Effect of Coulomb interaction on noise an transport properties of a weakly reflecting QPC

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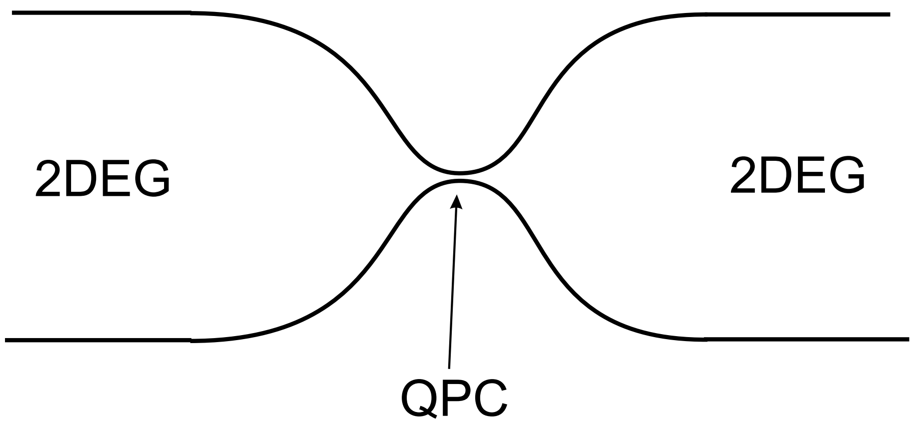
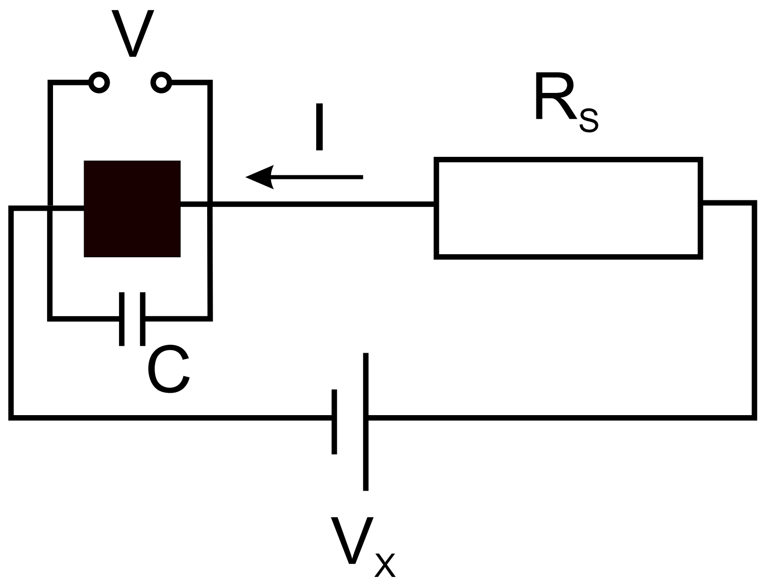
Outline

1. **Introduction and motivation**
2. **Effective action of a weakly reflecting Quantum Point Contact (QPC)**
3. **QPC embedded in electro-magnetic environment, correction to the current**
4. **Correction to the noise, non-equilibrium FDT**
5. **Quantum dot**

Model: QPC embedded in electro-magnetic environment





QPC is characterized by S-matrix $\hat{S}$

EM env. — resistor R_S and capacitor C

T_n — channel transmissions

all $T_n \ll 1$ — tunnel junction; **$1 - T_n \ll 1$ — weakly reflecting QPC**

<u>Our aim: current $I(V)$ and noise $S(\omega, V)$ for weakly reflecting QPC</u>

Tunnel junction, $I(V)$

Coulomb blockade and environment (see review by Ingold and Nazarov from 1992)

$$I(V) = \frac{1}{eR}\int dE\, E\, \frac{1-e^{-eV/T}}{1-e^{-E/T}} P(eV-E),$$

$$P(\omega) = \frac{1}{2\pi}\int dt\, e^{-F(t)-iK(t)+i\omega t},$$

$$F(t) = \frac{e^2 R_S}{2\pi}\int d\omega\, \frac{\omega \coth\frac{\omega}{2T}}{1+\omega^2 R_S^2 C^2}\frac{1-\cos\omega t}{\omega^2},$$

$$K(t) = \frac{e^2 R_S}{2\pi}\int d\omega\, \frac{1}{1+\omega^2 R_S^2 C^2}\frac{\sin\omega t}{\omega}$$

$$F(t) \propto \langle \hat{V}(t)\hat{V}(0) + \hat{V}(0)\hat{V}(t)\rangle$$

$$K(t) \propto \langle \hat{V}(t)\hat{V}(0) - \hat{V}(0)\hat{V}(t)\rangle$$

$I(V)$ for weakly reflecting QPC ?

