Full Counting Statistics in Superconducting Heterostructures

Wolfgang Belzig

Universität Konstanz

Circuit Theory
of
Crossed Andreev Reflection

Wolfgang Belzig

Universität Konstanz

Jan Petter Morten and Arne Brataas

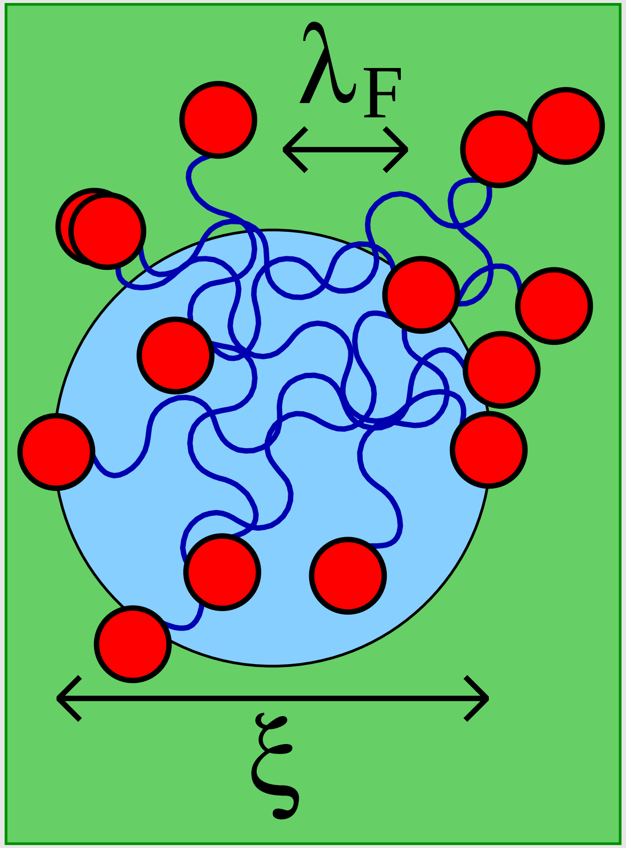
Norwegian University of Technology, Trondheim

Quantum Phenomena At Low Temperatures, Lammi (2006)

Content

- Nonlocality of Cooperpairs
- Crossed Andreev (CA) reflection
  (vs. Elastic Cotunneling (EC))
- Experimental Signatures
- Quasiclassical Approach (Circuit Theory)
- Results
- Conclusion/Outlook

Nonlocality of Cooperpairs



Size of a Cooperpair:

