Single-electron tunneling with strong mechanical feedback

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Can mechanical motion considerably modify transport properties of a SET device at weak coupling?

Nanoelectromechanical systems

NEMS – nanoscale devices which convert electrical current into mechanical energy or vice versa.

Experiments: precise measurements

attoNewtons of force (Stowe *et al* '97)

electrometry (Cleland and Roukes '98)

quantum of thermal conductance (Schwab *et al* '00)

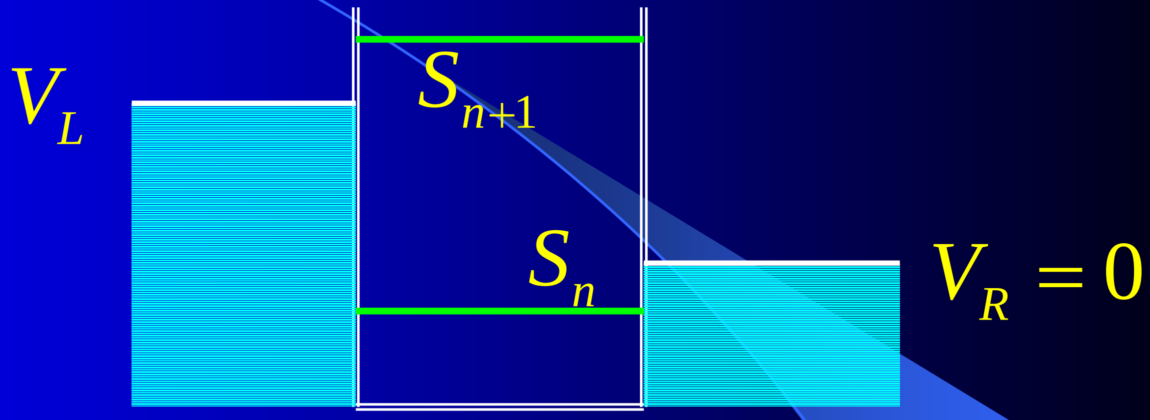
Casimir force (Chan *et al* '01)

Possible applications: nanoscale sensors and actuators

Coulomb blockade

$$S_n = W_{n+1} - W_n$$

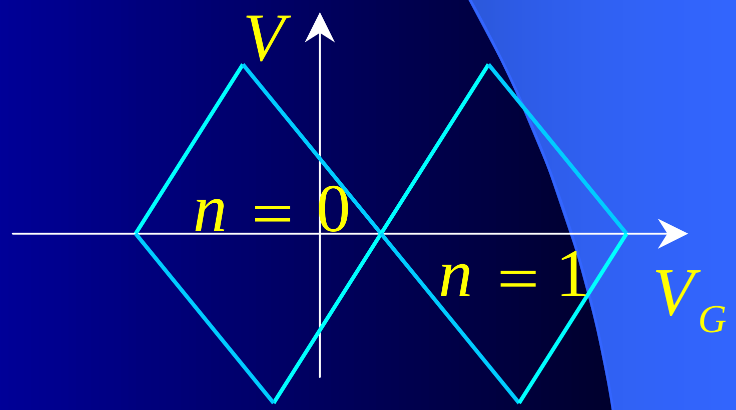
$$\approx (n + \frac{1}{2})\frac{e^2}{C_G} - eV_G$$



Conditions that current does not flow:

(a) $S_{n+1} > eV_L$ (b) $S_n < eV_L$

(c) $S_{n+1} > 0$ (d) $S_n < 0$

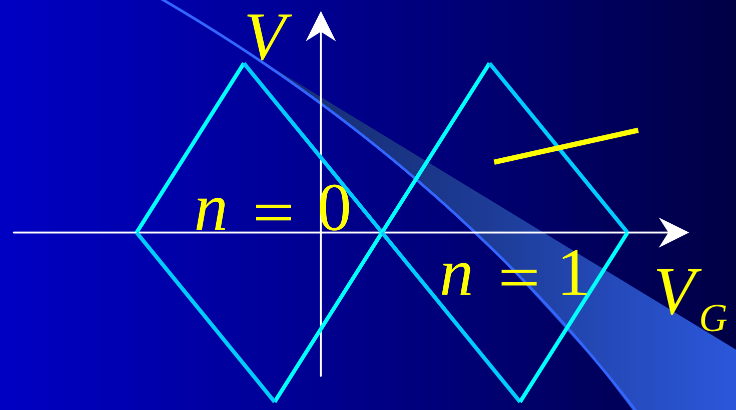


Linear dependence ⟹ Coulomb diamonds

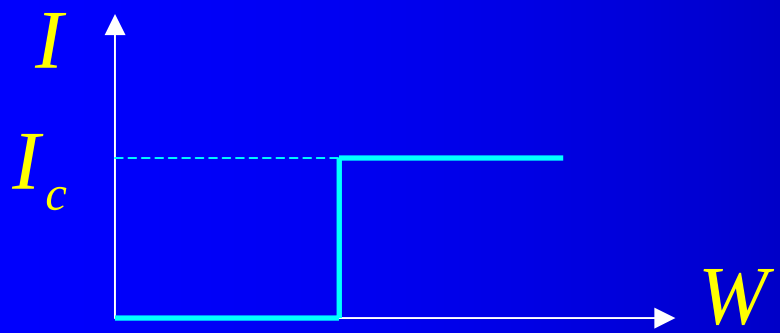
Coulomb blockade - Current

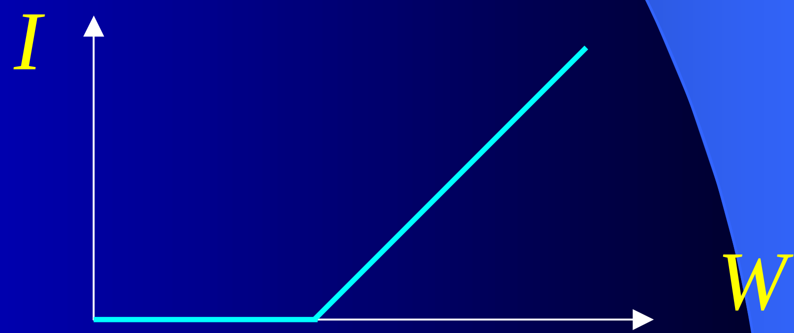
$$S_n = W_{n+1} - W_n$$

$$\approx (n + \frac{1}{2})\frac{e^2}{C_G} - eV_G$$



Current along the yellow line:





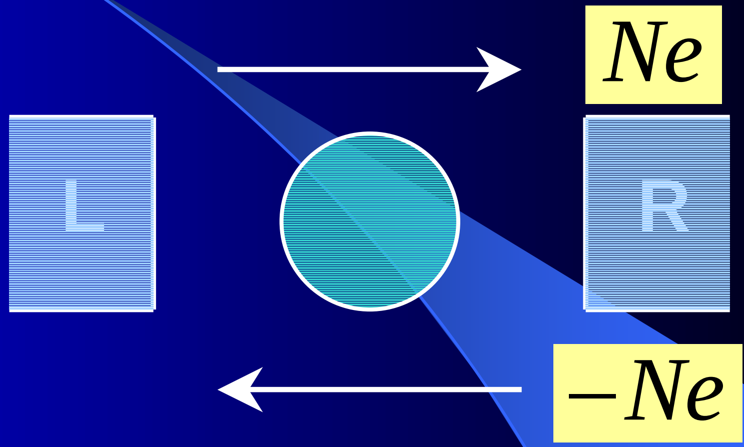
One resonant level

Continuum of levels

Shuttling

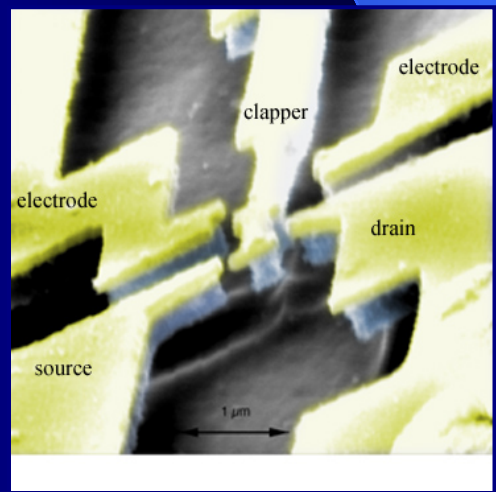
- Shuttling: First theoretical proposal by Gorelik *et al* ‘98

Electrostatically induced periodic motion of the shuttle (central island of the transistor)



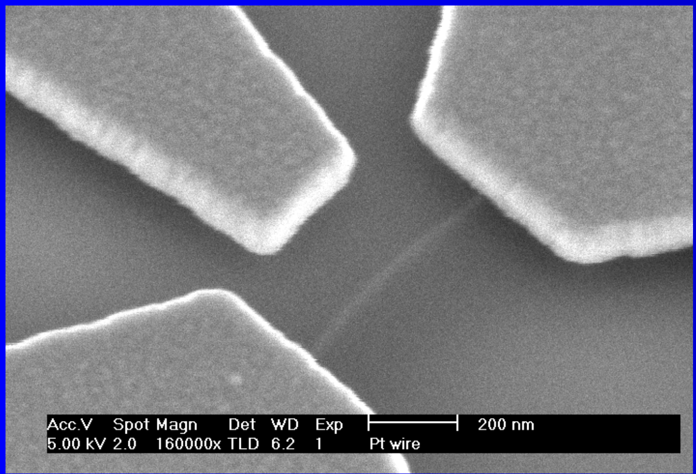
Experiments (all ac driven):

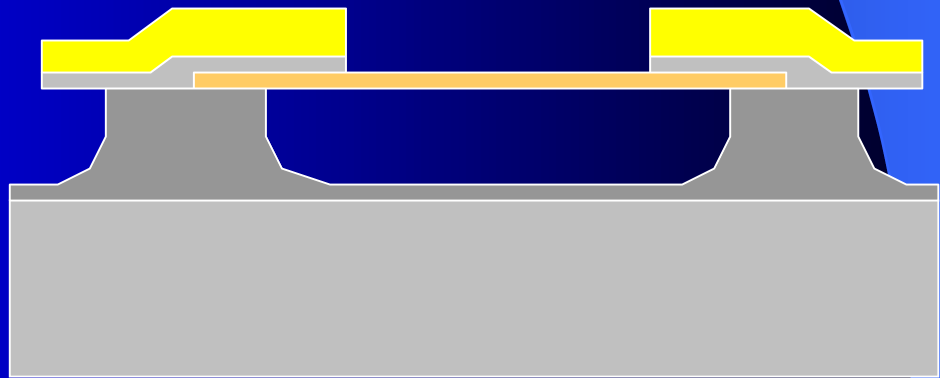
- Classical shuttle (Erbe *et al* ’98)
- Silver grain (Tuominen *et al* ’99)
- Fullerene molecule (Park *et al* ’00)
- ac driven cantilever (Erbe *et al* ’01)



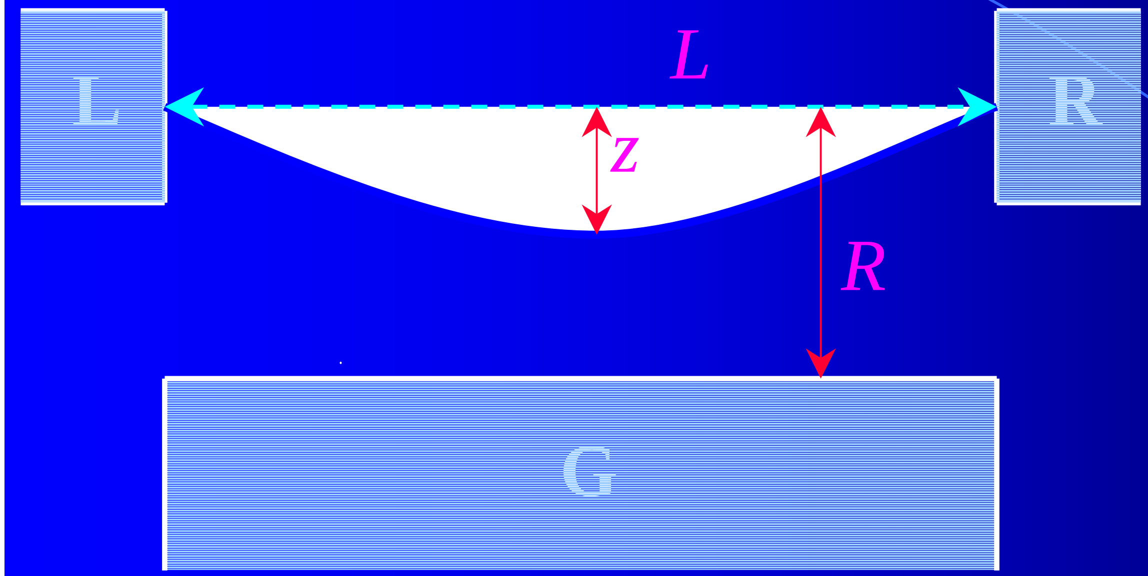
Suspended beams - experiments

- Phonon blockade (Weig *et al* ’04)
- Tuning eigenfrequencies with the gate (Sazonova *et al* ‘04)
- Phonon-mediated tunneling (Sapmaz et al, ‘05; Leroy et al, ’04)
- Quest for quantized mechanical motion (Knobel and Cleland, ’03; De la Haye *et al* ’04)





