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Cotunneling of Cooper pairs in superconducting SETs
(with local resistors)

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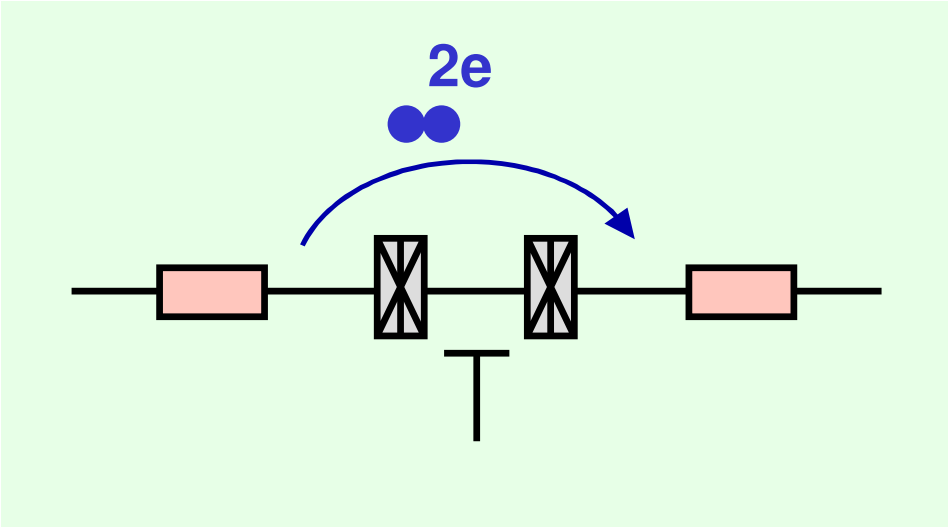
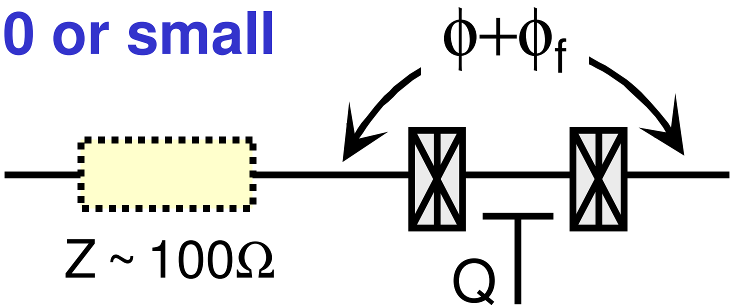


Physikalisch-Technische Bundesanstalt, Berlin und Braunschweig

Why resistors?

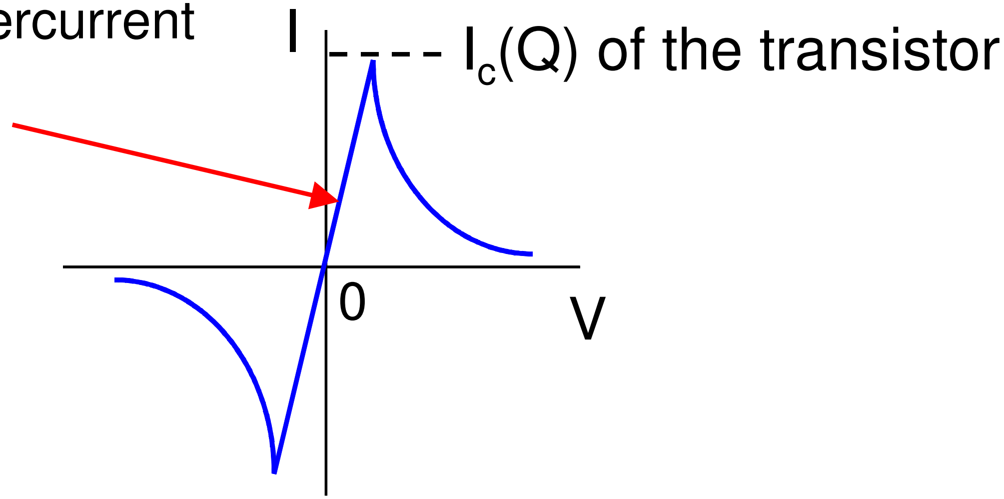
If R = 0 or small



Overall phase behaves either classically or slightly fluctuates around class. values
(phase slips possible)

