Saltdasfjorden - Norway

Low-Frequency Noise as a Source of Non-Gaussian Decoherence in Qubits

Y. M. Galperin[1]

Team: B. L. Altshuler,[2], J. Bergli,[1],D. V. Shantsev,[1] L. Faoro,[3] I. Martin[4]

[1]University of Oslo and Argonne National Laboratory
[2]Columbia University and NEC Research Institute
[3]Rutgers University
[4]Los Alamos National Laboratory

Discussions: Y. Nakamura, J. S. Tsai, Y. A. Pashkin, T. Yamamoto, O. Astafiev

Support:

Review of recent works:

B.L. Altshuler, D.V. Shantsev, YMG, in *Fundamental Problems of Mesoscopic Physics,* edited by I.V. Lerner *et al.*

B.L. Altshuler, J. Bergli, D.V. Shantsev, YMG, *PRL* 96, 097009 (2006).

L. Faoro, J. Bergli, B. L. Altshuler, YMG, *PRL* 95, 046805 (2005).

B.L. Altshuler, J. Bergli, D.V. Shantsev, YMG, *Europhys. Lett.* 71, 21 (2005)

J. Bergli, B.L. Altshuler, YMG, *cond-mat*/ 0603575.

I. Martin and YMG, *cond-mat*/0601556.

http://folk.uio.no/yurig

Outline

- Motivation
- Qubits, single Cooper pair box, etc
- Dynamical defects as a source of flicker-noise
- Decoherence by a single fluctuator
- Effect of many fluctuators
- Decoherence close to optimal point
- Resonant interaction between qubit and fluctuators
- Role of non-stationary fluctuations
- Conclusions

Motivation:

VOLUME 88, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JANUARY 2002

Charge Echo in a Cooper-Pair Box

Y. Nakamura,[1] Yu. A. Pashkin,[2,*] T. Yamamoto,[1] and J. S. Tsai[1]

A spin-echo-type technique is applied to an artificial two-level system that utilizes a charge degree of freedom in a small superconducting electrode. Gate-voltage pulses are used to produce the necessary pulse sequence in order to eliminate the inhomogeneity effect in the time-ensemble measurement and to obtain refocused echo signals. Comparison of the decay time of the observed echo signal with an estimated decoherence time suggests that low-frequency energy-level fluctuations due to the $1/f$ charge noise dominate the dephasing in the system.

What exactly is qubit?

Quantum two-level system equivalent to ½ spin

The qubit is described by effective Hamiltonian

$$\mathcal{H}_{\rm ctrl} = -\frac{1}{2}B_z\hat{\sigma}_z - \frac{1}{2}B_x\hat{\sigma}_x \ .$$

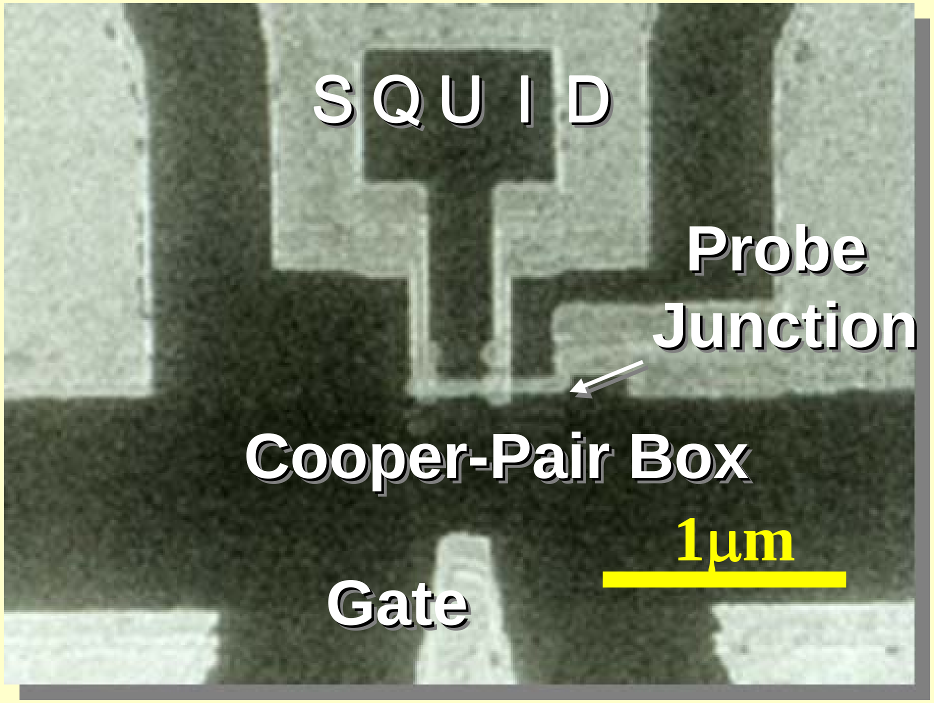
with tunable B_x and B_z to perform single-qubit operations.

A controllable interaction in the form

$$\mathcal{H}_{\rm ctrl}(t) = -\frac{1}{2}\sum_{i=1}^{N} B^i(t)\hat{\sigma}^i + \sum_{i\neq j} J_{ab}^{ij}(t)\hat{\sigma}_a^i\hat{\sigma}_b^j \ ,$$

(where a summation over spin indices $a, b = x, y, z$ is implied) to perform two-bit operations.