Modeling suspended nanotubes

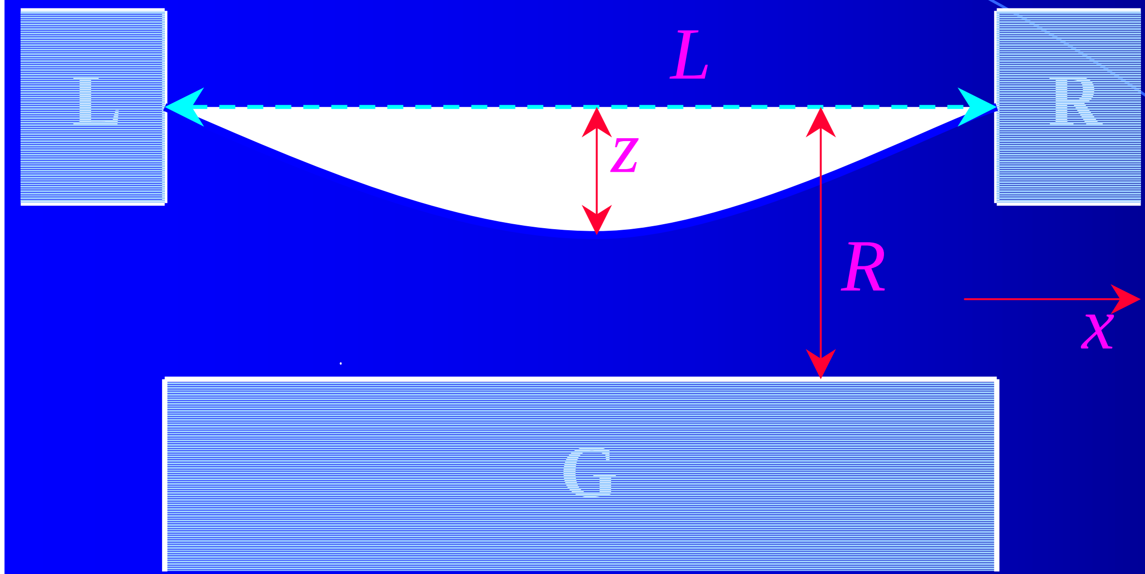


Sapmaz, Y.M.B., Gurevich, van der Zant '02

Simple-minded model:

- Interaction effects taken into account via charging energy;
- Mechanical degrees of freedom via classical theory of elasticity; nanotube modeled as an elastic rod

Elastic energy



Elastic modulus

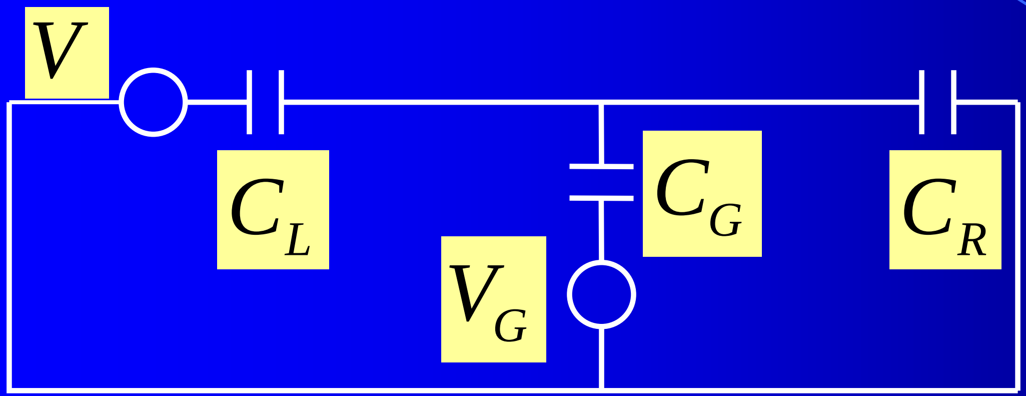
$$W_{el}\{z(x)\} = \int_0^L dx \left\{ \frac{EI}{2} z''^2 + \frac{T}{2} z'^2 \right\}$$

Residual stress

$$T = T_0 + \frac{ES}{2L} \int z'^2 dx$$

Stress induced by bending

Electrostatic energy



$$C_L, C_R \ll C_G$$

$$C_G = \int \frac{dx}{2\ln \frac{2(R - z[x])}{r}}$$

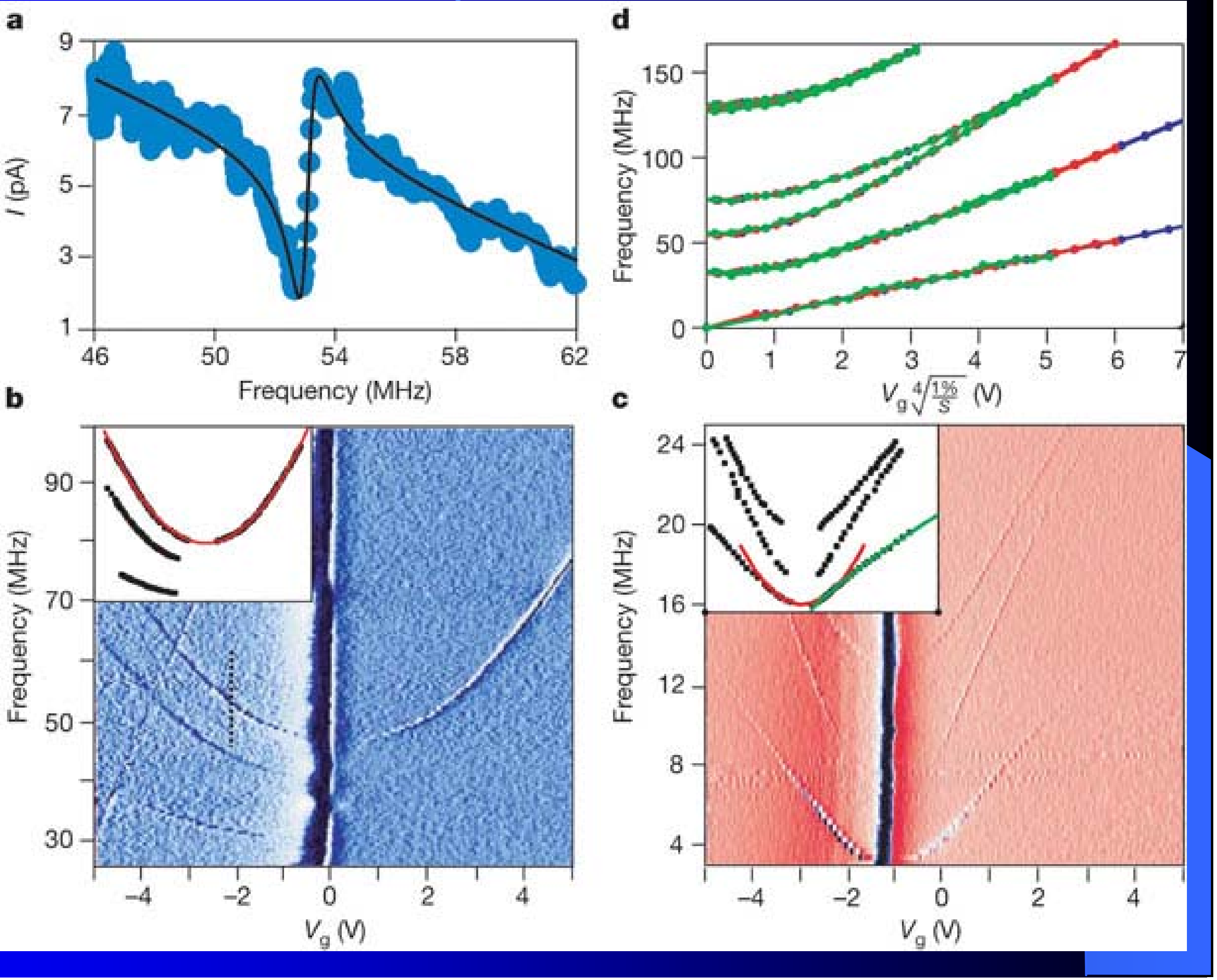
$$W_{e-st}\{z[x]\} = \frac{(ne)^2}{2C_G\{z\}} - neV_G \approx \frac{(ne)^2}{2C_0} - neV_G - \frac{(ne)^2}{L^2 R}\int z[x]\,dx$$

