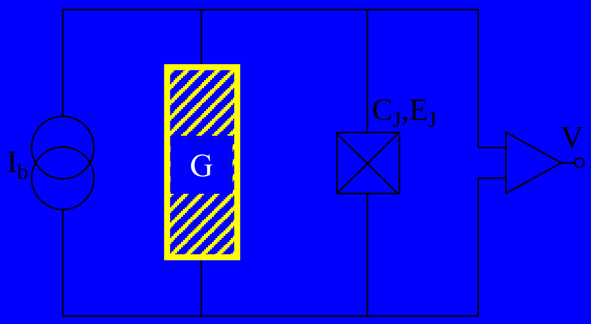
Ohmic Resistor

$$S_R[\phi] = \frac{h}{2\pi e^2 R} \int_C dt \int_C dt' \, \alpha(t-t') \, \phi(t)\phi(t')$$

$$\alpha(t) = \frac{\pi}{2(\hbar\beta)^2 \sinh^2(\pi t / \hbar\beta)}$$

Tunnel Junction

$$S_T[\phi] = \frac{-h}{\pi e^2 R_T} \int_C dt \int_C dt' \, \alpha(t-t') \, \sin^2\left[\frac{\phi(t)-\phi(t')}{2}\right]$$



bounce technique

$$Z = \int \mathcal{D}[\theta]\ e^{-S[\theta]}$$

$$S[\theta] = S_{JJ}[\theta] + S_G[\theta/2]$$

$$\Gamma \approx \Gamma_0\, e^{-S_G\left[\theta_{\mathrm{Bounce}}/2\right]}$$

$$g = \frac{h}{e^2 R} \ll \frac{E_J}{\hbar\omega_0}$$

Noise From Tunnel Junction

bounce $\theta(\tau) = \theta_0 \cosh^{-2}(\omega_0 \tau / 2)$

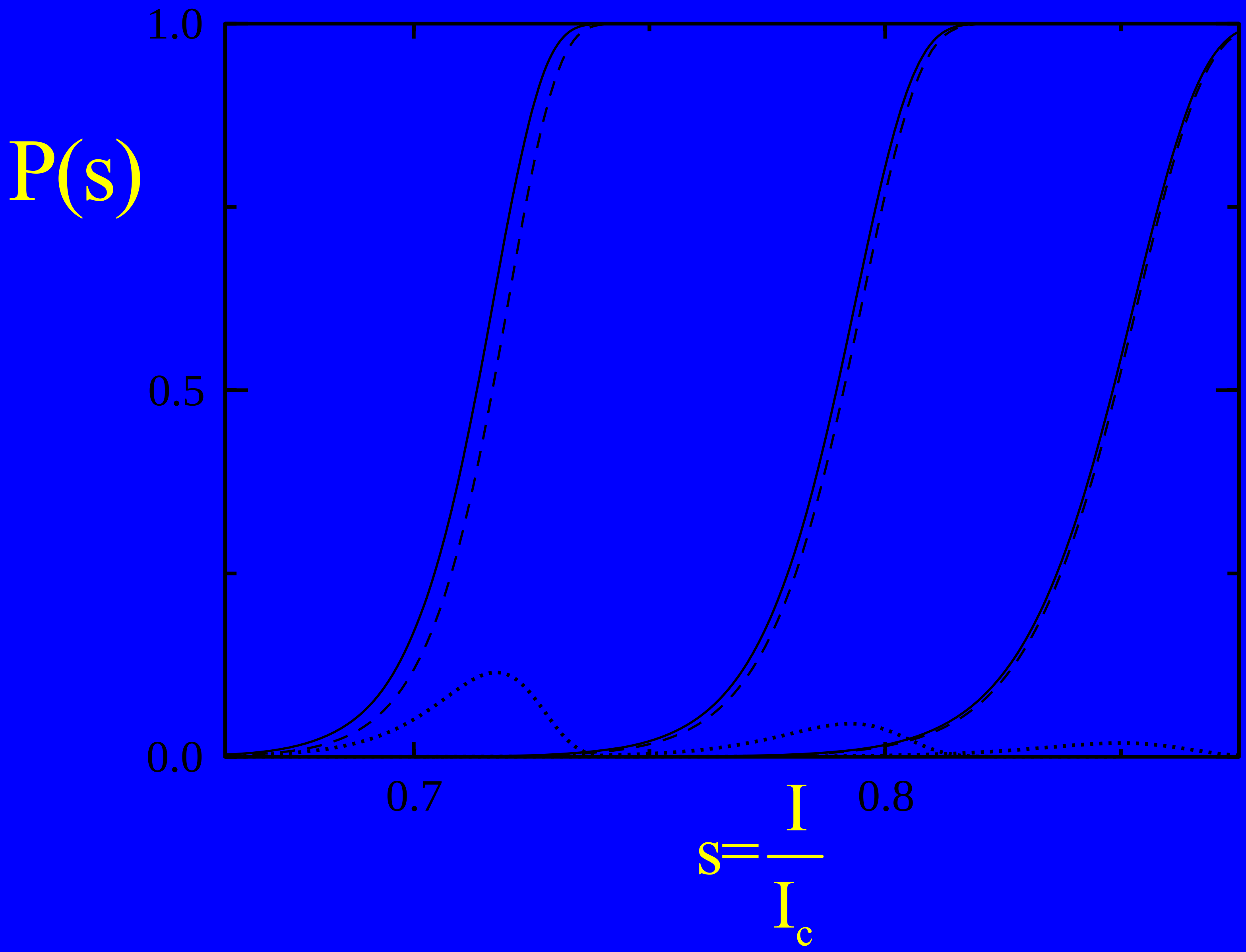
$$\Gamma = \Gamma_0 \exp\left[-\frac{6\varsigma(3)}{\pi^3} g_T \theta_0^2 \right] \exp\left[\left(168 \frac{\varsigma(3)}{\pi^3} + 960 \frac{\varsigma(5)}{\pi^5} + 1260 \frac{\varsigma(7)}{\pi^7} \right) g_T \theta_0^4 \right]$$