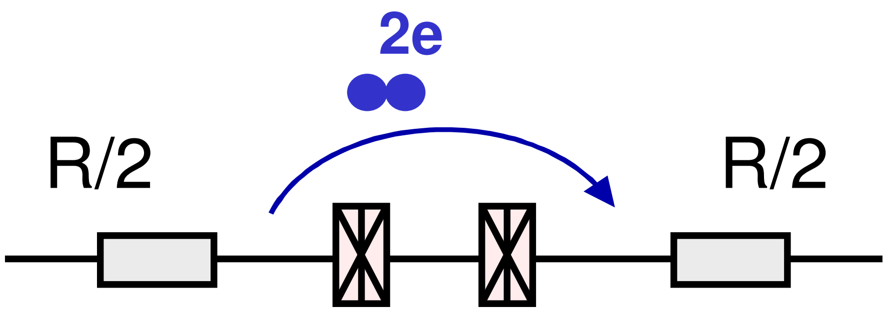
$\langle I_s(\phi)\rangle \neq 0$, at V = 0 (or at small values of V).

Either the vertical Josephson supercurrent
or inclined phase-diffusion branch



The Goal:

To investigate Cooper pair cotunneling (CPC), assuming transfer of individual pairs across the transistor.

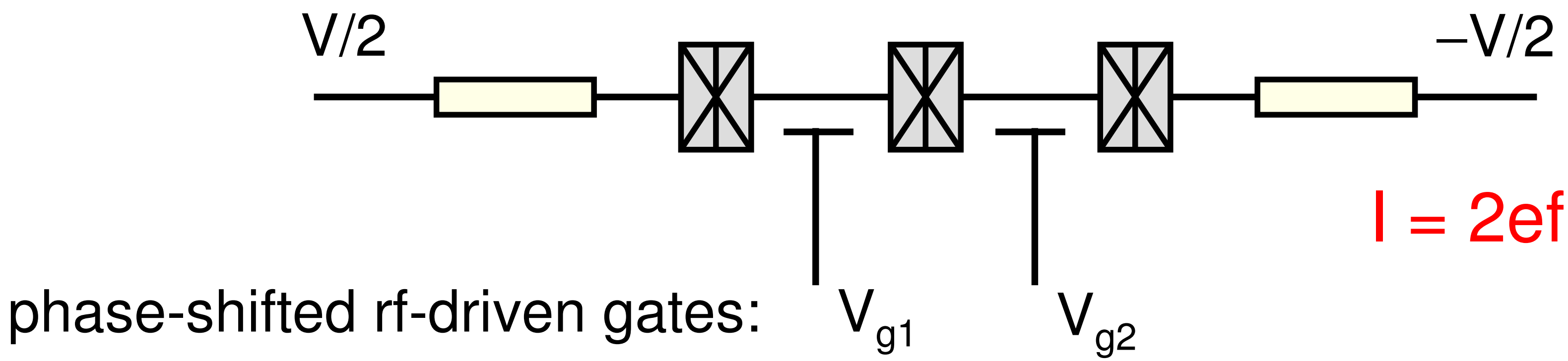


Such regime is established at appreciable dissipation due to resistors,

$$R \gtrsim R_Q \equiv \frac{h}{4e^2} \approx 6.45\,\mathrm{k}\Omega$$

Motivation

To realize a regime of pumping Cooper pairs in the array of small Josephson junctions with local resistors