$$F_n = \frac{(ne)^2}{LR}$$

Electrostatic force

Bending modes

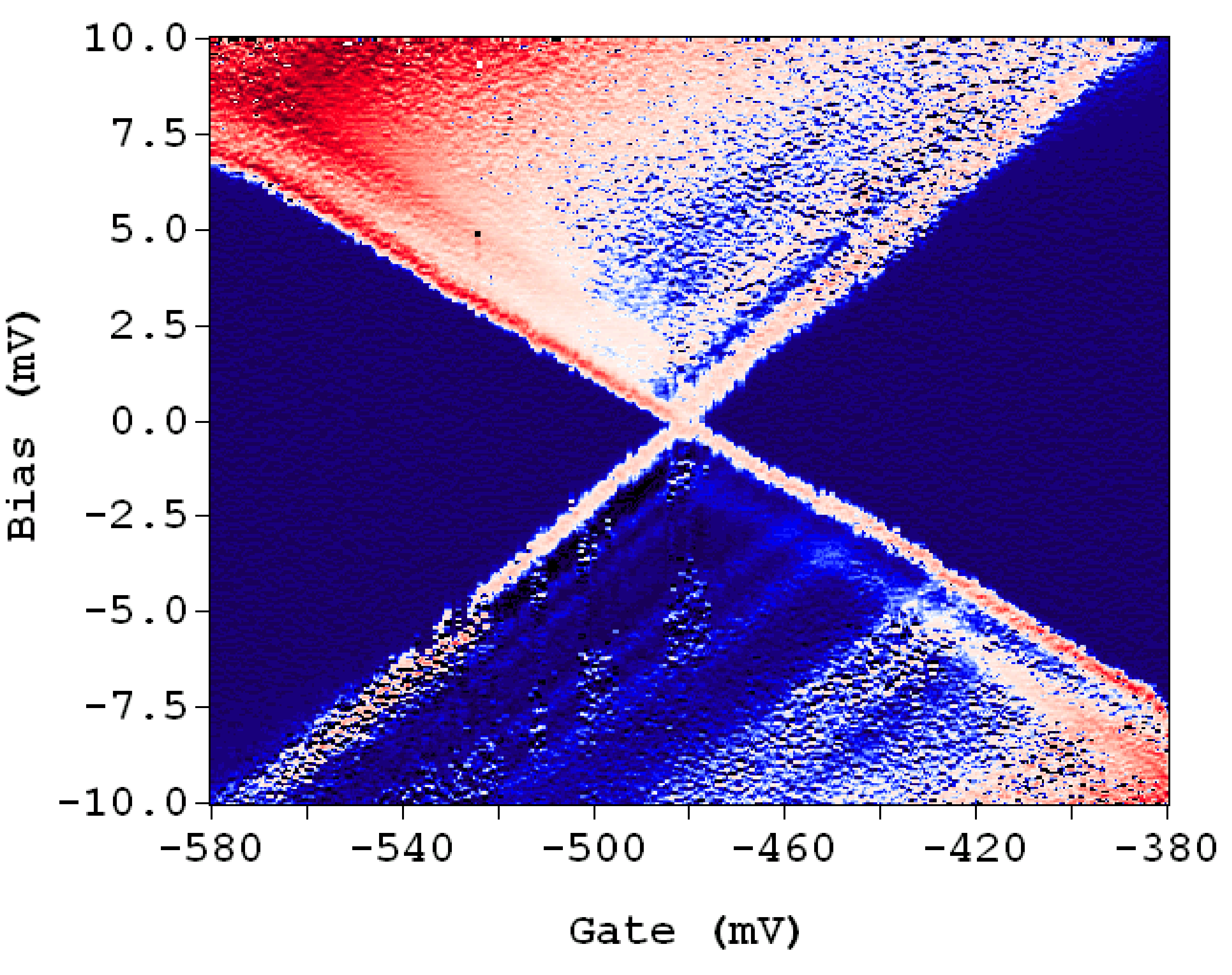
Sazonova et al '04



Stability diagram – Delft experiments

Sapmaz, Jarillo-Herrero, Y.M.B., Dekker, van der Zant, '05

L=140nm, T=300mK



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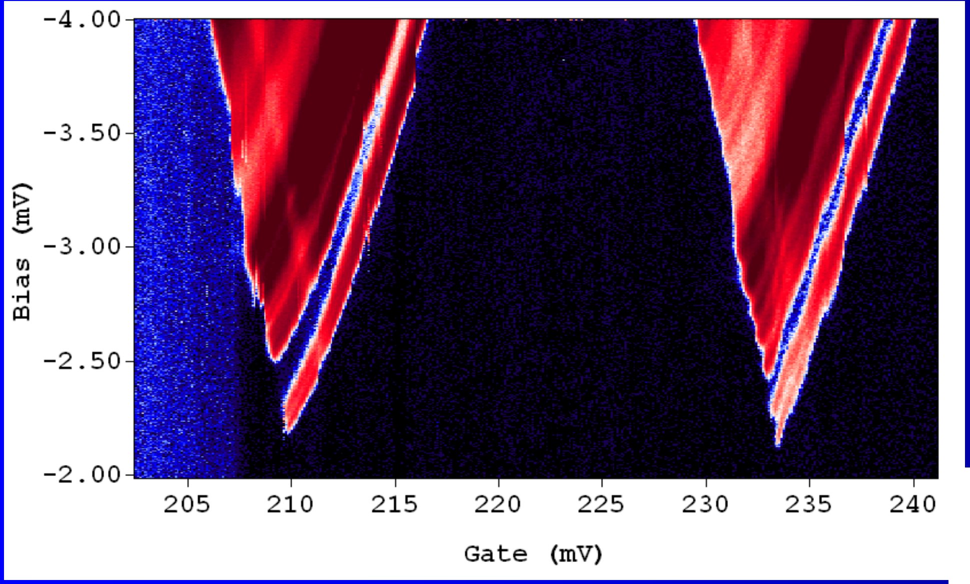
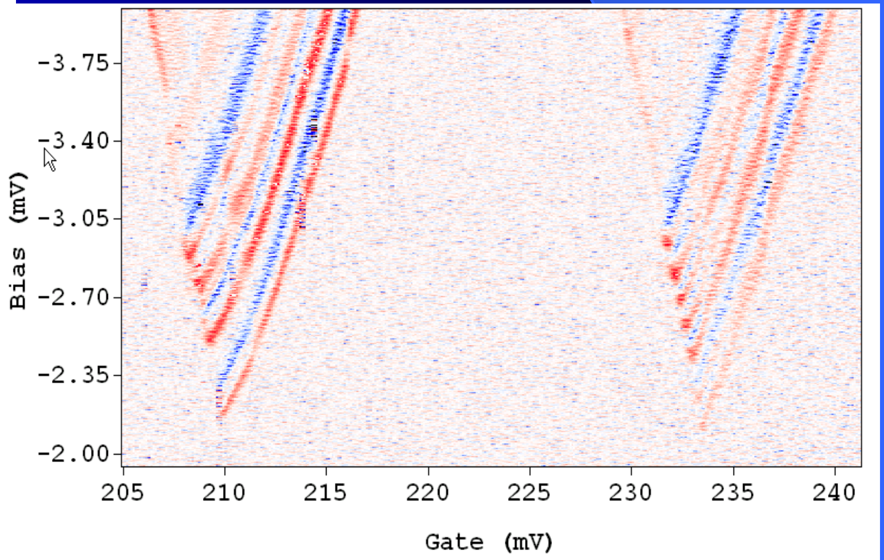


ln *I*

dI / dV

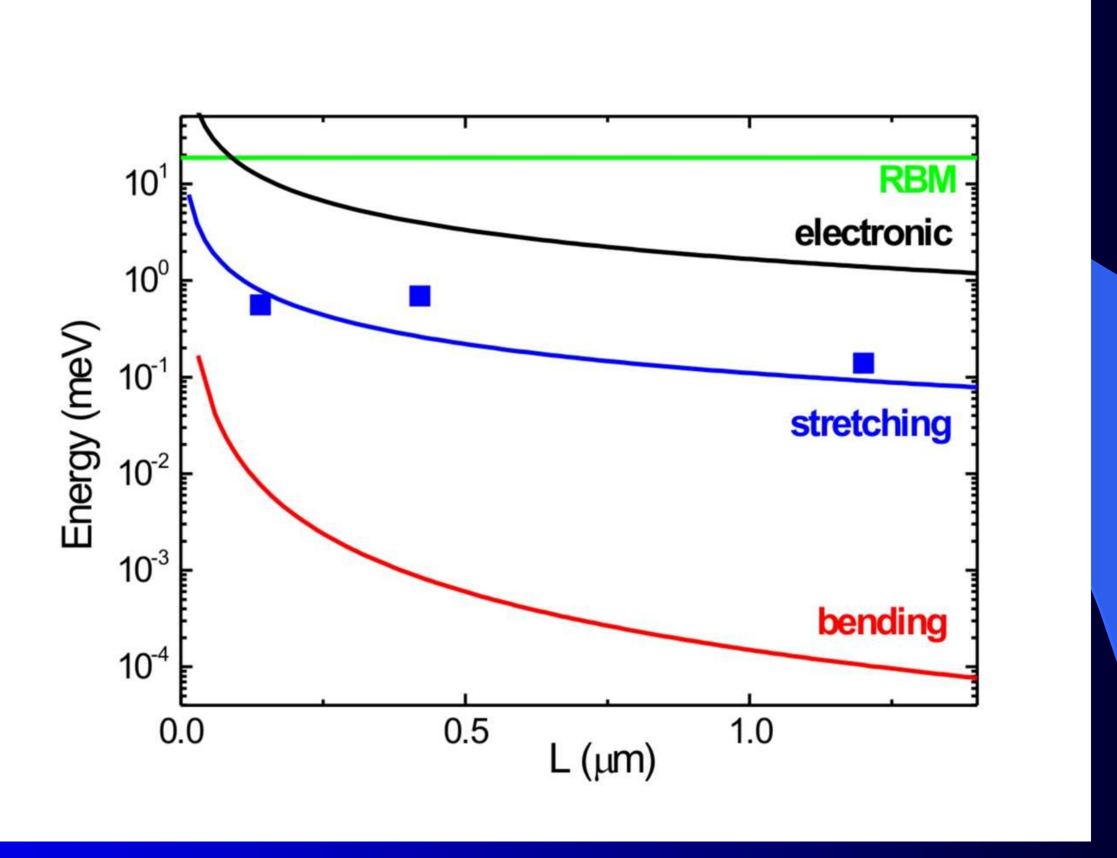


Inelastic tunneling?

Eigenmodes – Delft experiments

Sapmaz, Jarillo-Herrero, Y.M.B., Dekker, van der Zant, ‘05

What modes?



Longitudinal stretching modes

Transport parameters and regimes

SET coupled to a single-mode oscillator

Force F

$$\lambda = \frac{F^2}{M\hbar\omega^3}$$

- coupling parameter

$\lambda \ll 1$ - weak coupling:

- Ground state of SET not affected by phonons (standard Coulomb blockade);
- Phonon-assisted tunneling.