$$\xi_0 = \frac{\hbar v_F}{2\Delta} \text{ (clean limit)}$$

$$\xi = \sqrt{\frac{\hbar D}{2\Delta}} \text{ (dirty limit)}$$

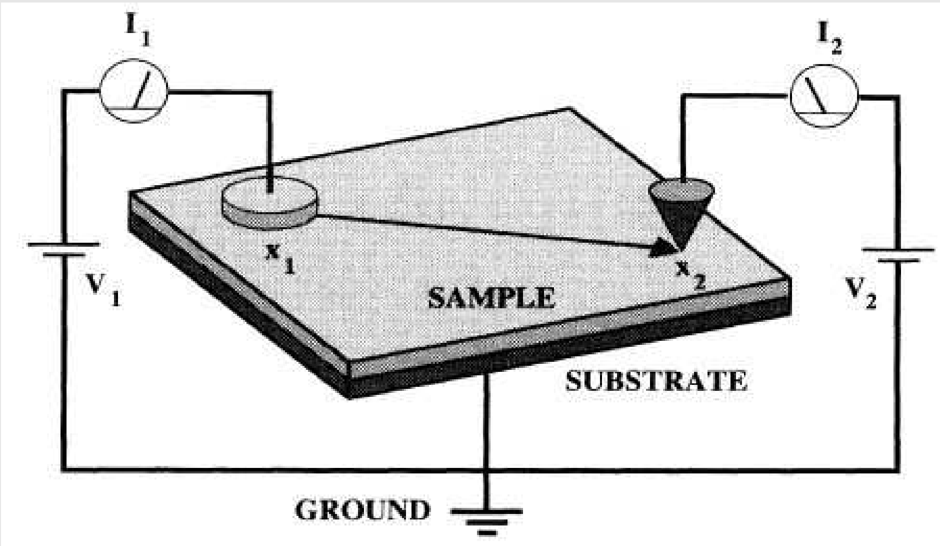
Spin-entangled electrons

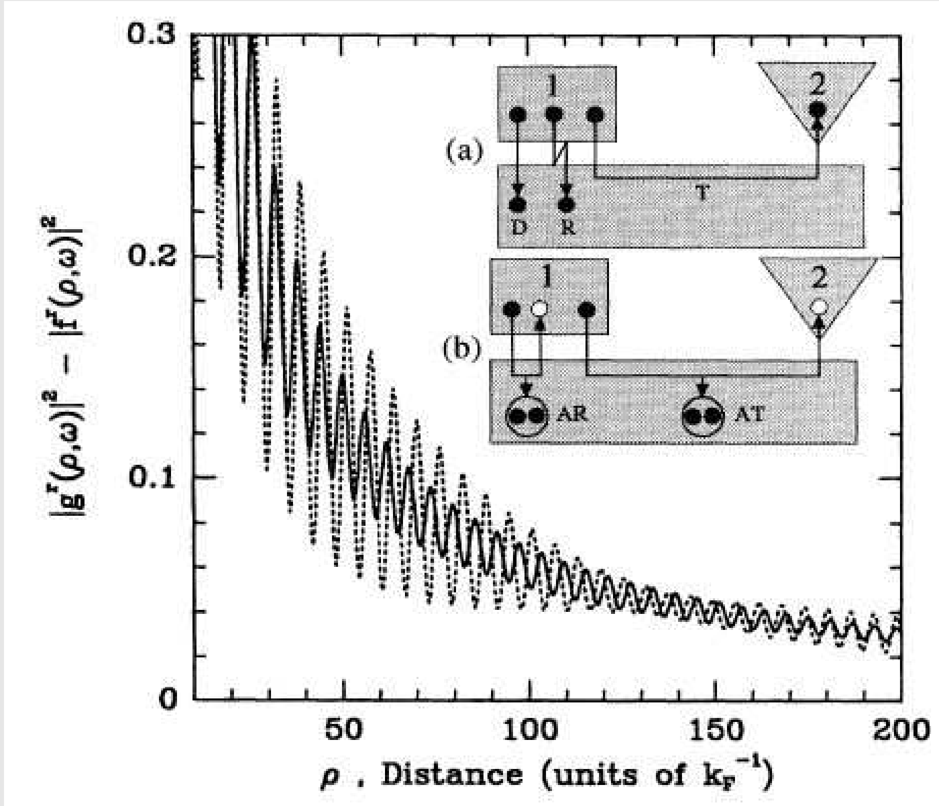
$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\right)$$

Question:
Detection by nonlocal transport setup?

Theoretical Proposals

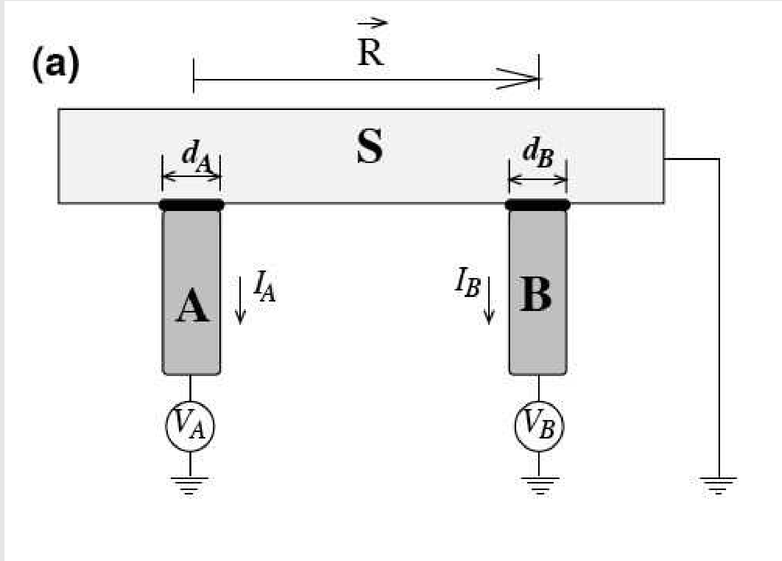
Spatial correlations
[Byers, Flatte, PRL 95]





- fast oscillations on scale $1/k_F$
- slow decay of envelope on scale $\xi_0 = \hbar\Delta/v_F$
- double STM -> difficult to realize

Separated normal contacts
[Falci, Feinberg, Hekking, EPL 01]
e.g. lithographically defined



conductance matrix:

$$\begin{pmatrix} I_A \\ I_B \end{pmatrix} = \begin{pmatrix} G_{DA}^A + G_{CA} + G_{EC} & G_{CA} - G_{EC} \\ G_{CA} - G_{EC} & G_{DA}^B + G_{CA} + G_{EC} \end{pmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix}$$

Tunneling Hamiltonian approach:
limited to lowest order perturbation theory

$$\begin{pmatrix} G^\sigma_{\mathrm{EC}} \\ G^\sigma_{\mathrm{CA}} \end{pmatrix} \approx \frac{2\pi^3 e^2}{\hbar} |T_{\mathrm{A}} d_{\mathrm{A}}|^2 \, |T_{\mathrm{B}} d_{\mathrm{B}}|^2 N_S^2(0) N_{\mathrm{A}}^\sigma(0) \frac{e^{-2R/\pi\xi}}{(k_{\mathrm{S}} R)^2} \begin{pmatrix} N_{\mathrm{B}}^\sigma(0)\cos^2(k_{\mathrm{S}} R) \\ N_{\mathrm{B}}^{-\sigma}(0)\sin^2(k_{\mathrm{S}} R) \end{pmatrix}$$