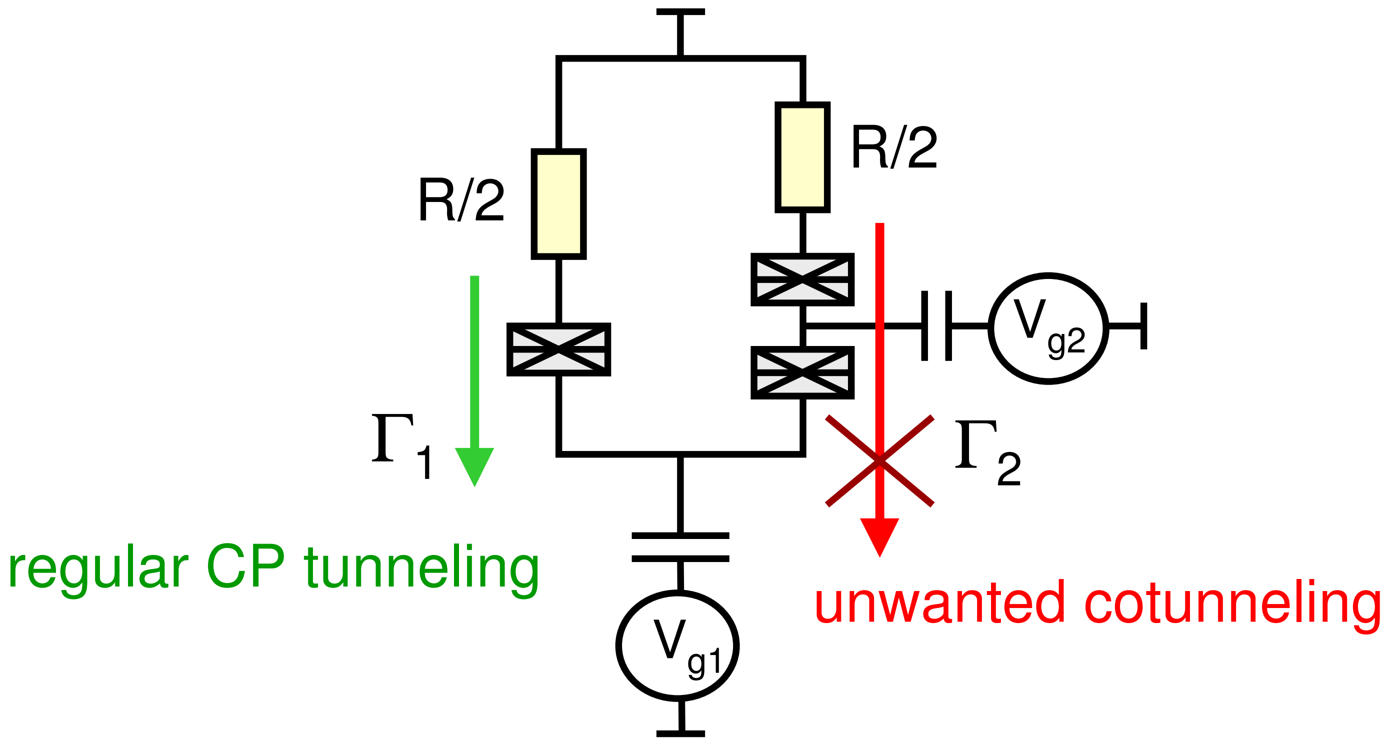


Such CP R-pump is potentially faster than the normal SET counterpart.

What is the benefit of using resistors in the pump?

1. They should suppress the tunneling across the whole device at non-zero (small) V - the so-called N-tunneling.

2. They should suppress the so-called (N–1)-cotunneling:



3. They should decohere the islands and improve charge quantization (i.e., the pumping accuracy) even for $E_J \leq E_c$.