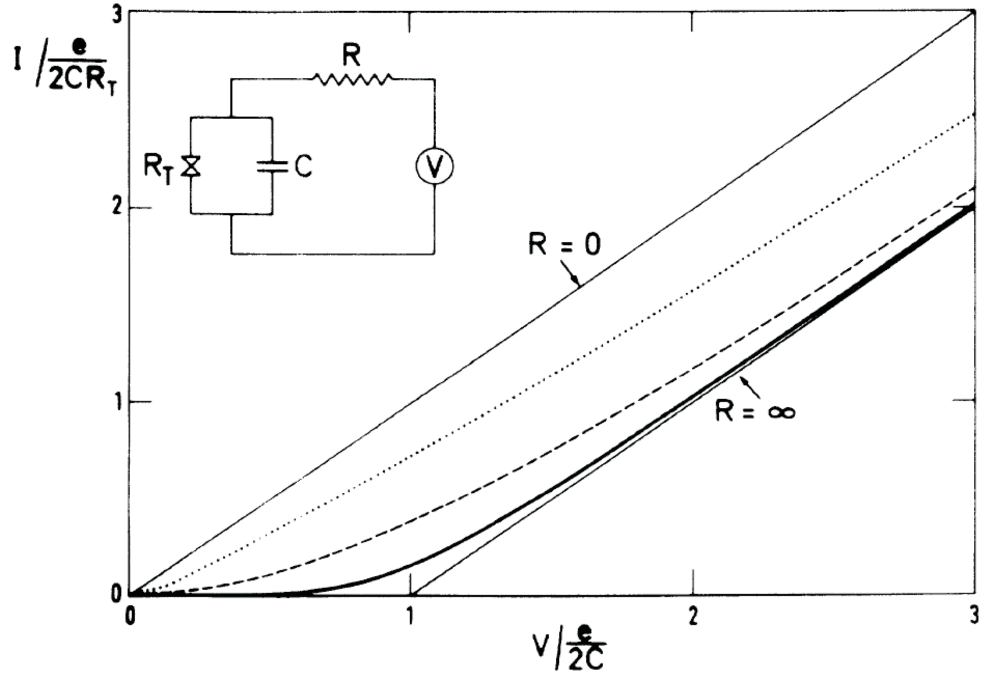


FIG. 2. The I-V characteristic of a tunnel junction coupled to an environment characterized by a resistance R (see inset) for R/R_Q=0, 0.1, 1, 10, and ∞.

Figure is taken from Devoret *et al*, PRL 1990

Noise of a tunnel junction, non-equilibrium FDT

$$S(\omega) = \int dt\, e^{i\omega t} \left(\left\langle \hat{I}(t)\hat{I}(0) + \hat{I}(0)\hat{I}(t) \right\rangle - 2\langle I \rangle^2 \right)$$

Equilibrium FDT, arbitrary system $S(\omega) = 2\,\mathrm{Re}\left[G(\omega)\right] \omega \coth \frac{\omega}{2T}$

Non-equilibrium FDT for tunnel junctions

$$S(\omega) = eI\left(\frac{\omega}{e} + eV\right) \coth \frac{\omega + eV}{2T} + eI\left(\frac{\omega}{e} - eV\right) \coth \frac{\omega - eV}{2T}$$

Valid for

1. **No interaction, NIN, SIN, SIS tunnel junctions (Dahm** *et al*, **PRL 1969)**
2. **Normal tunnel junction embedded in el.-mag. environment (Lee, Levitov, PRB 1996)**
3. **Cotunneling through quantum dots (see e.g. Sukhorukov, Burkard, Loss, PRB 2001)**
4. **SIN tunnel junctions, Andreev reflection ($e \to 2e$) and other systems**

Weakly reflecting QPC, simple arguments

N conducting channels, $1 - T_n \ll 1$ for $n = 1, \dots, N$

Split current in two parts

$$I(V) = \frac{e^2 N}{\pi} V + \delta I$$

$e^2 N V / \pi$ — contribution of transmitted electrons,
$\delta I \propto \sum_n 1 - T_n$ — small contribution of reflected electrons, $\delta I < 0$

Assumptions:

1. transmitted electrons — statistics of an Ohmic resistor
2. reflected electrons, rare events $\Rightarrow$ statistics of electrons transm. trough tunnel junction
3. no correlation between transmitted and reflected electrons

$$S(\omega) = \frac{2e^2 N}{\pi} \omega \coth \frac{\omega}{2T} - \sum_{\pm} e\delta I \left(\frac{\omega}{e} \pm eV \right) \coth \frac{\omega \pm eV}{2T}$$

Is it correct? It turns out to be <u>almost correct</u>

Previous work, summary

1. Formal analogy between a single channel QPC and Luttinger liquid. Bosonization technique. Quantum dot with two single channel QPC

   Flensberg, PRB 1993; Furusaki Matveev, PRL 1995, PRB 1995;

   multichannel case: Aleiner, Brouwer, Glazman, Phys. Rep. 2002

$$G = \frac{e^2}{\pi}\frac{N_1 N_2}{N_1 + N_2} - c_N \frac{e^2}{\pi}\left(\frac{E_C}{T}\right)^{\frac{1}{N_1+N_2}}$$

2. $1/g$ expansion, $g \gg 1$

   Golubev, Zaikin PRL 2001; single channel QPC – A.L. Yeyati *et al* PRL 2001

$$G(T) = G_0 - \frac{e^2}{\pi}\frac{\sum_n T_n(1-T_n)}{\sum_n T_n}\ln\frac{1}{TRC}$$

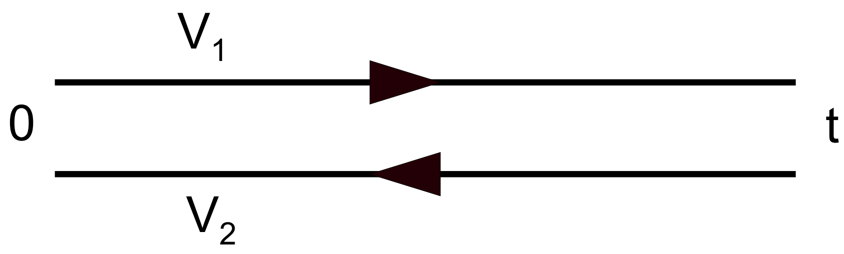
3. $g \gg 1$, RG approach, Full Counting Statistics

   Kindermann, Nazarov, PRL 2003;

   quantum dot — Bagrets, Nazarov, PRL 2005

Formalism of effective action

Keldysh contour, fluct. potentials $V_{1,2}$



classical field $V^{+}=(V_1+V_2)/2$

quantum field $V^{-}=V_1-V_2$

phases $\varphi^{\pm}(t) = \int_0^t dt' \, eV^{\pm}(t')$

Action of environment

$$
\begin{aligned}
iS_{env} &= i \int dt \, C \frac{\dot{\varphi}^{+}\dot{\varphi}^{-}}{e^2} \\
&\quad + \frac{i}{e^2 R_S} \int \varphi^{-}(eV_x - \dot{\varphi}^{+}) \\
&\quad + \frac{1}{2e^2 R_S} \int dt dt' \frac{\pi T^2 \varphi^{-}(t)\varphi^{-}(t')}{\sinh^2 \pi T(t-t')}
\end{aligned}
$$

Total action $\quad iS[\varphi^{+},\varphi^{-}] = iS_{env}[\varphi^{+},\varphi^{-}] + iS_{QPC}[\varphi^{+},\varphi^{-}]$

Current $\quad I(t) = -e \int \mathcal{D}\varphi^{\pm} \, \frac{\delta S_{QPC}[\varphi^{\pm}]}{\delta \varphi^{-}(t)} \, e^{iS[\varphi^{+},\varphi^{-}]}$

Noise $\quad \langle I(t_1) I(t_2) \rangle - I^2 = -2ie^2 \int \mathcal{D}\varphi^{\pm} \, \frac{\delta^2 S_{QPC}[\varphi^{\pm}]}{\delta \varphi^{-}(t_1) \delta \varphi^{-}(t_2)} \, e^{iS[\varphi^{+},\varphi^{-}]}$