-> $G_{CA} = G_{EC}$ in many channel limit

More Theoretical Proposals

Spin-polarized contacts [Falci, Feinberg, Hekking, EPL 01]

$$\begin{pmatrix} G_{EC} \\ G_{CA} \end{pmatrix} \sim \begin{pmatrix} N_\uparrow^A N_\uparrow^B + N_\downarrow^A N_\downarrow^B \\ N_\uparrow^A N_\downarrow^B + N_\downarrow^A N_\uparrow^B \end{pmatrix} \sim \begin{pmatrix} 1 + 2P^A P^B \\ 1 - 2P^A P^B \end{pmatrix}$$

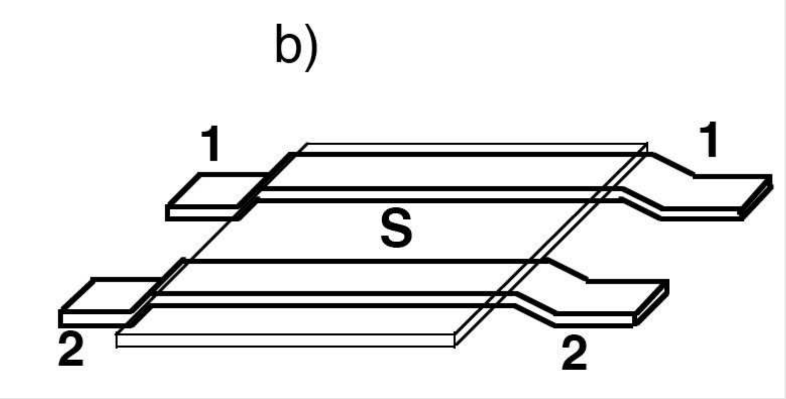
-> depends on magnetic configuration (parallel vs. antiparallel)

Diffusion inside S [Feinberg PRB 03; Chtchelkatchev JETPL 03]
mean free path l; coherence length $\xi = \sqrt{\xi_0 l/3}$