Content of the talk:

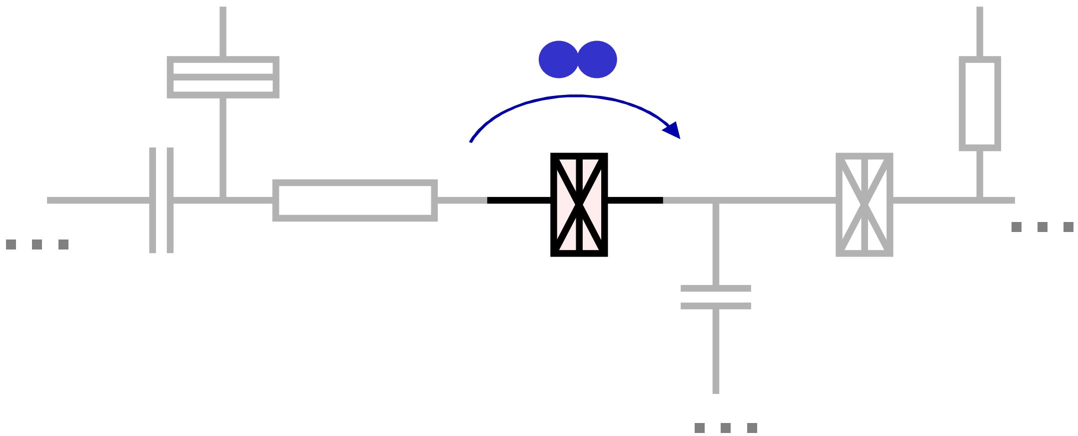
Theory	~ 60%
Experiment	~ 40%

How to describe CP cotunneling?

first, regular CP tunneling...

Tunneling of a Cooper pair across one junction

Hamiltonian: $H = H_{env} + H_{es} - E_J \cos\phi, \qquad E_J = (\hbar/2e)\, I_c$



Phase variable ϕ is conjugate variable to the transferred charge Q, i.e. $[\phi, Q] = 2ei$.

$$\cos\phi = \frac{1}{2}\left(e^{i\phi} + e^{-i\phi}\right)$$

The most straightforward procedure for finding the CP current is the perturbation theory.

Probability (rate) of single pair tunneling in positive direction:

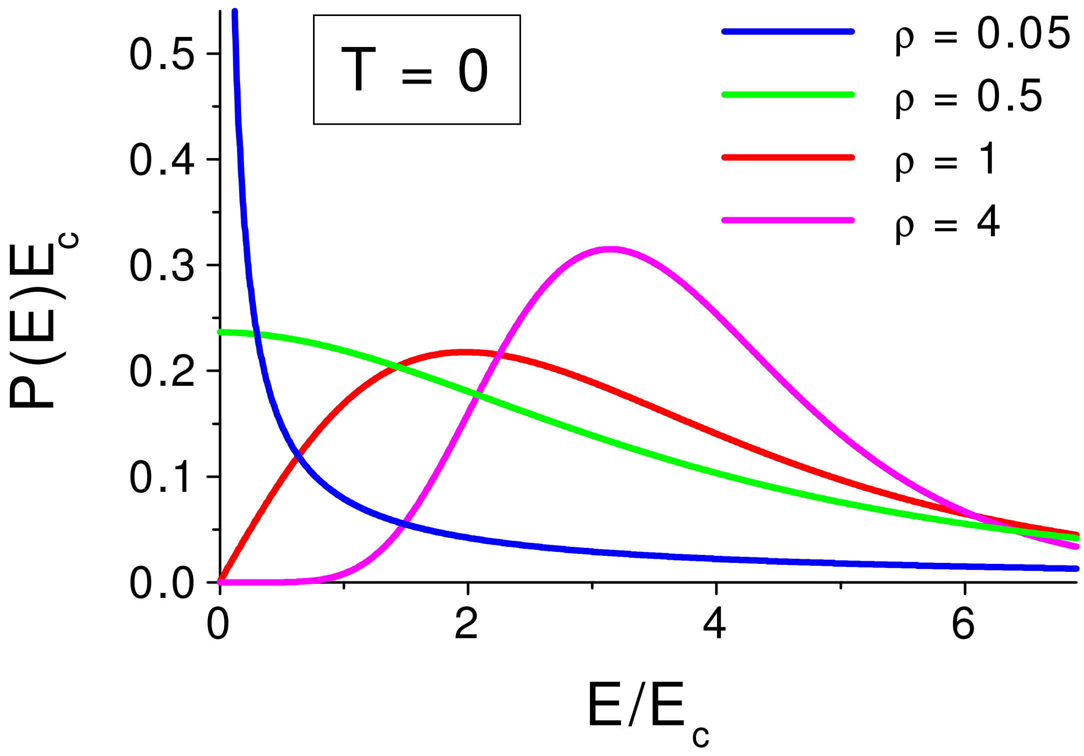
$$\Gamma \propto \left| \left\langle \mathrm{init} \left| -\frac{1}{2} E_J e^{i\phi} \right| \mathrm{fin} \right\rangle \right|^2$$ - the Golden rule.