Rate suppression by 2nd cumulant

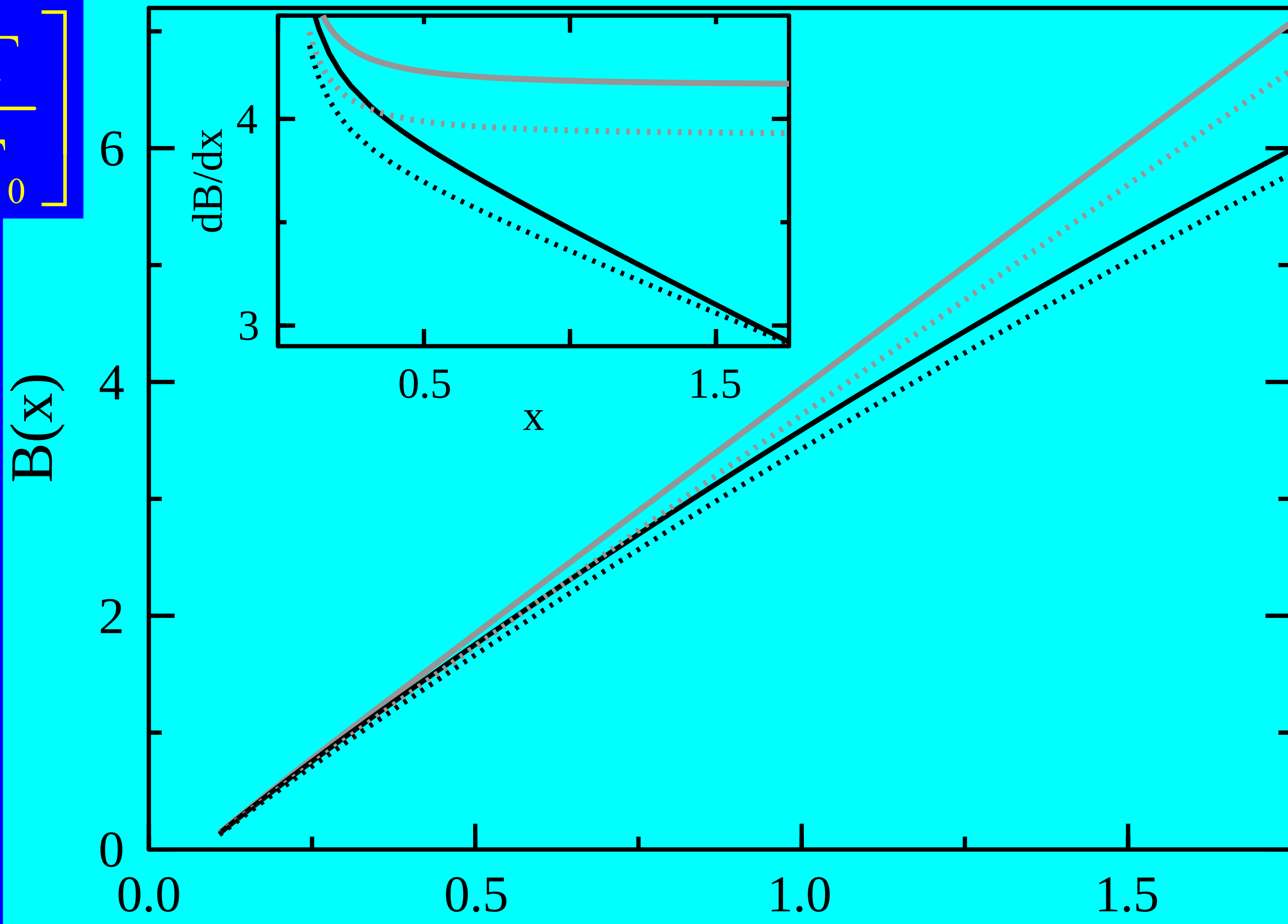
Rate enhancement by 4th cumulant

Switching Probability for Tunnel Junction and Ohmic Resistor

(with identical 2nd cumulant)



$$-\ln\left[\frac{\Gamma}{\Gamma_0}\right]$$



$$x = \frac{1-s^2}{s^2}$$

Summary

Josephson Junctions

Basis for highly sensitive detectors

On- chip detection of non-Gaussian noise
Device characterization

Ankerhold & HG, Phys. Rev. Lett. 95, 186601 (2005)

Ankerhold to be published