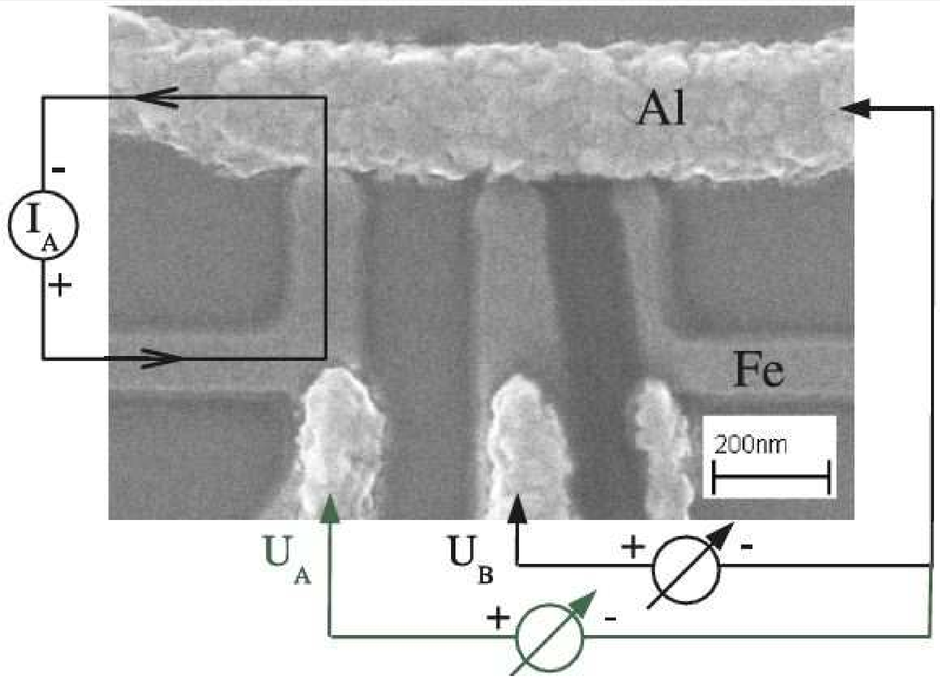
Effective renormalization

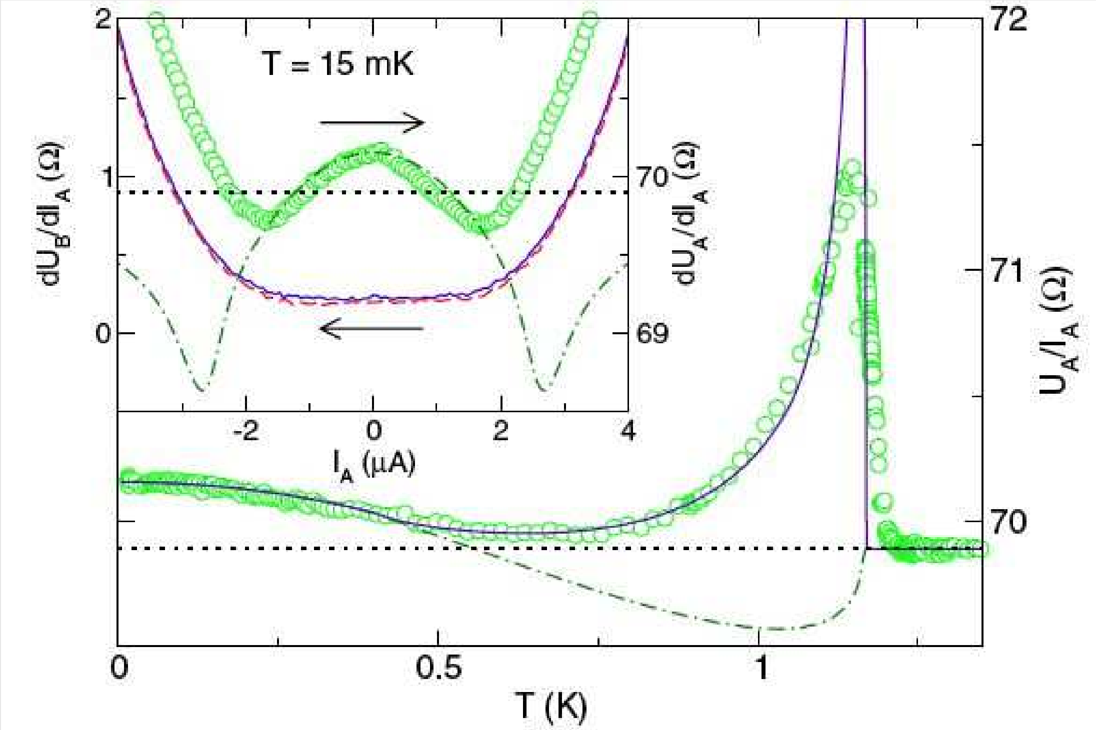
$$G_{EC/CA}^{dirty} = G_{EC/CA}^{clean} \frac{R}{l} e^{-R/\xi}$$

-> nonlocal conductance enhanced
-> cancellation remains



Ferromagnetic contacts [Beckmann, Weber, Löhneysen, PRL 04]