Action of a QPC

Very general form

$$iS = \mathrm{Tr}\ln \check{G}[V^+(t), V^-(t)]$$

Formal expression in terms of S-matrix (see e.g. the review by Levitov, cond-mat/0210284)

$$iS_{QPC} \;=\; \mathrm{tr}\ln\left[\hat{E}\delta(x-y) + \left(e^{i\frac{\hat{\varphi}^-(x)}{2}}\hat{S}^\dagger e^{i\hat{\varphi}^-(x)}\hat{S}e^{i\frac{\hat{\varphi}^-(x)}{2}} - \hat{E}\right)e^{i\hat{\varphi}^+(x)}f_T(x-y)e^{-i\hat{\varphi}^+(y)}\right]$$

$$\hat{S} = \begin{pmatrix} \hat{r} & \hat{t}' \\ \hat{t} & \hat{r}' \end{pmatrix}, \quad e^{i\hat{\varphi}^\pm} = \begin{pmatrix} e^{i\varphi_L^\pm} & 0 \\ 0 & e^{i\varphi_R^\pm} \end{pmatrix}, \quad f_T(t) = \int \frac{dE}{2\pi}\,\frac{e^{-iEt}}{1+e^{E/T}}$$

$\hat{E}$ is the identity matrix

Action of a QPC, explicit expression

Weakly reflecting QPC, $1 - T_n \ll 1$ (Golubev, Galaktionov, Zaikin, PRB 2005)

$$
\begin{aligned}
iS_{QPC} = & -\frac{iN}{\pi}\int dt\,\dot{\varphi}^+(t)\varphi^-(t) + \frac{N}{2\pi}\int dtdt'\frac{\pi T^2}{\sinh^2 \pi T(t-t')}\varphi^-(t)\varphi^-(t') \\
& + \frac{i\mathcal{R}}{\pi}\int dt\,\dot{\varphi}^+(t)\sin\varphi^-(t) - \frac{\mathcal{R}}{\pi}\int dtdt'\frac{\pi T^2}{\sinh^2 \pi T(t-t')}\varphi^-(t)\sin\varphi^-(t') \\
& + \frac{2\mathcal{R}}{\pi}\int dtdt'\frac{\pi T^2\,\mathrm{e}^{i[\varphi^+(t)-\varphi^+(t')]}}{\sinh^2 \pi T(t-t')}\,\mathrm{e}^{\int d\tau\left[T\coth\pi T(t-\tau) - T\coth\pi T(t'-\tau)\right]\varphi^-(\tau)}\,\sin\frac{\varphi^-(t)}{2}\sin\frac{\varphi^-(t')}{2}
\end{aligned}
$$

Here $\mathcal{R} = \sum_n 1 - T_n$.

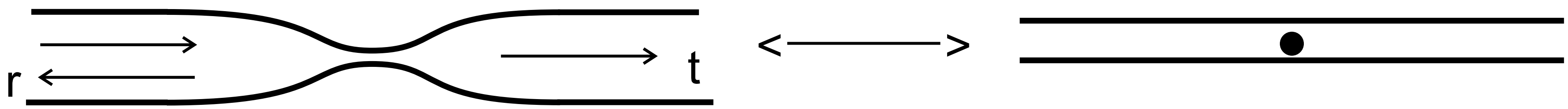
Tunnel junction, $T_n \ll 1$, expansion in T_n (Eckern, Schön, Ambegaokar, PRB 1984)

$$
iS_T = -\frac{ig}{2\pi}\int dt\dot{\varphi}^+(t)\sin\varphi^-(t) + \frac{g}{\pi}\int dtdt'\frac{\pi T^2\cos[\varphi^+(t)-\varphi^+(t')]}{\sinh^2\pi T(t-t')}\sin\frac{\varphi^-(t)}{2}\sin\frac{\varphi^-(t')}{2}
$$

Here $g = 2\sum_n T_n = 2\pi/e^2 R$.

Luttinger liquid and QPC

Analogy between a single channel QPC and Luttinger liquid with an impurity (Flensberg, PRB 1993; Furusaki Matveev, PRL 1995, PRB 1995; multichannel case: Aleiner, Brouwer, Glazman, Phys. Rep. 2002)



no reflection $\Rightarrow$ clean Luttinger liquid $\Rightarrow$ loop cancelation theorem $\Rightarrow$ the action is quadratic in phases

We find

$$iS_c = -\frac{iN}{\pi}\int dt\,\dot{\varphi}^+(t)\varphi^-(t) + \frac{N}{2\pi}\int dt dt' \frac{\pi T^2}{\sinh^2 \pi T(t-t')}\varphi^-(t)\varphi^-(t')$$

Current

Weakly reflecting QPC, $\mathcal{R} \ll 1$

$$I = \frac{e^2(N - \mathcal{R})}{\pi} V - 2e\mathcal{R}T^2 \int_0^\infty dt \, \frac{e^{F(t)} \sin[K(t)] \sin eVt}{\sinh^2 \pi T t}$$

Tunnel junction, $g \ll 1$ (comparison)

$$I = \frac{V}{R} - egT^2 \int_0^\infty dt \, \frac{e^{-F(t)} \sin[K(t)] \sin eVt}{\sinh^2 \pi T t}$$

The two results are similar.
One needs to interchange $F(t) \leftrightarrow -F(t)$ and $\sum_n T_n \leftrightarrow \sum_n 1 - T_n$.