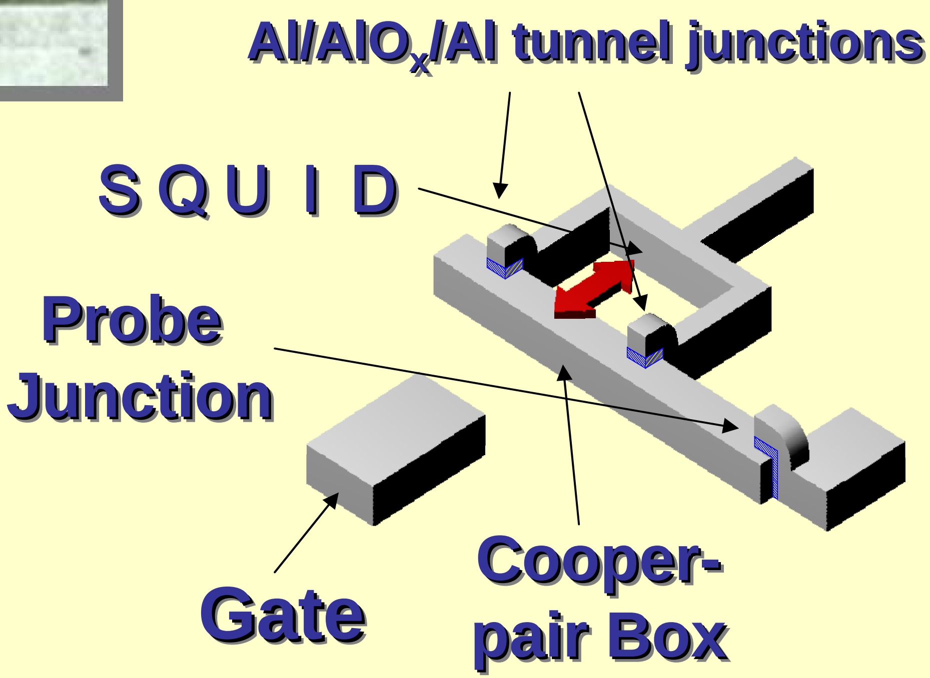
Josephson Charge Qubit



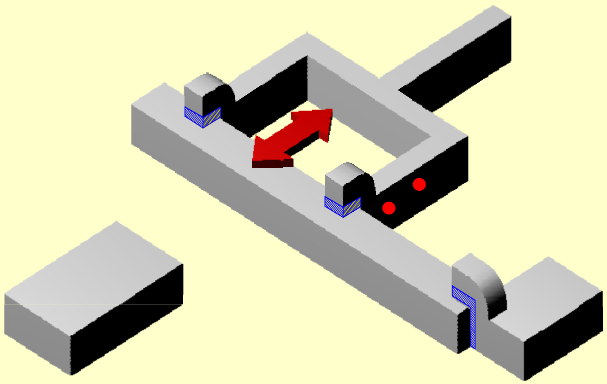
Artificial ½-spin –
Josephson qubit
(NEC, Japan)

Nature 2003, 2004

Europhysics Prize, 2004



Single Cooper Pair Box



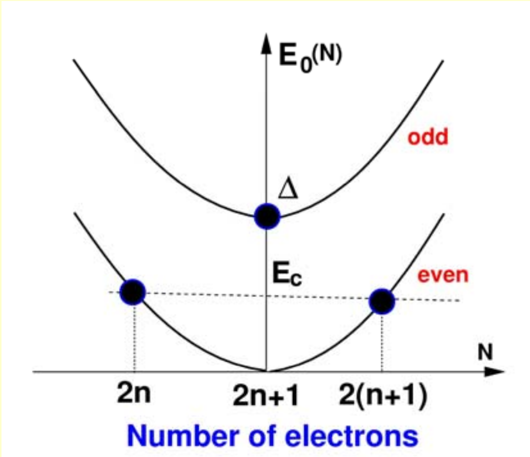
$$E_0(N) = E_c(N - \alpha V_g)^2 + \Delta_N$$

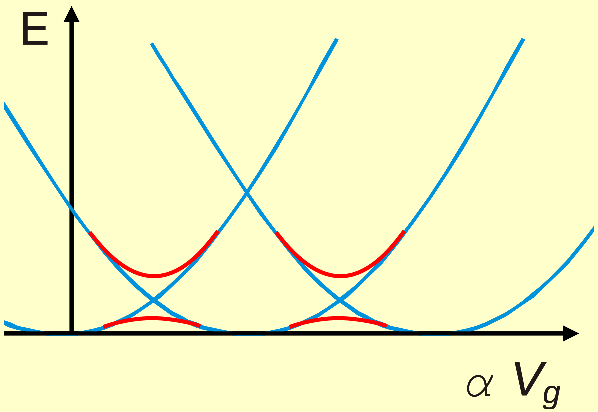
$$\Delta_N = \begin{cases} 0, & N = 2n \\ \Delta, & N = 2n+1 \end{cases}$$

At $\alpha V_g = 2n + 1$ ground state is degenerate with respect to addition of 1 CP

The degeneracy is lifted by the Josephson tunneling of CP, the energy splitting being the Josephson energy, E_J.

The energy is tuned by the magnetic flux in the SQUID loop.





Effective Hamiltonian

$$\tilde{\mathcal{H}} = -\frac{1}{2}\underbrace{E_c(1-\alpha V_g)}_{B_z}\sigma_z - \frac{1}{2}\underbrace{E_J}_{B_z}\sigma_x$$

The qubit can be manipulated through the gate voltage, V_g, and magnetic flux in the SQUID loop, which modulates E_J.

Crucial question:

How long such system evolves coherently after AC excitation?

Studies:

Typical ESR or NSR experiments:

free induction decay, echo, etc.

Theoretical models for decoherence.

Decoherence: Spin-Boson Model

A.J. Leggett et al, *Rev. Mod. Phys.* v.59, 1 (1987).
U. Weiss, ``Quantum Dissipative Systems'', 2nd ed., (Word Scientific, Singapore, 1999).
A. Shnirman, Y. Makhlin, and G. Schon, *Physica Scripta* v.T102, 147 (2002)
D. Loss and D. DiVincenzo, *cond-mat/030411*

½-spin linearly coupled with a set of oscillators

$$\mathcal{H}_{\text{s-b}} = \sigma_z \hat{\mathcal{X}}\,, \quad \hat{\mathcal{X}} = \sum_j C_j \left(\hat{b}_j + \hat{b}_j^\dagger\right)$$

Decoherence is expressed through ***noise spectrum***

$$S_{\mathcal{X}}(\omega) \equiv \left\langle \left[\hat{\mathcal{X}}(t), \hat{\mathcal{X}}(0)\right]_+ \right\rangle_\omega = 2J(\omega)\coth\frac{\omega}{2T}\,.$$