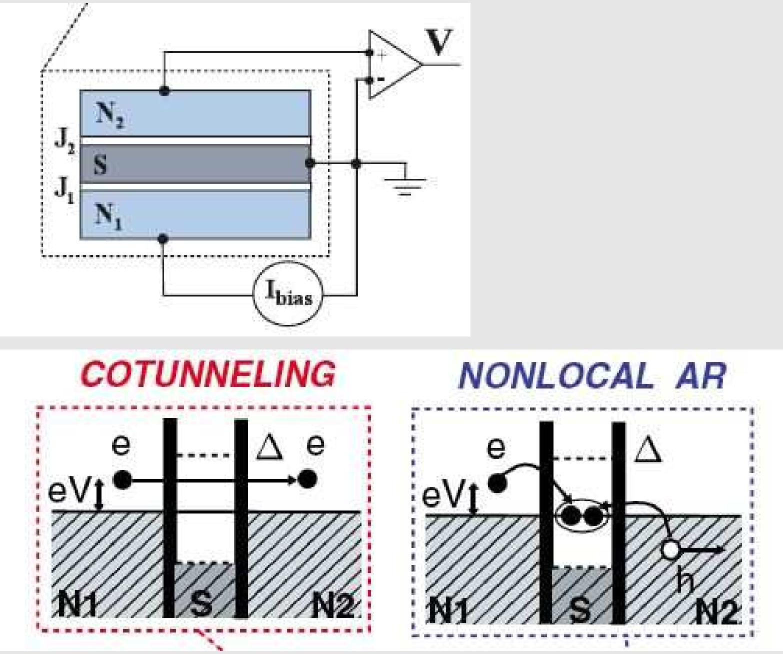


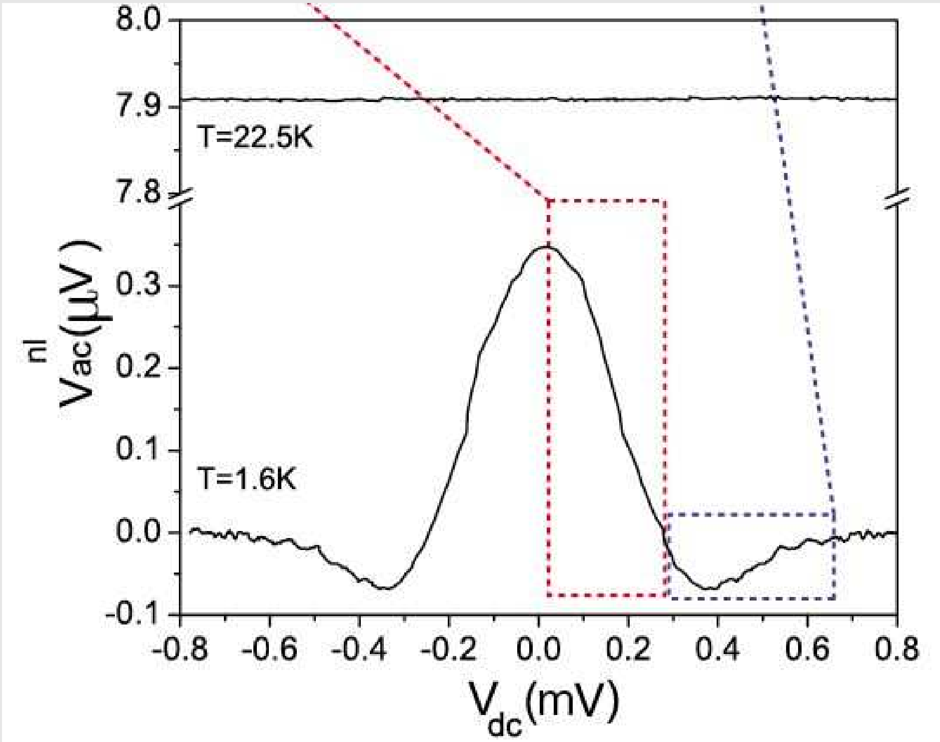


- nonlocal (spin-)signal observed
- sign corresponds to (dominant) elastic cotunneling

Experiment II

Energy resolved detection scheme
(Russo, Kroug, Klapwijk, Morpurgo PRL 2005)





- Energy dependence on scale of Thouless energy
- Magnitude, sign, and voltage dependence not explained

Question: Can this effect be described
by the quasiclassical theory

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Answer:
Of course!

Quasiclassical theory

General theory (Gorkov equation) for matrix GF $\check{G}(x,x')$

$$\begin{pmatrix} -i\partial_t - h(x) & \Delta(x) \\ -\Delta^*(x) & i\partial_t - h^*(x) \end{pmatrix} \check{G}(x,x') = \delta(x-x')$$

realistic system -> too complicated to be useful

Quasiclassical approximation (Eilenberger equation)

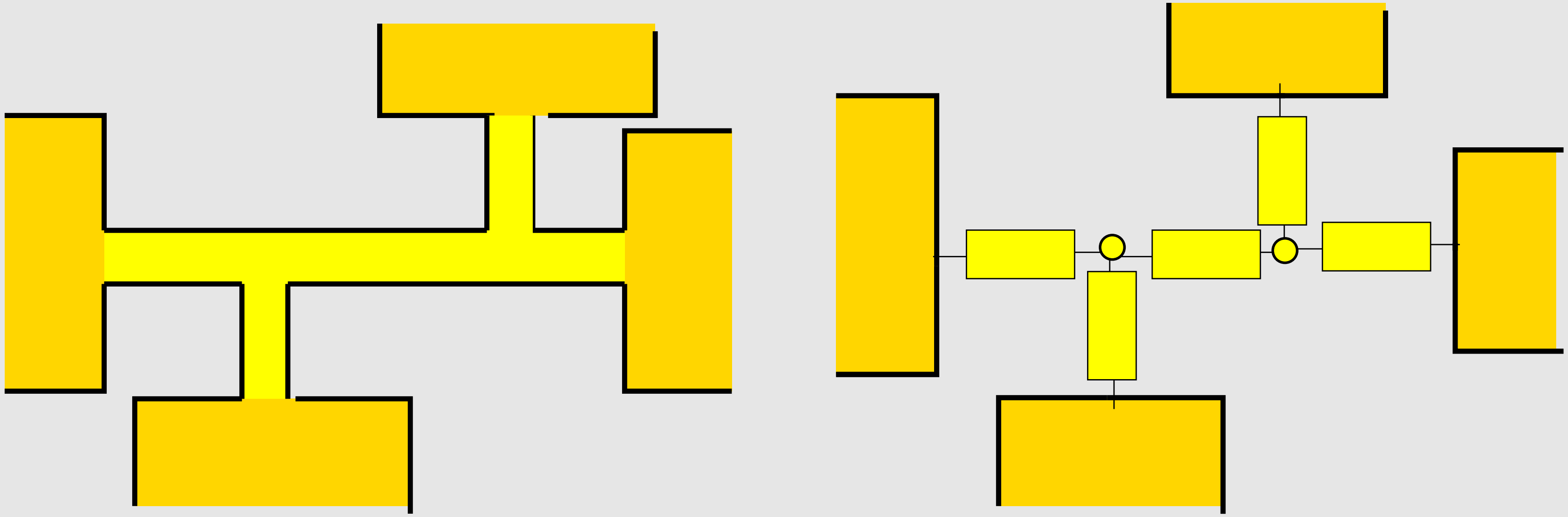
$$\left[\begin{pmatrix} -i\partial_t - h_{qc}(x) & \Delta(x) \\ -\Delta^*(x) & i\partial_t - h_{qc}^*(x) \end{pmatrix}, \check{g}_{qc}(r) \right] = 0$$

-> compact effective equation for CMS-Green's function

Only approximation: $\xi_0, l, L \gg 1/k_F$

Circuit theory formulation

Discretized version of quasiclassical equations by 'electrical circuit'