In the limit $g \gg 1 \quad F(t)$ can be ignored $\Rightarrow$ duality between reflected and transmitted electrons holds.

Limiting cases

1. **High voltages**

$$I = \frac{e^2(N-\mathcal{R})}{\pi}V - \frac{e^2\mathcal{R}}{\pi}\frac{e}{2C}$$

Offest is smaller by factor $\mathcal{R}/N$

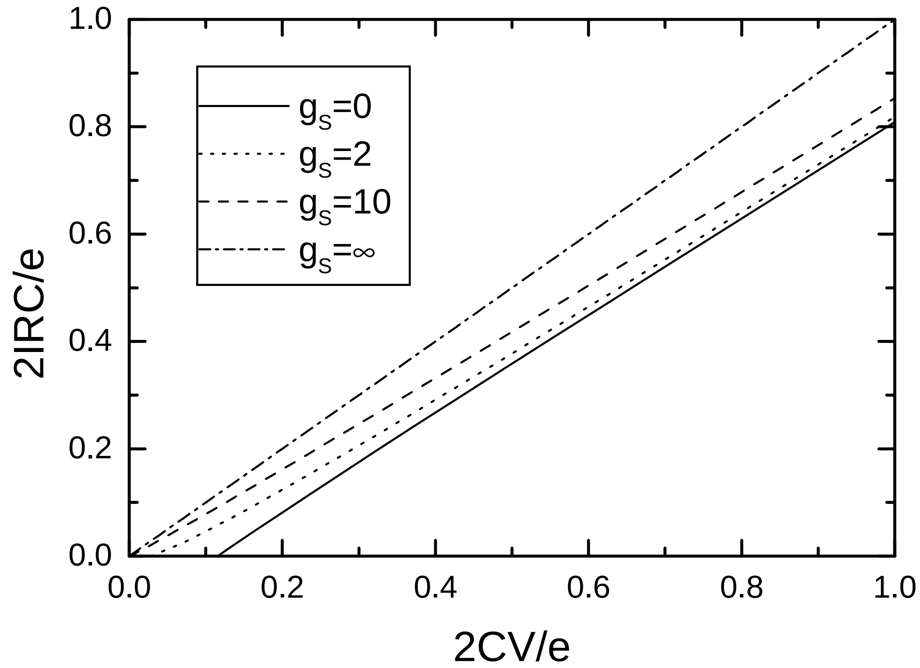
2. **Low voltage $eV \ll 1/\tau_{RC}$, but $eV \gg T$**

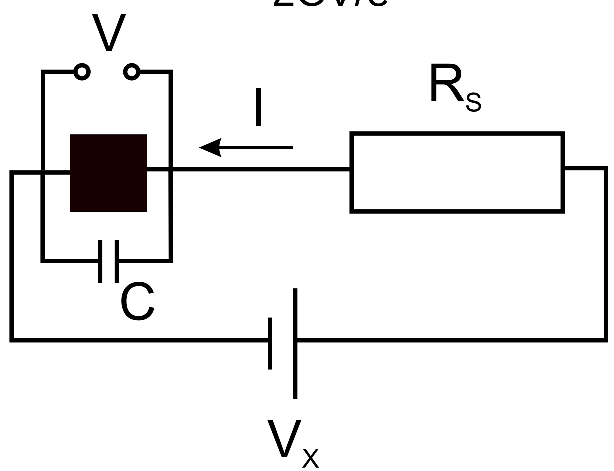
$$\frac{dI}{dV} = \frac{e^2N}{\pi} - \frac{e^2\mathcal{R}}{\pi\Gamma\left(1-\frac{2}{2N+g_S}\right)}(eV\tau_{RC})^{-\frac{2}{2N+g_S}}$$