[Averin et al. (1990); Falci et al. (1991); Ingold and Nazarov (1992)]

Regime of **incoherent** tunneling of pairs at $(E_J/E_c)(R_Q/R)^{1/2} << 1$

Rate of tunneling: $$\Gamma = \frac{\pi}{2\hbar} E_J^2 P(\Delta E)$$

energy gain

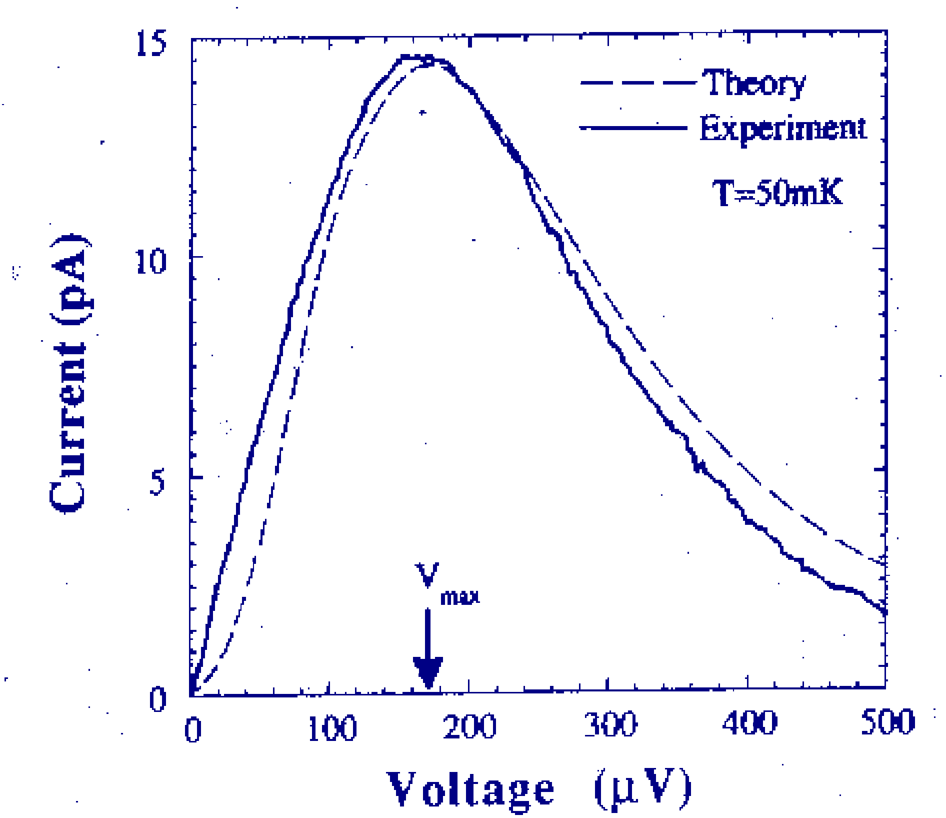


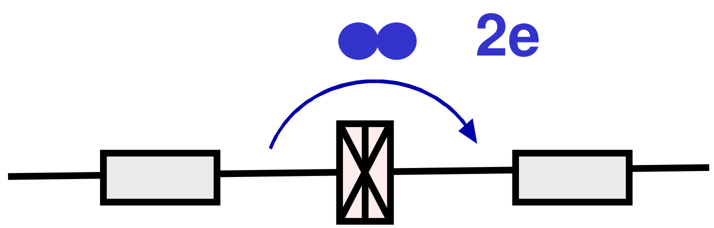
$$\rho = \frac{R}{R_Q}, \quad R_Q = \frac{h}{4e^2},$$

$$E_c = \frac{e^2}{2C}$$

Experiment with a single junction

L.S. Kuzmin et al., PRL 67, 1161 (1991).





$$\Gamma = \frac{\pi}{2\hbar} E_J^2 P(2eV)$$