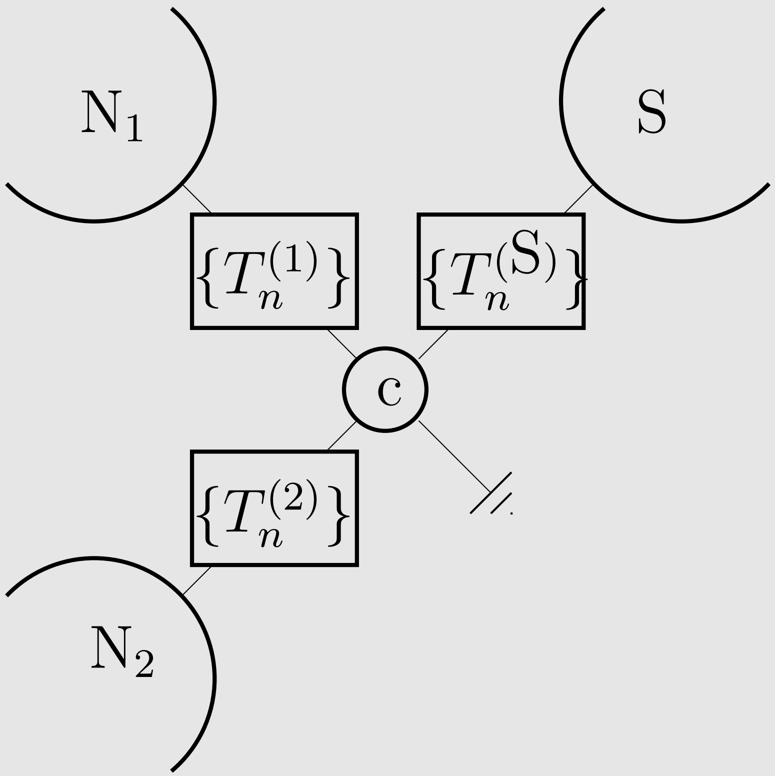


Matrix current between two general nodes [Nazarov 99]
(transmission eigenvalues $\{T_n\}$):

$$\check{I}_{12} = -\frac{e^2}{\pi} \sum_n \frac{T_n \left[\check{G}_1, \check{G}_2\right]}{4 + T_n \left(\left\{\check{G}_1, \check{G}_2\right\} - 2\right)}$$

Our (toy) setup

Three terminal structure (one S and two N)



Main assumption:

- all leads are coherently coupled to central node
- connectors characterized by set of transmission eigenvalue $\{T_n^i\}, i = 1, 2, S$
- finite propagation in node leads to Thouless energy $E_{Th} = \hbar D/L^2$
- $E_{Th} \ll \Delta$

Identification of contributions

Energy dependent spectral conductances

$$\begin{aligned} I_1(E) = \mathrm{Tr}\hat{\sigma}_3(\check{I}_1)_K \;\; = \;\; & \frac{G_{\mathrm{EC}}(E)}{e}\left[f_2(E) - f_1(E)\right] \\ & + 2\frac{G_{\mathrm{DA}}(E)}{e}\left[1 - f_1(E) - f_1(-E)\right] \\ & + \frac{G_{\mathrm{CA}}(E)}{e}\left[1 - f_1(E) - f_2(-E)\right] \end{aligned}$$

Fermi factors indicate different processes

Simple analytical results: Tunneling limit, low energy

All $\{T_n^i\} \ll 1$ and $E \ll E_{Th}$

$$G_{\mathsf{EC}} = \frac{g_1 g_2}{2} \frac{2\,(g_1+g_2)^2 + g_{\mathsf{S}}^2}{[(g_1+g_2)^2 + g_{\mathsf{S}}^2]^{3/2}}$$

$$G_{\mathsf{CA}} = \frac{g_1 g_2}{2} \frac{g_{\mathsf{S}}^2}{[(g_1+g_2)^2 + g_{\mathsf{S}}^2]^{3/2}}.$$

- $G_{\mathsf{EC}} > G_{\mathsf{CA}}$: elastic cotunneling dominates!
- limit $g_S \gg g_{1,2}$: $G_{\mathsf{EC}} = G_{\mathsf{CA}}$, exact cancellation

General analytical result

Spectral properties of central node -> pairing angle $\theta_c(E)$

Determines conductance factors: $G_{\text{T/L,i}}(\{T_n^{(i)}\}, \theta_c) > 0$

Spectral conductances

$$G_{\text{DA}}(E) = \frac{1}{4}\left(\frac{G_{\text{T,1}}\left(G_{\text{T,2}} + G_{\text{T,S}}\right)}{G_{\text{T,1}} + G_{\text{T,2}} + G_{\text{T,S}}} - \frac{G_{\text{L,1}}G_{\text{L,2}}}{G_{\text{L,1}} + G_{\text{L,2}}}\right)$$