Here $J(\omega)$ the bath spectral density,

$$J(\omega) \equiv \pi \sum_j C_j\,\delta(\omega - \omega_j)\,,$$

The free induction signal:

$$\hat{\Phi} \equiv i \sum_j (2C_j/\omega_j) \left(\hat{b}_j^\dagger - \hat{b}_j \right) ,$$

$$\mathcal{P}(t) = \mathrm{Tr} \left(e^{-i\hat{\Phi}(t)} \, e^{i\hat{\Phi}(0)} \hat{\rho}_b \right)$$

$$\hat{\Phi}(t) = e^{i\mathcal{H}_b t} \hat{\Phi} e^{-i\mathcal{H}_b t},$$

subscript b refers to the thermal bath, $\hat{\rho}_b$ being its density matrix.

$$\mathcal{K}(t) = -\ln \mathcal{P}(t) = \frac{8}{\pi} \int_0^\infty \frac{d\omega \, J(\omega)}{\omega^2} \left[\sin^2 \frac{\omega t}{2} \coth \frac{\omega}{2T} + \frac{i}{2} \sin \omega t \right]$$

The noise is assumed to be Gaussian

↓

Everything is determined by the power spectrum of the noise !!!

However, for 1/f noise the above integral is divergent.

Sub-Ohmic spin-boson model:

A. Shnirman, Y. Makhlin, and G. Schön, Physica Scripta **T102**, 147 (2002).

Decoherence is expressed through the fluctuation spectrum of the environment similarly to the spin-boson model.

This fluctuation spectrum is then assumed to show $1/\omega$-behavior and the integral is cut-off at some reasonable frequency.

Problem with *1/f* – type noise:

This noise is not

1) Gaussian	higher order moments are important
2) Markovian	history is important

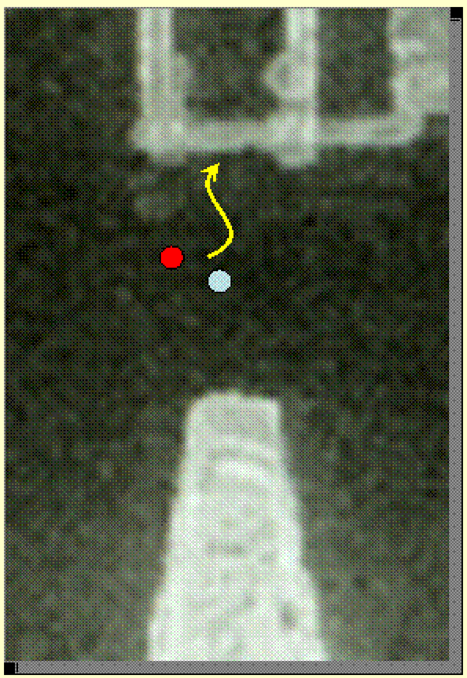
Decoherence and energy relaxation: Spin-Fluctuator Model

Fluctuators: structural defects, charge traps, which can exist in dielectric parts of the device

The fluctuators randomly switch between their states due to interaction with extended modes of environment – phonons or electrons.

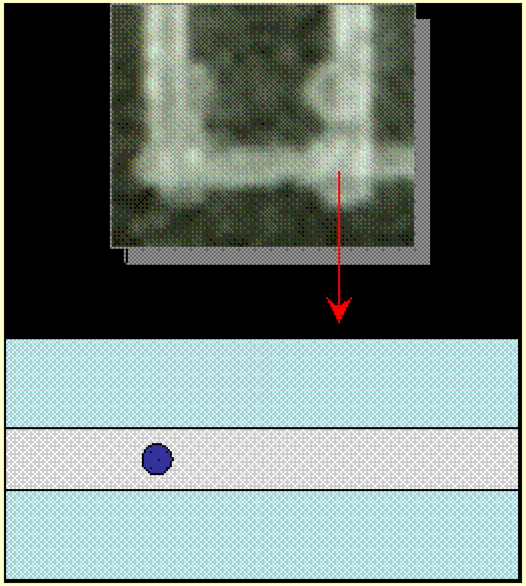
Switching $\Rightarrow$ random fields $\Rightarrow$ decoherence

Modulation of induced charge



$$-\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x$$

Modulation of critical Josephson current



Fluctuators during last 30 years

Pioneering ideas:

TLS in amorphous media:	P.W. Anderson, B. I. Halperin, and C. M. Varma, Phil. Mag. **25**, 1 (1972) W. A. Phillips, J. Low Temp. Phys. **7**, 351 (1972)
Application to charge noise:	A. Ludviksson, R. Kree, A. Schmid, PRL 52, 950 (1984) Sh.M. Kogan, K.E. Nagaev, Sol. St. Comm., 49, 387 (1984)

Extensive work in between:

Fluctuator-induced noise in point contacts and Josephson junctions:	V.I. Kozub, 1984 (several papers); C. Rogers and R. Buhrman, PRL 53, 1272 (1984) + other experiments YMG, V.G. Karpov, V.I.Kozub, 1989; YMG, V.I. Gurevich, V.I. Kozub, 1989 + other theoretical and experimental activities

Application to qubits:

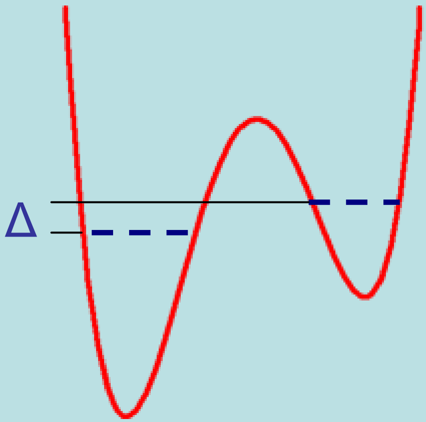
Charge noise:	E. Paladino, L. Faoro, G. Falci, and R. Fazio, PRL 88, 228304 (2002)
Fluctuations of J_c:	J. M. Martinis, S. Nam, J. Aumentado, K. M. Lang, and C. Urbina, PRB 67, 94510 (2003)