I-V curve:

$$I = 2e\,[\Gamma(V) - \Gamma(-V)]$$

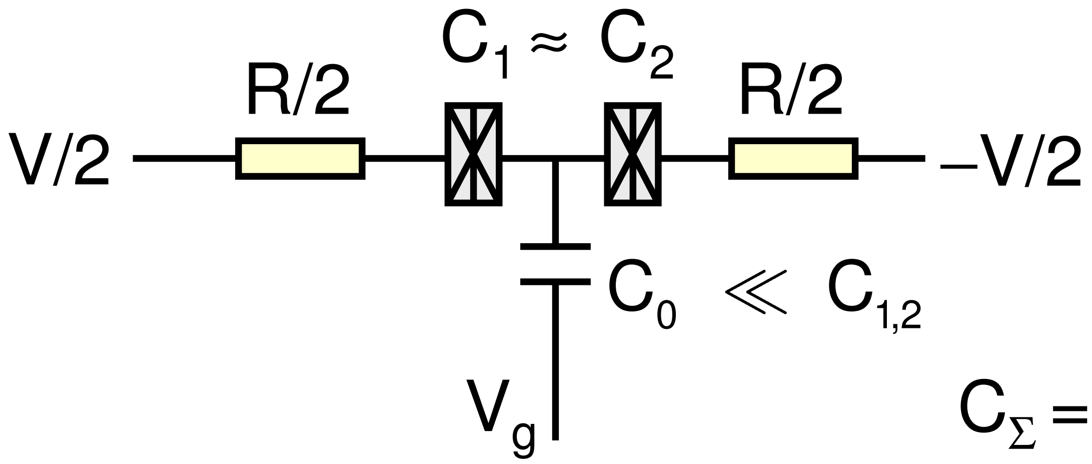
$E_J/E_c \sim 0.01$

$R = 22k\Omega$

...tunneling in 2-junctions system (transistor)?

Superconducting SET - parameters and assumptions

nominal critical currents: $I_{c1} \approx I_{c2} \approx I_{c0}$



$$C_\Sigma = C_1 + C_2 + C_0$$

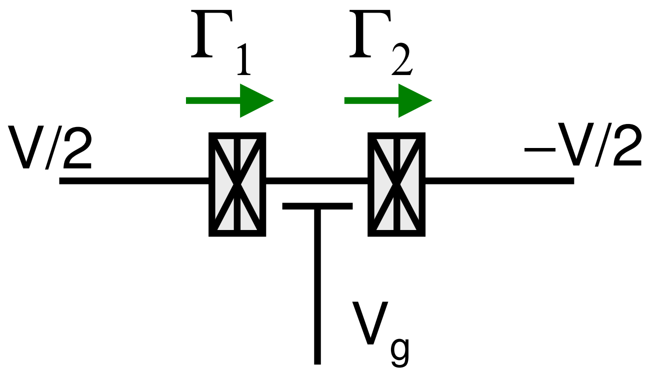
$$E_{J0} \equiv \left(\frac{\hbar}{2e}\right) I_{c0} \lesssim E_c \equiv \frac{e^2}{2C_\Sigma}$$, small parameter $\frac{E_{J0}}{8E_c}$