3. **Zero bias conductance, $T \ll 1/\tau_{RC}$**

$$G = \frac{e^2}{N} - \frac{e^2\mathcal{R}}{2\sqrt{\pi}}\frac{\Gamma\left(1-\frac{1}{2N+g_S}\right)}{\Gamma\left(\frac{3}{2}-\frac{1}{2N+g_S}\right)}(T\tau_{RC})^{-\frac{2}{2N+g_S}}$$

$G(T)$ diverges at $T\tau_{RC} \le (\mathcal{R}/N)^{\frac{2N+g_S}{2}}$
(c.f. Bagrets, Nazarov PRL 2005)





Noise

Exact result at $\mathcal{R} \ll 1$ and arbitrary N and g_S

$$S(\omega,V) = \frac{2e^2N}{\pi}\,\omega\coth\frac{\omega}{2T} + \sum_{\pm} 2e\delta I\left(\frac{\omega}{e} \pm eV\right)\coth\frac{\omega}{2T} - \sum_{\pm} e\delta I\left(\frac{\omega}{e} \pm eV\right)\coth\frac{\omega \pm eV}{2T}$$

Effective non-linear conductance

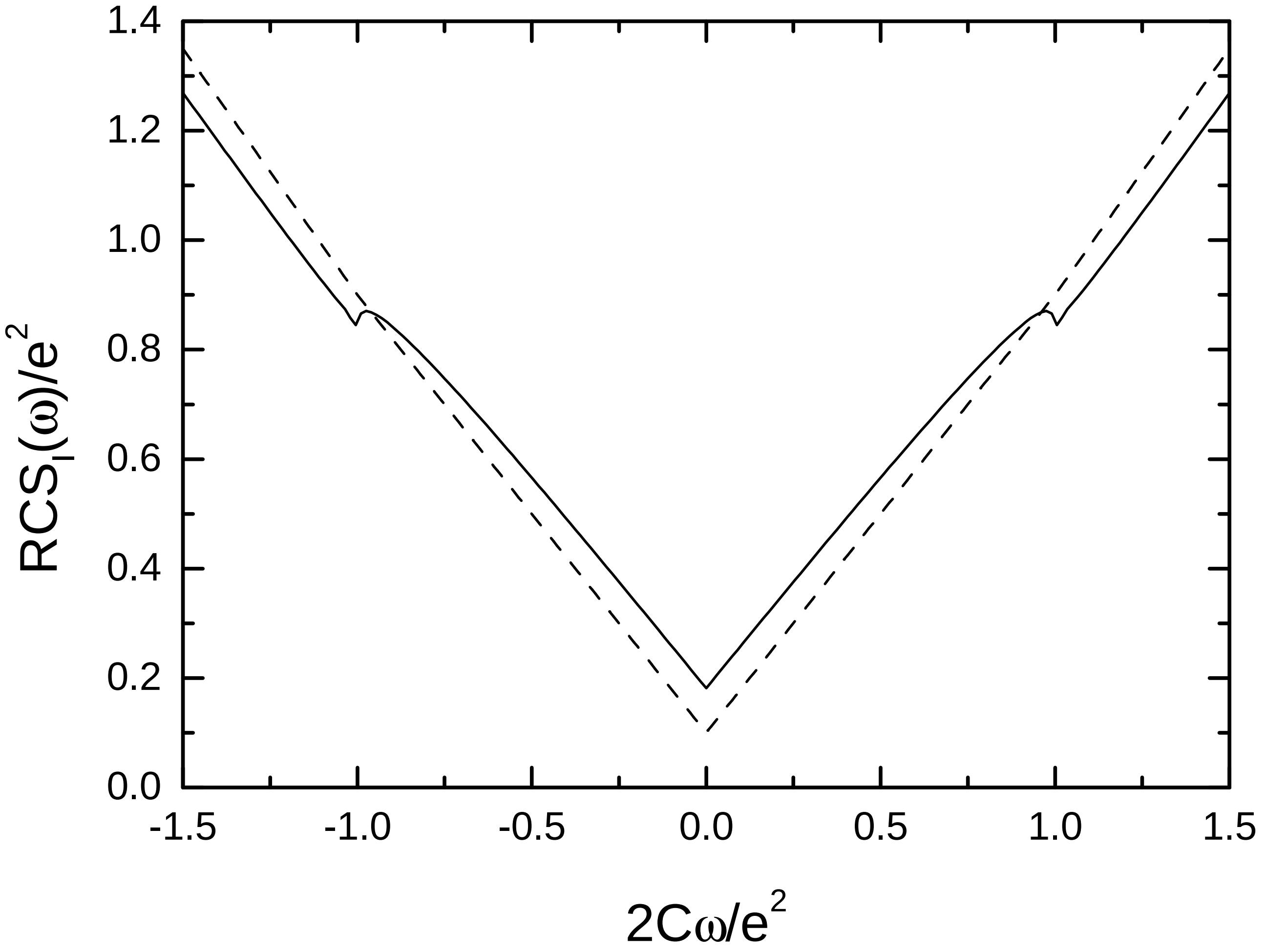
$$\tilde{G}(\omega,V) = \frac{e^2N}{\pi} + \frac{e}{\omega}\left[\delta I\left(V + \frac{\omega}{e}\right) - \delta I\left(V - \frac{\omega}{e}\right)\right]$$