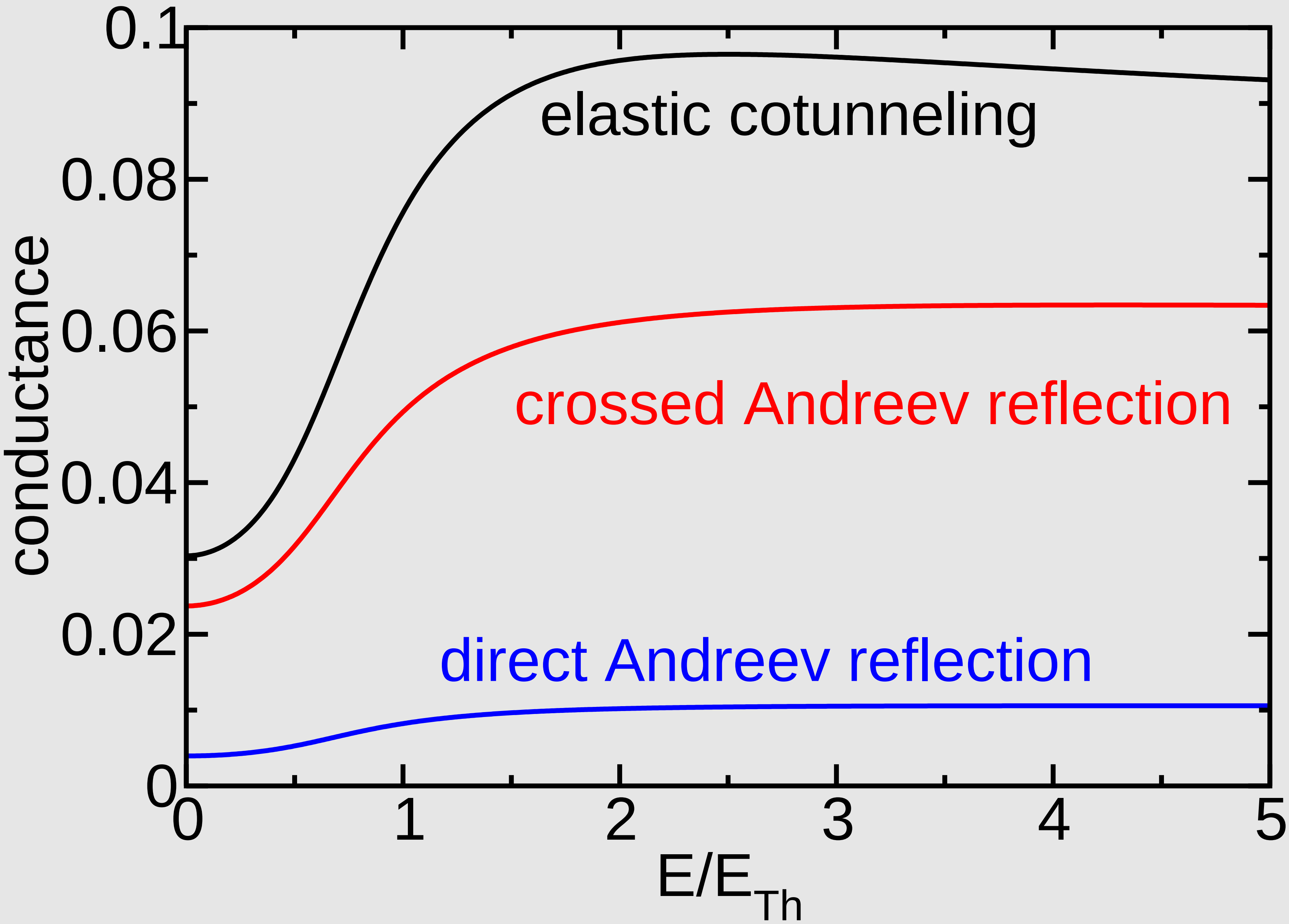
$$G_{\substack{\text{EC}\\\text{CA}}}(E) = \frac{1}{2}\left(\frac{G_{\text{L,1}}G_{\text{L,2}}}{G_{\text{L,1}} + G_{\text{L,2}}} \pm \frac{G_{\text{T,1}}G_{\text{T,2}}}{G_{\text{T,1}} + G_{\text{T,2}} + G_{\text{T,S}}}\right)$$

- (unfortunate) general result: $G_{\text{EC}} > G_{\text{CA}}$
- however: energy dependence on scale of Thouless energy

Numerical results

Perfect contact to S: $\{T_n^S\} = 1$ tunneling contact to N: $\{T_n^S\} \ll 1$

Asymmetric junctions $g_S \gg g_{1,2}$



Summary/Outlook

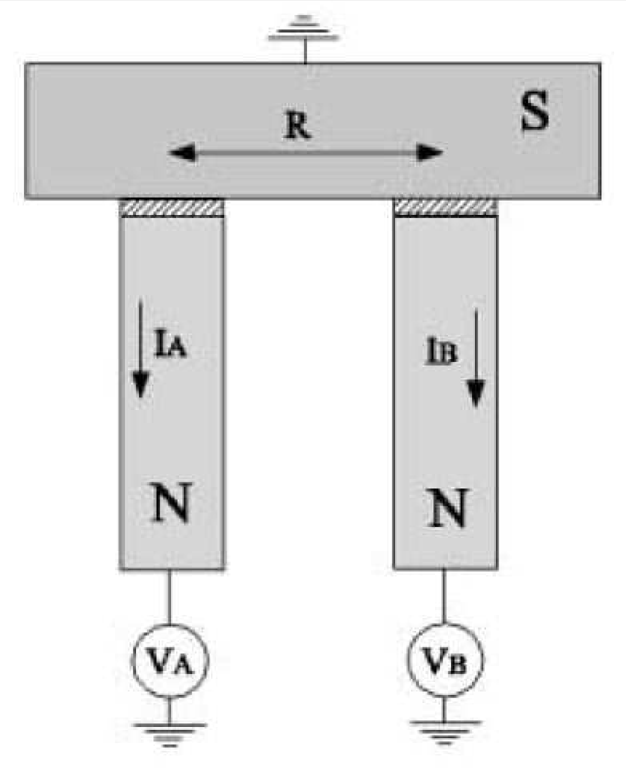
- quasiclassical description of nonlocal Andreev reflection
- energy dependence on Thouless energy
- elastic cotunneling dominates (for present setup)