$$eV, k_B T \ll E_c < \Delta$$, qp tunneling suppressed

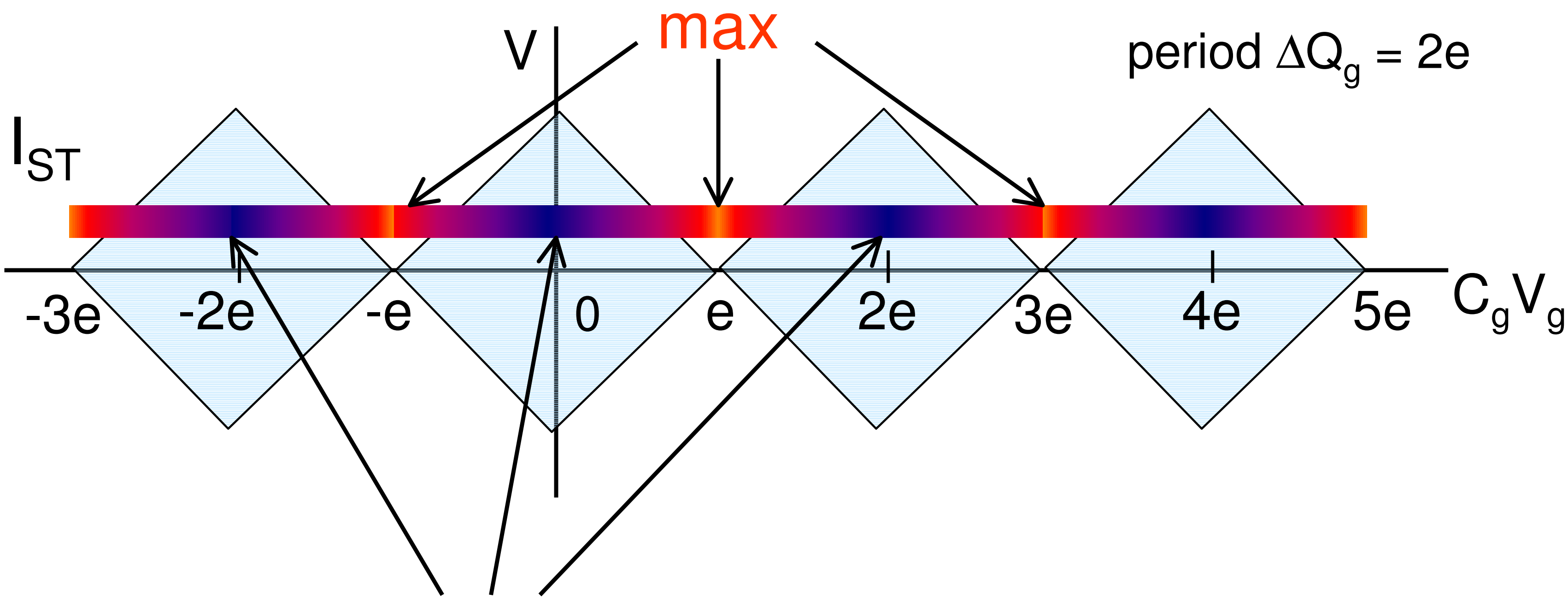
$$R \gtrsim R_Q$$

2 regimes...