Hamiltonian:

$$\mathcal{H} = -(B/2)\,\sigma_z + (1/2)F(t)\sigma_x$$ qubit

fluctuator $$+ (1/2)\sum E_i \tau_z^{(i)} + \mathcal{H}_{\text{env}} + \mathcal{H}_{F-\text{env}}$$

interaction $$+ \sum_i \left(v_i\, \sigma_z^{(i)} \tau_z^{(i)} + \dots \right)$$



$$E_i = \sqrt{\Delta_i^2 + \Lambda_i^2}$$

$$v_i = g(r_i)A(\mathbf{n}_i)(B_z/B)(\Delta_i/E_i)$$

$$\mathcal{H}_{\text{env}} = \sum_\mu \omega_\mu \left(\hat{b}^\dagger \hat{b} + \frac{1}{2} \right)$$

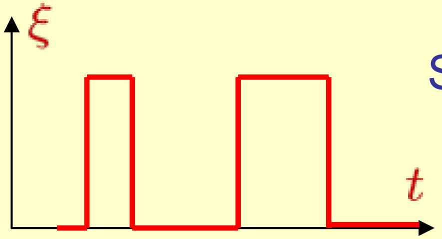
$$\mathcal{H}_{F-\text{env}} = \sum_{i\mu} C_{i\mu} \tau_x^{(i)} \left(\hat{b}_\mu + \hat{b}_\mu^\dagger \right)$$

Simple classical model:

Classical low-frequency fluctuations $\xi_i(t)$ acting upon the qubit:

$$\mathcal{H}_{qF} = X_1(t)\,\sigma_z\,, \quad X_1(t) = \sum_i v_i \xi_i(t)$$

Uncorrelated random telegraph processes: $\xi_i(t) = 0 \text{ or } \xi_i(t) = 1$.



Switching times are distributed according to Poisson distribution

$$\langle \xi_i(t)\xi_k(t')\rangle = \delta_{ik} e^{-2\gamma_i |t-t'|}$$

The switching rates, γ, are calculated in the 2nd order in the interaction between the fluctuator and the thermal bath:

$$\gamma_i = (1/2)\gamma_0(T)\,(\Lambda_i/E_i)^2$$

Finally, for the qubit and a fluctuator we have:

$$\mathcal{H} = \frac{1}{2}\left[E_0 + \mathcal{X}(t)\right]\sigma_z + \frac{1}{2}F(t)\sigma_x + \frac{1}{2}\sum_i E_i \tau_z^{(i)}$$

where $E_0 = B + \mathcal{X}_1(0)\,, \quad \mathcal{X}(t) = \mathcal{X}_1(t) - \mathcal{X}_1(0)\,.$

Density matrix of the resonantly-manipulated qubit:

$$\hat{\rho} = \begin{pmatrix} n & -if\, e^{i\Omega t} \\ if^*\, e^{-i\Omega t} & 1-n \end{pmatrix}.$$

Von Neumann equation:

$$\frac{\partial n}{\partial t} = -2\gamma_q (n - n_0) - F \,\mathrm{Re}\, f \,,$$
$$\frac{\partial f}{\partial t} = i\left[E_0 + \mathcal{X}(t) - \Omega\right] f - \gamma_q f + \frac{F}{2}(2n - 1).$$

F – Rabi frequency, $\mathcal{X}(t)$ - random deviation of eigenfrequency

Stochastic differential equation

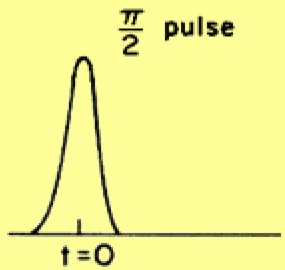
The result can be expressed in terms of the phase-memory functional

$$\Psi[\beta(t'),t] = \left\langle \exp\left(i \int_0^t \beta(t') \mathcal{X}(t')\, dt' \right) \right\rangle_{\xi_i}$$

$\beta(t)$ depends on the manipulation protocol

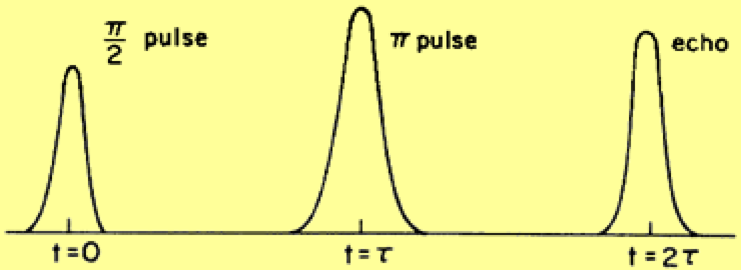
Protocols:

"Free induction"



$$\beta(t) = \Theta(t)$$

"Echo"



$$\beta(t) = \begin{cases} 0 & \text{for} \quad t \le 0\,, \\ 1 & \text{for} \quad 0 < t \le \tau_{12}\,, \\ -1 & \text{for} \quad \tau_{12} < t\,. \end{cases}$$

Similar to spectral diffusion in magnetic systems

R. Klauder and P. W. Anderson, Phys. Rev. **125**, 912 (1962).

and glasses

J. L. Black and B. I. Halperin, Phys. Rev. B **16**, 2829 (1977).

In the latter case fluctuators are structural two-level systems

Single Fluctuator

Simplification: $\xi^2(t) = 0,1$ is a determined quantity

The phase-memory functional obeys the differential equation

$$\frac{d^2\psi}{d\psi^2} + \left(2\gamma - \frac{d \ln \beta}{dt} - iv\beta\right) - iv\gamma\beta^2 = 0$$

with initial conditions $\psi(0) = 0\,,\quad \frac{d\psi}{dt}\,|_{t=0} = \frac{iv}{2}\beta\,|_{t=-0}$

One can easily find exact solution of a simple manipulation protocol when $\beta = \pm 1$