Non-equilibrium FDT for weakly reflecting QPC

$$S(\omega,V) = 2\,\tilde{G}(\omega,V)\,\omega\coth\frac{\omega}{2T} - \sum_{\pm} e\delta I\left(\frac{\omega}{e} \pm eV\right)\coth\frac{\omega \pm eV}{2T}$$

! Note $\quad \tilde{G}(\omega,V) \neq \frac{e}{2\omega}\left[I\left(V + \frac{\omega}{e}\right) - I\left(V - \frac{\omega}{e}\right)\right]$

Noise, frequency dependence



Noise at $T \to 0$, $N = 1$, $\mathcal{R} = 0.1$, $V = e/2C$.
solid line — $g_S = 2$, dashed line — $g_S = \infty$ (no interaction)

Noise at low frequencies

At $\omega \to 0$

$$S(\omega, V) = \left(\frac{e^2 N}{\pi} + 2 \frac{d\,\delta I(V)}{dV} \right) 4T - 2e\,\delta I(V)$$

Similar relation holds for Luttinger liquid with a weak impurity (Fendley, Saleur, PRB 1996)

<u>Shot noise</u> regime $\omega \to 0$, $eV\mathcal{R} \gg T$: $\quad S(\omega, V) = -2e\,\delta I(V)$

High voltage, positive offset

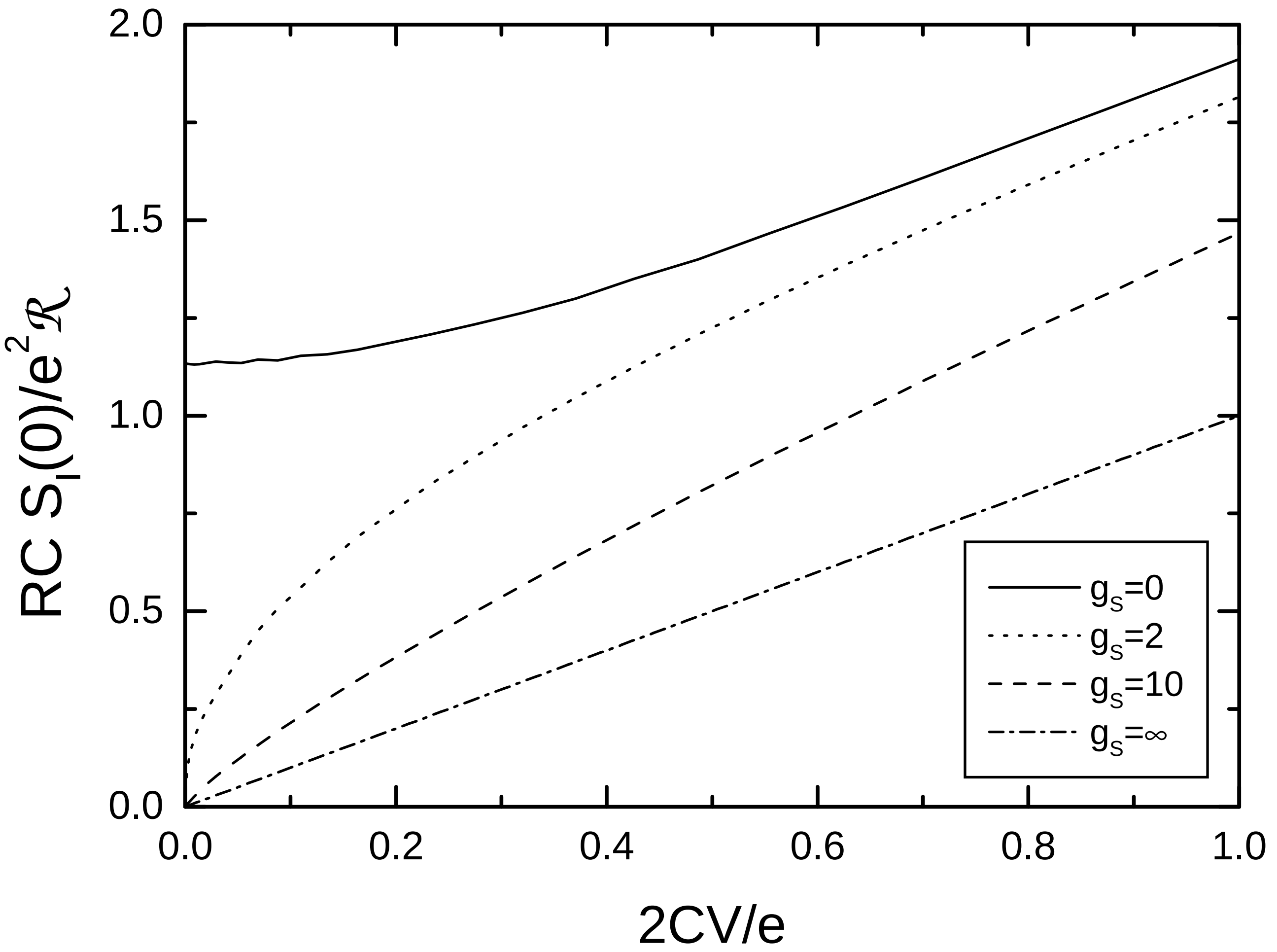
$$S(\omega, V) = \frac{2e^3 \mathcal{R}}{\pi} \left(V + \frac{e}{2C} \right)$$