$\lambda \gg 1$ - strong coupling:

- Changes in the oscillator position due to one electron: polarons; Franck-Condon physics;
- Instabilities; negative differential resistance.

Transport parameters and regimes

Friction *vs* oscillator frequency

$Q \ll 1$ - overdamped

- Amplitude decays quickly
- Equilibrium phonons

$Q \gg 1$ - underdamped

- Amplitude decays slowly;
- Non-equlibrium phonons

Transport in NEMS

Vibrations (phonons) affect electrons:
- Position dependence of tunneling matrix element;
- Position-dependent electron energies

Could we have a strong effect of phonons for transport at weak coupling?

Yes, if there are many phonons (large amplitude)!

Electrons affect phonons:
- Tunneling in and out produces a stochastic driving force.

High quality factor: amplitude is large, and this produces strong feedback even for weak coupling.

Components of the model

Y.M.B., Usmani, Nazarov '04

Mechanical oscillator weakly coupled to a SET-device

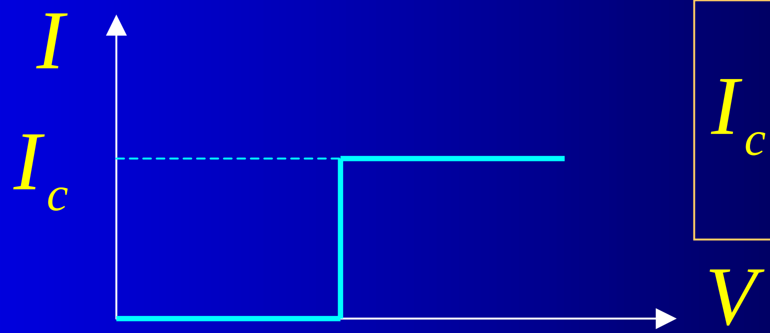
$Q \gg 1$ - underdamped $\qquad \omega \ll \Gamma$ - adiabatic

Coupling: charge-dependent force acting on the oscillator

Only two charge states *n=0* & *n=1* $\qquad F = F_1 - F_0$

Tunnel rates: $\Gamma^{-}_{L,R} = \Gamma_{L,R}(\Delta E - Fx)$

Without coupling:



$$I_c = 2e\frac{\Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R}$$

Distribution function

$P_n(x,v,t)$ - obeys master equation

x – position
v - velocity

$$\frac{\partial P_n}{\partial t} + \left(v\frac{\partial}{\partial x} + \frac{\partial}{\partial v}\frac{F}{M}\right)P_n = St[P]$$

$$F = -M\omega^2 x - M\gamma v + F_n$$

Adiabaticity: reduce to Fokker-Planck equation

$$\frac{\partial P}{\partial t} + v\frac{\partial P}{\partial x} - \omega^2 x\frac{\partial P}{\partial v} - \left[\frac{\omega}{Q} + \Psi(x)\right]\frac{\partial}{\partial v}(vP) = D(x)\frac{\partial^2 P}{\partial v^2}$$

Built-in dissipation

Dissipation due to tunneling

Diffusion in velocity space

Distribution function

$$\frac{\partial P}{\partial t}+v\frac{\partial P}{\partial x}-\omega^2 x\frac{\partial P}{\partial v}-\left[\frac{\omega}{Q}+\Psi(x)\right]\frac{\partial}{\partial v}(vP)=D(x)\frac{\partial^2 P}{\partial v^2}$$

$$D(x)=\frac{F^2}{M^2}\frac{\Gamma^+(x)\Gamma^-(x)}{\Gamma_t^3}$$

$$\Psi(x)=\frac{F^2}{M^2}\frac{\Gamma^-(x)\partial_x\Gamma^+(x)-\Gamma^+(x)\partial_x\Gamma^-(x)}{\Gamma_t^3}$$

$$\Psi(x)\propto\delta(W+Fx)$$

Dissipation happens when the oscillator moves SET over the threshold

Distribution function

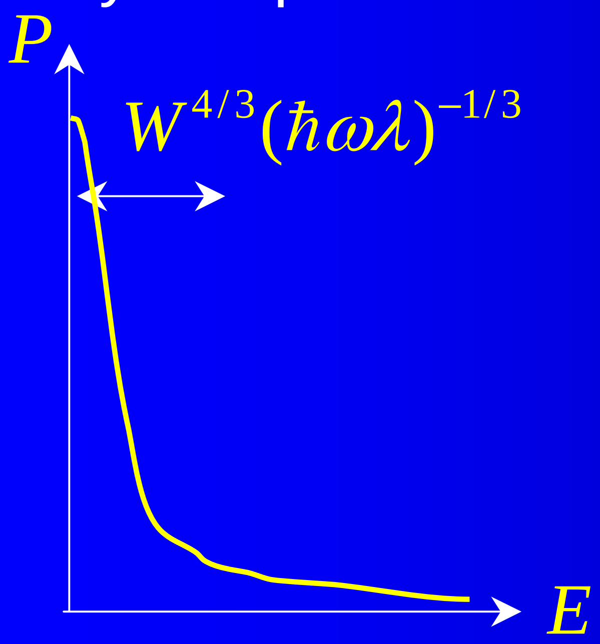
Underdamped: separate slow (energy) and fast (phase) variables

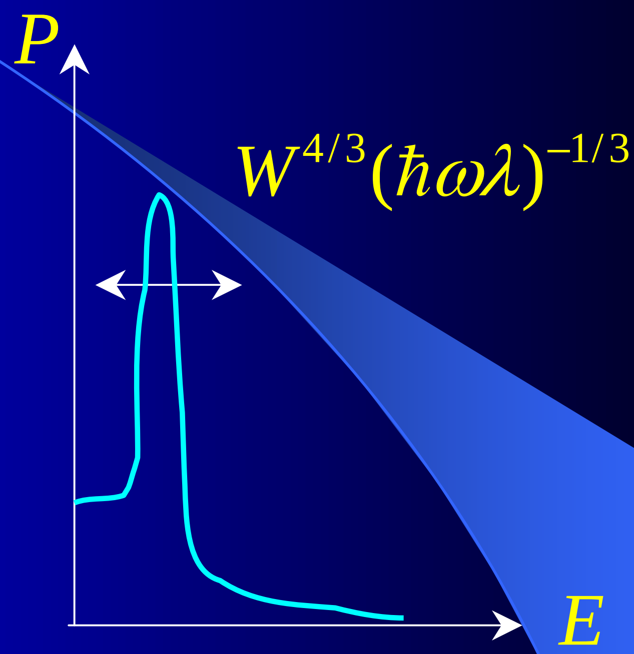
$$x = \frac{1}{\omega}\sqrt{\frac{2E}{M}}\sin\theta; \quad v = \sqrt{\frac{2E}{M}}\cos\theta$$

$$P(E) \propto \exp\left[-\int_0^E dE'\left(\frac{\omega}{2MQD_1(E')} + \frac{\Psi_1(E')}{MD_1(E')}\right)\right]$$

Distribution function

Very sharp features!