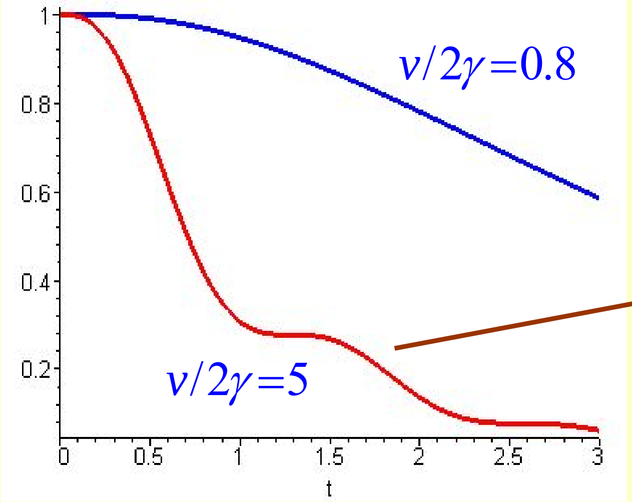
Two parameters: switching rate, γ, and coupling strength, v

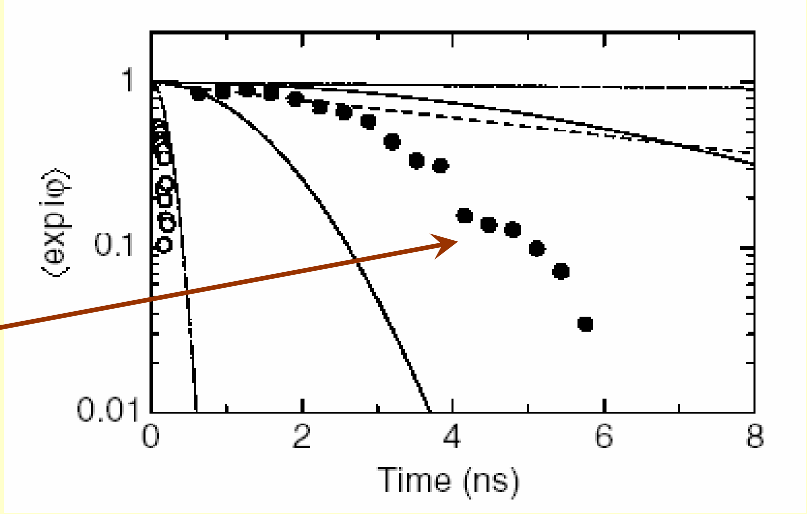
Strong coupling, $v \gg \gamma$:

$$\psi(t) = e^{-2\gamma\tau}\left(1 + \frac{2\gamma}{v}\sin v\tau\right)$$

Plateaus at $v\tau \approx 2\pi k + 2\gamma/v$

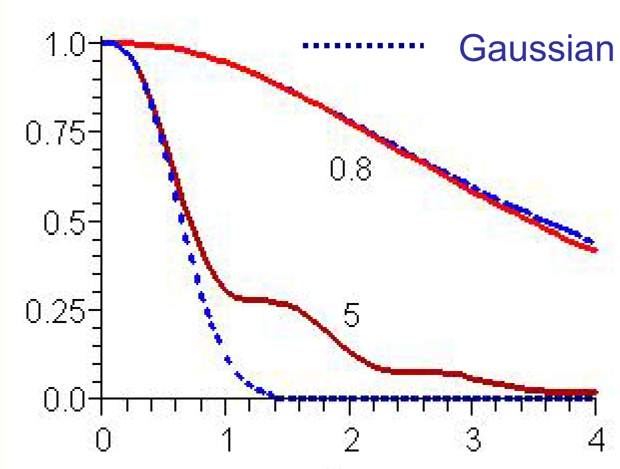
Weak coupling, $v \ll \gamma$: $\psi(t) = e^{-v^2\tau/4\gamma}$





Nakamura et al., PRL 2002

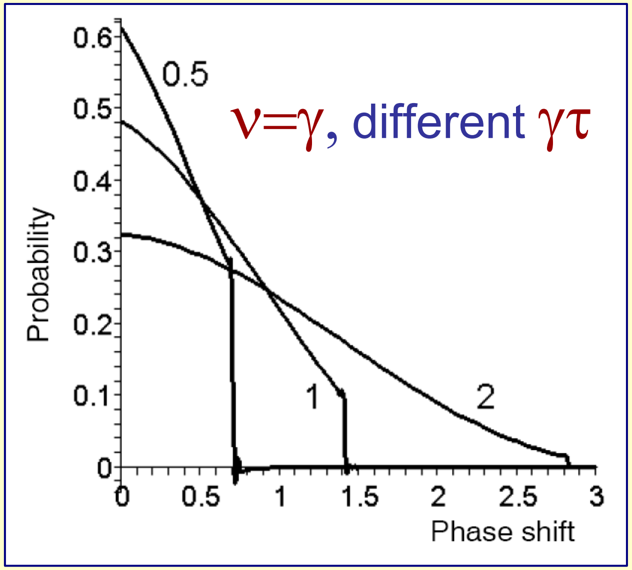
How good is the Gaussian assumption for a single fluctuator?



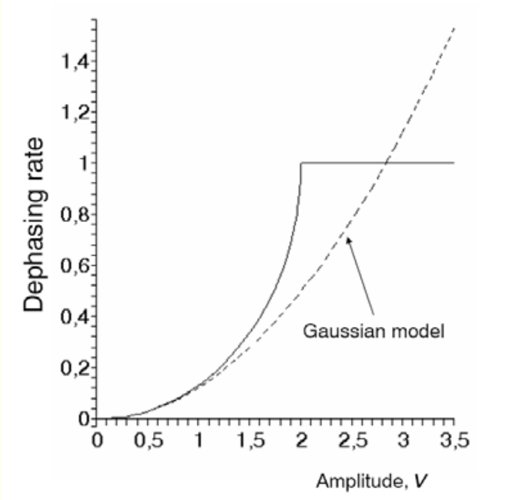
Gaussian approximation is good only for weakly coupled fluctuators.

Why this approximation does not work for "strong" fluctuators?