Open question:

- other methods to detect nonlocal Andreev reflection
- detecting the entanglement (Bell-type measurement)

Alternative method: cross correlations

Tunneling Hamiltonian [Bignon, Houzet, Pistolesi, Hekking, EPL 04]



Tunnel Hamiltonian approach:

Current cross correlations

$$S_{12} = 2eG_{CA}|V_1 + V_2| - 2eG_{EC}|V_1 - V_2|$$

Sign of cross correlations indicate type of process

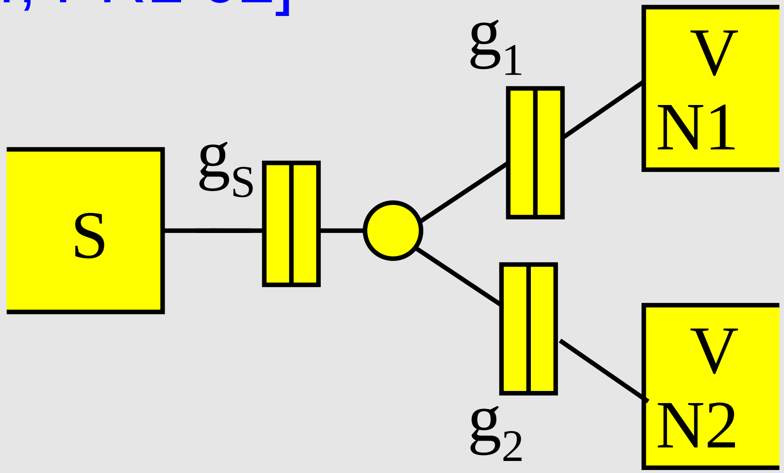
positive = crossed Andreev

negative = elastic cotunneling

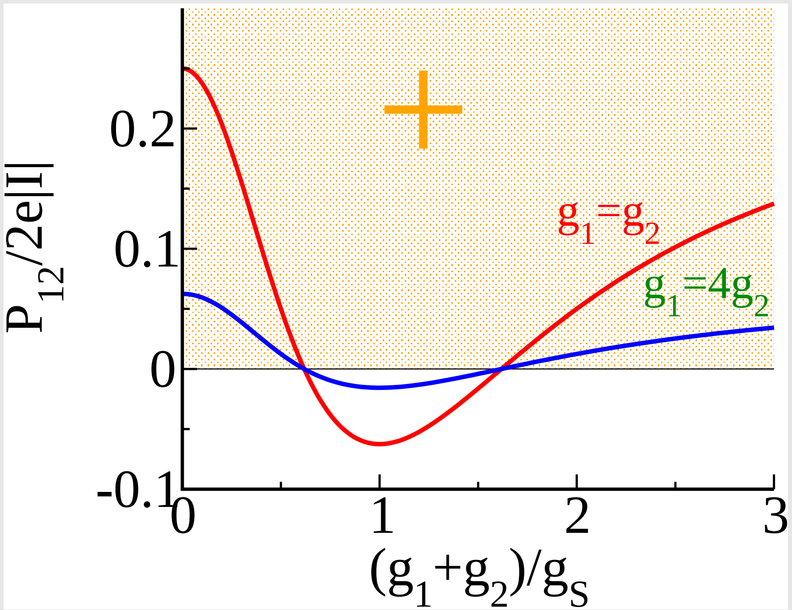
Reverse outlook: superconducting beam splitter

Only tunnel junctions [Börlin, Belzig, Bruder, PRL 02]

- Three tunnel junctions connected to common node
- no dephasing $T \ll V \ll E_{Th}, \Delta$
- crosscorrelations



$$P_{12} = 2\langle \delta I_1(t) \delta I_2(0) \rangle_{\omega=0}$$



- for $g_N \ll g_S$ or $g_N \gg g_S$ positive crosscorrelations -> crossed Andreev
- negative crosscorrelations around $g_N = g_S$ (even in the absence of elastic cotunneling)

Summary/Outlook

- quasiclassical description of nonlocal Andreev reflection
- energy dependence on Thouless energy
- elastic cotunneling dominates (for present setup)
- positiv cross correlation in the coherent regime

Open question:

- full counting statistics
- detecting the entanglement (Bell-type measurement)