Weak mechanical feedback:
Amplitude small

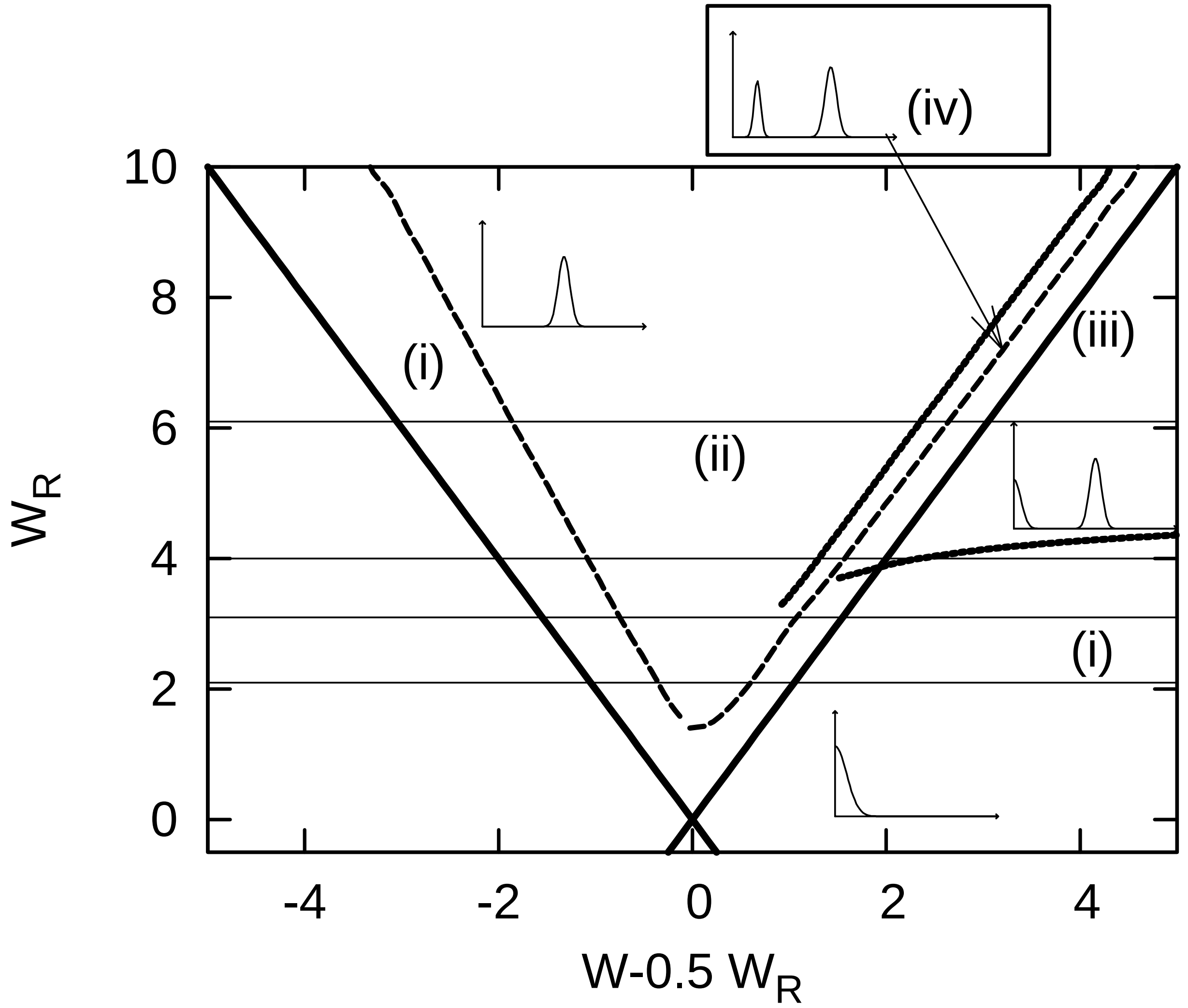
Strong mechanical feedback:
Amplitude large

Strong feedback: requires negative “quality factors”

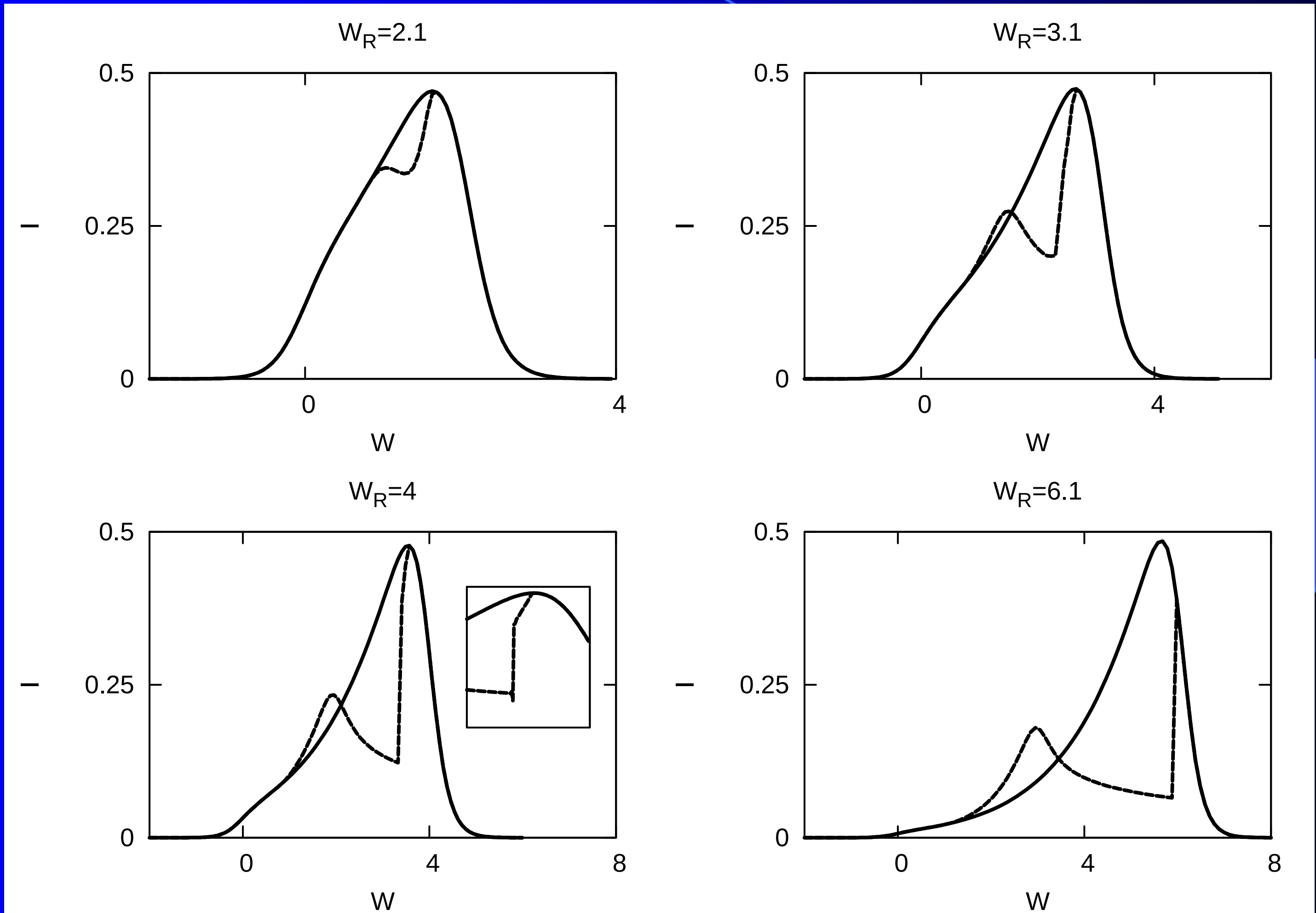
Requires that one of the rates decreases with energy