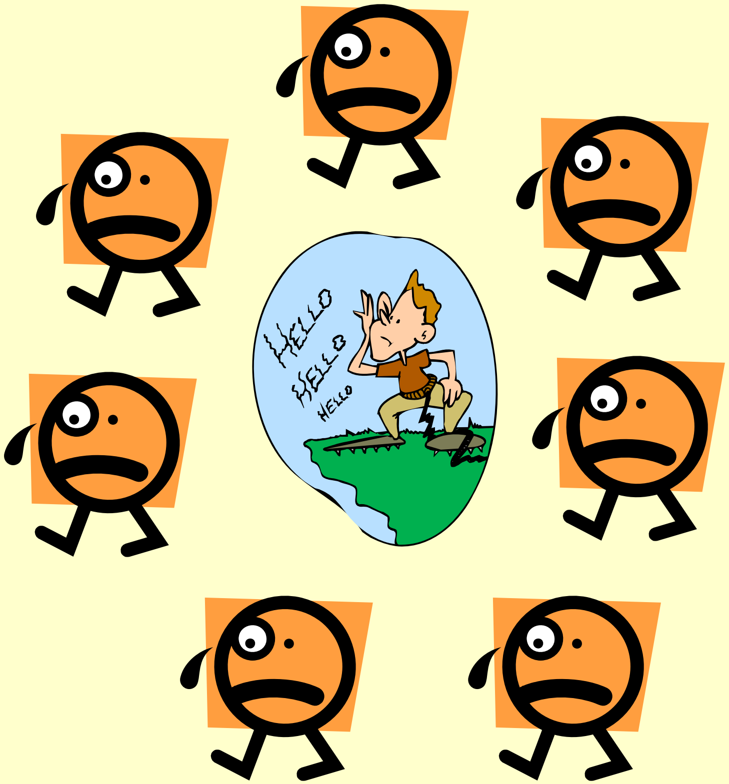
The distribution function of the phase shift is essentially non-Gaussian since the phase shift is limited by the quantity $2v\tau$



Resulting amplitude dependence of the dephasing rate



Many fluctuators (decoherence by 1/f noise)



Many fluctuators with exponentially broad distribution of switching rates produce 1/f noise.

- Can such noise be considered as Gaussian?
- What kind of model should describe decoherence by 1/f noise?
- Is the decoherence directly related to the 1/f noise?

Microscopic model leading to 1/f noise - many uncorrelated fluctuators

$$\Psi[\beta(t),t] = \prod_i \psi^{(i)}(t) = e^{\sum_i \ln \psi^{(i)}(t)} \equiv e^{-\mathcal{K}(t)}\,.$$

For $N >> 1$

$$\mathcal{K}(t) \approx \sum_i \left[1 - \psi^{(i)}(t)\right] = \mathcal{N} \langle 1 - \psi(t) \rangle_F$$

Holtsmark method: S. Chandrasekhar, Rev. Mod. Phys. **15**,1 (1943).

To calculate the average one needs distributions of fluctuators' decay rates and coupling constants

Properties of distributions:

- Only the fluctuators with $E \le kT$ are important, the rest are frozen in their ground states
- Relaxation rates: since $\Lambda \propto e^{-\lambda r}$ the distribution of the *logarithm of* Λ should be uniform

$$P(E,\theta) = \frac{P_0}{\sin\theta} \qquad \sin\theta \equiv \frac{\Lambda}{E}$$

- Distribution of v depends both on the interaction range and the location of fluctuators. In a bulk system, assuming that $v = g/r^3$ fluctuators are randomly distributed in space we get

$$\mathcal{P}(E,\theta,v) = \frac{\eta}{v^2 \sin\theta}, \quad \eta = \frac{g}{r_T^3}, \; r_T = (P_0 T)^{-1/3}$$

η is the typical coupling to a thermal fluctuator.

General expression:

$$\mathcal{K}(t) = \eta \int \frac{du}{u^2} \int_0^{\pi/2} \frac{d\theta}{\sin\theta} \left\{ 1 - \psi\left[\beta, t \middle| u\cos\theta, \gamma_0 \sin^2\theta\right] \right\}$$

Echo signal:

Switching probability

$$\mathcal{K}(\tau) \sim \begin{cases} \eta\tau(\gamma_0\tau)\,, & \gamma_0\tau \ll 1 \quad \text{Markovian} \\ \eta\tau\,, & \gamma_0\tau \gg 1 \quad \text{Non-Markovian} \end{cases}$$

$$T_2^{-1} \approx \min\{\eta, \sqrt{\eta\gamma_0}\}$$

At $\tau \gg 1/\gamma_0$ decoherence is due to *optimal* fluctuators with

$$v(r_{\text{opt}}) \approx \gamma_0(T)$$

Different from those which mainly contribute to the noise spectrum

For an exponentially-broad distribution of relaxation rates

$$
\begin{aligned}
S_{\mathcal{X}}(\omega) &= 2\int_0^\infty dt\, e^{i\omega t}\langle\langle \mathcal{X}(t)\mathcal{X}(0)\rangle_\xi\rangle_F \\
&= 2\cos^2\Theta \left\langle \frac{2\gamma_i}{\omega^2+\gamma_i^2}\cdot v_i^2 \right\rangle_F \\
&= \pi\frac{\eta v(r_{\mathrm{min}})}{2\omega}
\end{aligned}
$$

Noise spectrum is determined by closet fluctuators

Interplay between decoherence and noise spectrum can depend on actual distribution of fluctuators in in the device

Energy relaxation

Motivation:

PRL **93,** 267007 (2004) PHYSICAL REVIEW LETTERS week ending 31 DECEMBER 2004

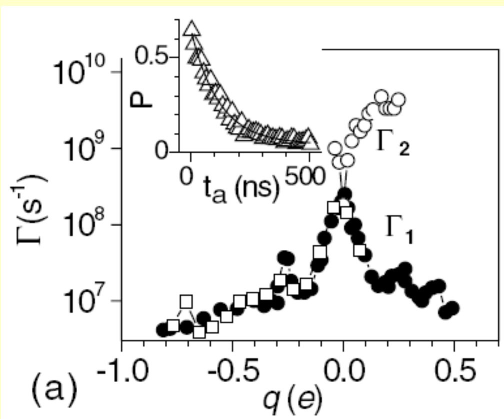
Quantum Noise in the Josephson Charge Qubit