Low voltage, $eV\tau_{RC} \ll 1$

$$S(\omega, V) = \frac{2e^2 \mathcal{R}}{\pi \tau_{RC}} \frac{(eV\tau_{RC})^{1 - \frac{2}{2N + g_S}}}{\Gamma\left(2 - \frac{2}{2N + g_S}\right)}$$

At low voltage shot noise may be strongly enhanced by interaction!

Shot noise



$T \to 0$, $N = 1$, $\mathcal{R} \ll 1$

Quantum dot with two weakly reflecting QPC

Idea: second junction is the environment resistor for the first one and the other way around

Zero bias conductance at $T\tau_D \gg 1$, but $T\tau_{RC} \ll 1$

$$G = \frac{e^2}{\pi}\frac{N_1 N_2}{N_1 + N_2} - \frac{e^2}{2\sqrt{\pi}}\frac{\Gamma\left(1 - \frac{1}{2(N_1+N_2)}\right)}{\Gamma\left(\frac{3}{2} - \frac{1}{2(N_1+N_2)}\right)}\frac{N_2^2\mathcal{R}_1 + N_2^2\mathcal{R}_1}{(N_1+N_2)^{2-\frac{1}{N_1+N_2}}}\left(\frac{2e^{\gamma}E_C}{\pi^2 T}\right)^{\frac{1}{N_1+N_2}}$$

In the limit $N_1 = N_2 = 1$ agrees with Furusaki, Matveev (PRL,PRB 1995)
At arbitrary N_1, N_2 also agrees with Aleiner, Brouwer, Glazman (Phys. Rep. 2002)

Shot noise of a QD with strong inelastic relaxation and in the limit $eV\tau_{RC} \ll 1$

$$S(\omega, V) = \frac{4\, e^{\frac{\gamma}{N_1+N_2}}}{\pi^2\Gamma\left(2 - \frac{1}{N_1+N_2}\right)}\frac{N_2^2\mathcal{R}_1 + N_2^2\mathcal{R}_1}{(N_1+N_2)^{2-\frac{1}{N_1+N_2}}}e^2 E_C\left(\frac{\pi eV}{2E_C}\right)^{1-\frac{1}{N_1+N_2}}$$

Summary

1. The effective action for a weakly reflecting QPC is derived.
2. The effect of electro-magnetic environment is considered.
   First order in $\mathcal{R} = \sum_n 1 - T_n$ correction to the current is derived.
   The result in nonperturbative in $1/g$ and $1/g_S$.
3. First order in $\mathcal{R}$ correction to the noise is obtained.
   Non-equilibrium FDT for a weakly reflecting QPC is formulated
   Strong enhancement of the shot noise by interaction at low voltages is predicted.
4. Results are generalized to quantum dots.
5. In the absence of interaction and for a weakly reflecting QPC all cumulants of the current at all frequencies are found analytically.

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