Distribution function

Example: $$\Gamma^{\pm}_{L,R} = 2e^{a_{L,R}(W - W_{L,R} + Fx)} f_F(\pm(W - W_{L,R} + Fx))$$



Current for strong mechanical feedback



Solid: no motion; dashed: with feedback

Noise for strong mechanical feedback

Dimensional analysis: $S \sim I^2 t$

Common situation: the only time scale $t \sim \Gamma^{-1}$

$I \sim e\Gamma \Rightarrow S_P \sim eI$ Poisson value of shot noise

NEMS: the longest time scale $t \sim Q/\omega_0$

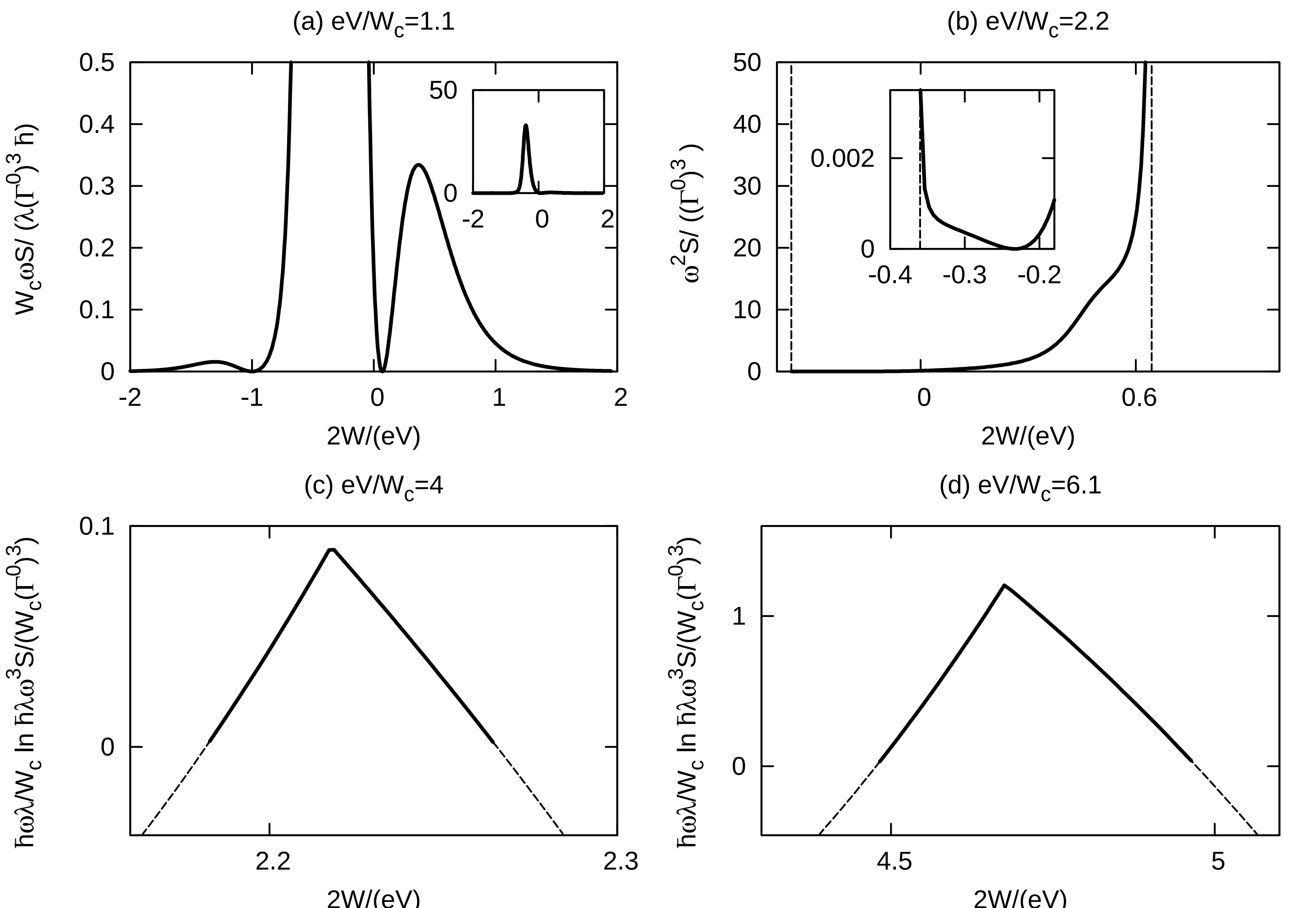
$S \sim eI\Gamma Q/\omega_0$ Strong enhancement!

One-peak distribution function with zero amplitude:

$$S/S_P \sim (\Gamma/\omega_0)^2(\hbar\omega_0\lambda/W)$$

Two peaks: further enhancement due to switching

Noise for strong mechanical feedback



Conclusions

- Current can be considerably modified by mechanical degrees of freedom even for weak electron-phonon coupling
- This only happens if the tunnel rates essentially depend on energy
- Giant enhancement of noise, even without strong mechanical feedback