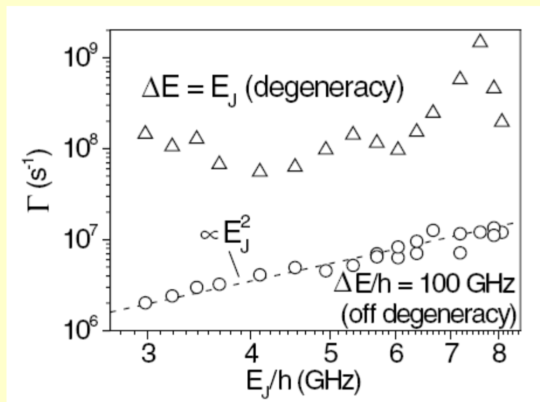
O. Astafiev,[1,*] Yu. A. Pashkin,[1,†] Y. Nakamura,[1,2] T. Yamamoto,[1,2] and J. S. Tsai[1,2]

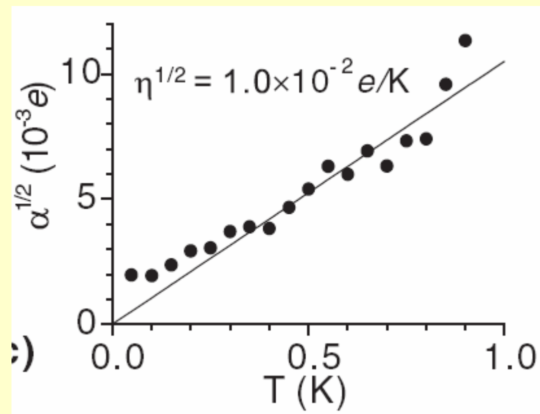
PRL **96,** 137001 (2006) PHYSICAL REVIEW LETTERS week ending 7 APRIL 2006

Temperature Square Dependence of the Low Frequency $1/f$ Charge Noise in the Josephson Junction Qubits

O. Astafiev,[1,2,*] Yu. A. Pashkin,[1,2,†] Y. Nakamura,[1,2] T. Yamamoto,[1,2] and J. S. Tsai[1,2]



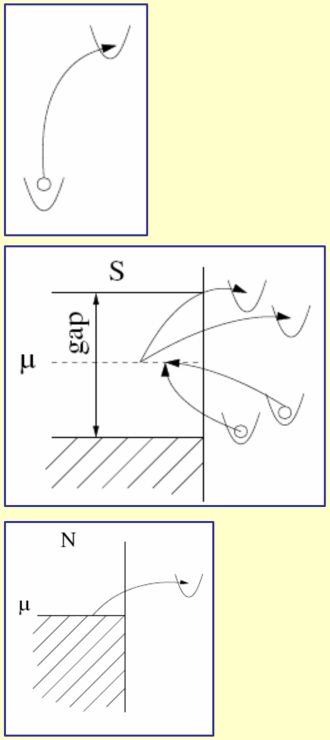




Energy relaxation rate was measured versus device parameters and temperature

Model

Superconductor



Three scenarios:

The first two are possible only if the part of the device is normal

The third one is important if the electrode is superconducting:

crossed Andreev reflection – coherent tunneling of a Cooper on two localized sites

Microscopic theory: transition rate of the qubit is

interaction Hamiltonian

$$\Gamma_\downarrow = 2\pi \sum_{i,f} \rho_i^0 |\langle +, f| T e^{-i\int_0^t H_I(t')dt'} |-, i\rangle|^2.$$

occupancy of the initial state of the bath

Model for calculation of T_1

Assumption: Tunneling between the gate and the trap depends on the state of the qubit: $\hat{t} = t_0 + \tilde{t}\sigma_z$

$$\mathcal{H} = -\frac{\delta E_c}{2}\sigma_z - \frac{E_J}{2}\sigma_x$$

Qubit

Fluctuator + gate

$$+ \sum_\alpha \epsilon_\alpha c^\dagger_\alpha c_\alpha + \frac{1}{2}\sum_{\alpha\neq\beta} U_{\alpha\beta}\, c^\dagger_\alpha c_\alpha c^\dagger_\beta c_\beta + t_0 \sum_{\alpha\neq\beta}(c^\dagger_\alpha c^\dagger_\beta + c_\alpha c_\beta)$$

$$+ \left[v\sum_\alpha c^\dagger_\alpha c_\alpha + \tilde{t}\sum_{\alpha\neq\beta}(c^\dagger_\alpha c^\dagger_\beta + c_\alpha c_\beta)\right]\sigma_z$$

Coupling between qubit and fluctuator

After diagonalizing the qubit Hamiltonian one obtains both dephasing and direct transitions leading to T_2 and T_1

Calculation explains quadratic temperature dependence of the energy relaxation rate experimentally observed by Astafiev *et al*.

They also observed non-Gaussian behavior for the case of pronounced decoherence.

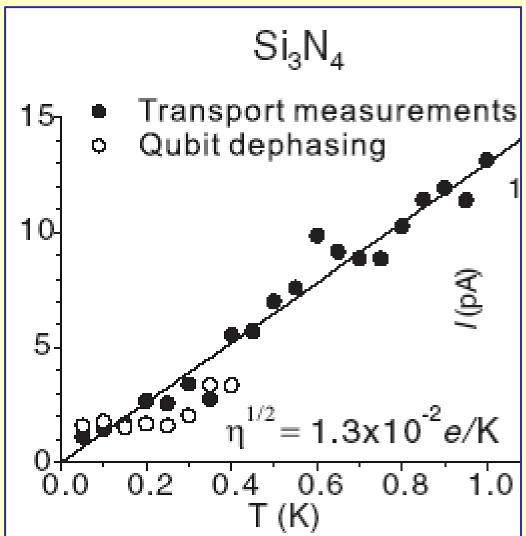
PRL **96**, 137001 (2006) PHYSICAL REVIEW LETTERS week ending 7 APRIL 2006

Temperature Square Dependence of the Low Frequency $1/f$ Charge Noise in the Josephson Junction Qubits

O. Astafiev,[1,2,*] Yu. A. Pashkin,[1,2,†] Y. Nakamura,[1,2] T. Yamamoto,[1,2] and J. S. Tsai[1,2]

[1]*NEC Fundamental and Environmental Research Laboratories, Tsukuba, Ibaraki 305-8501, Japan*

[2]*The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-0198, Japan*



Relevant work:

A. Grishin, I. V. Yurkevich, and I. V. Lerner, Phys. Rev. B **72**, 060509 (2005). Renormalization of DOS

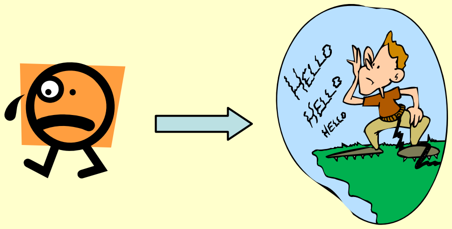
A. Shnirman, G. Schön, I. Martin, and Y. Makhlin, Phys. Rev. Lett. **94**, 127002 (2005). Phenomenological model

L. Faoro and L. B. Ioffe, Phys. Rev. Lett. **96**, 047001 (2006). Kondo impurities in Josephson junction

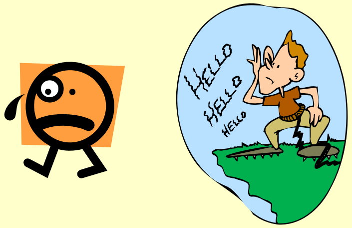
What I did not have time to tell about

- What happens close to optimal point?

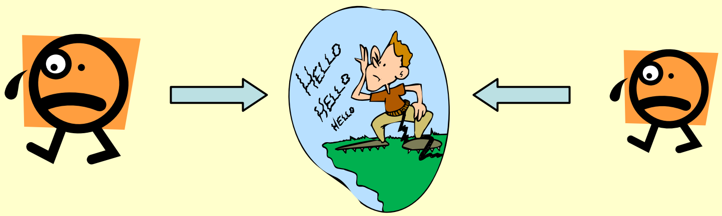
Far from the optimal point



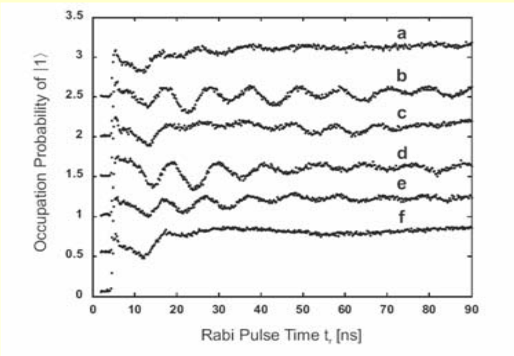
At the optimal point



But

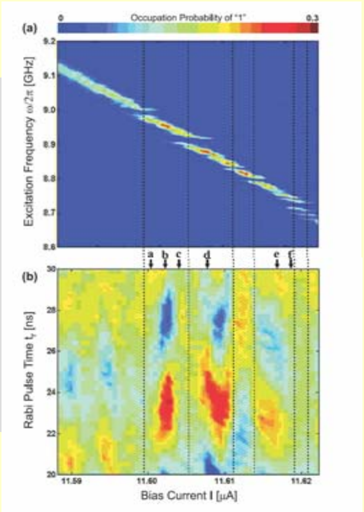


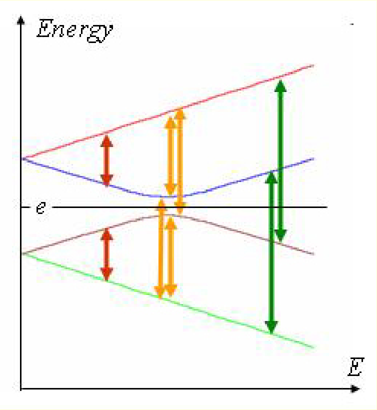
- Resonant interaction (Josephson phase qubit, Simmonds et al., 2004):



Significant changes of the Rabi amplitudes as a function of bias current

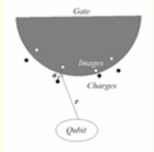
The results were interpreted as “spurious resonances”, induced by the fluctuators



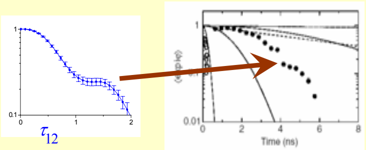


Summary and Conclusions

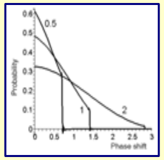
We considered extended spin-fluctuator model for



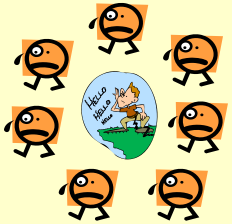
1. Echo & energy relaxation



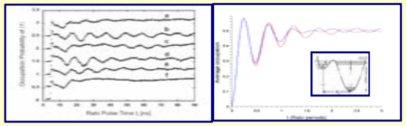
2. Distribution of single-shot readouts



3. Decoherence by many fluctuators



4. Fluctuator-mediated Rabi oscillations

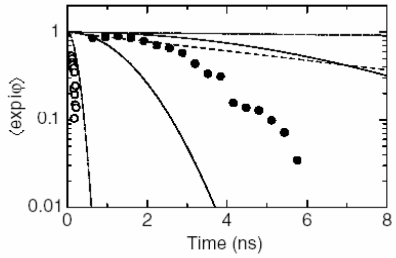


5. Decoherence @ optimal point

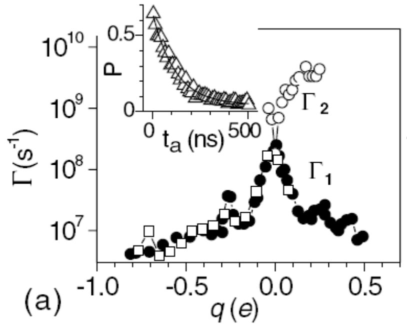
6. Decoherence due to “quenched” fluctuators

The model

explains observed features



shows pronounced non-Gaussian behavior – there is NO direct relation between the decoherence and noise spectrum



Decoherence and 1/f noise are determined by different fluctuators.

No direct connection between the decoherence and flicker noise.