Regime of sequential tunneling (ST) of pairs in superconducting SET:

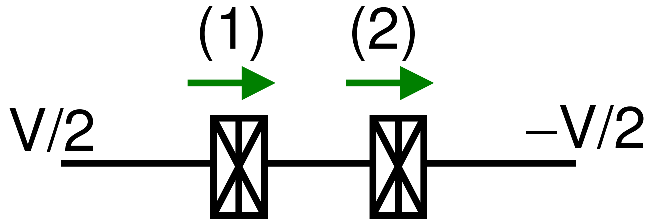


Stability diagram for Cooper pairs (T = 0):



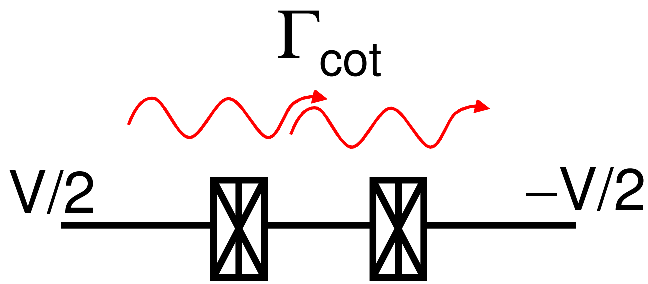
ST current is blocked, but cotunneling (across 2 junct.) possible...

Sequential tunneling:



$$\Gamma_{1,2} \propto E_J^2$$

Cotunneling:



Tunneling across both junctions
in one step and without “landing” on the island!

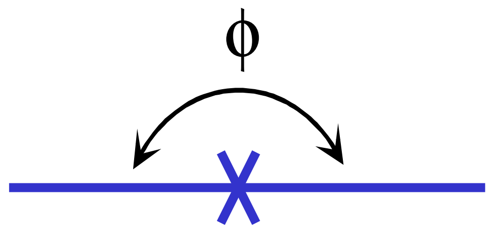
$$\Gamma_{cot} \propto E_J^4$$

Classification (according to Averin-Nazarov):

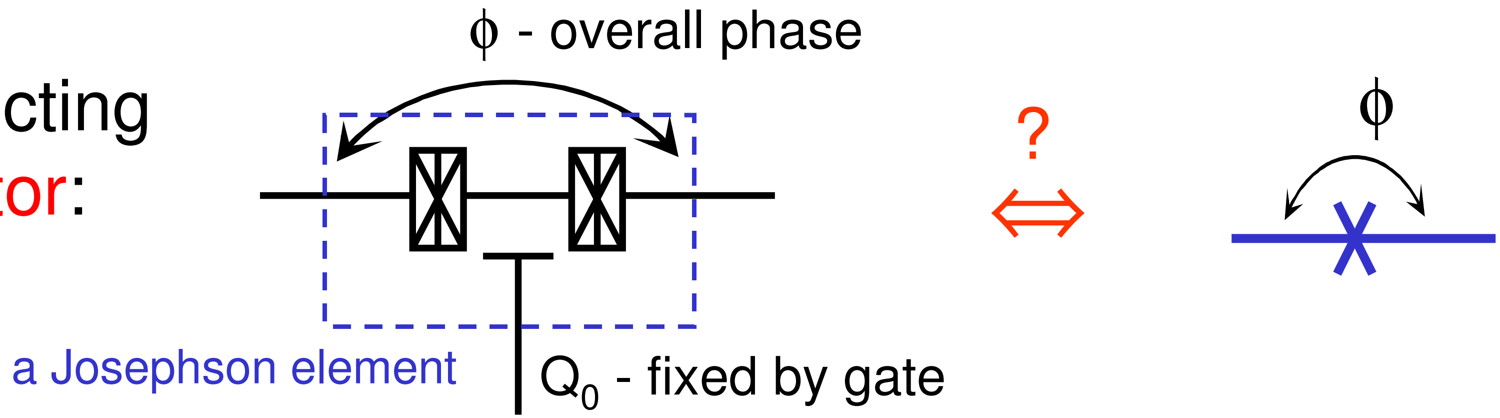
elastic cotunneling of charge (cf. normally inelastic cotunneling in SET)

What is difference between tunneling across ONE junction and TWO junctions?

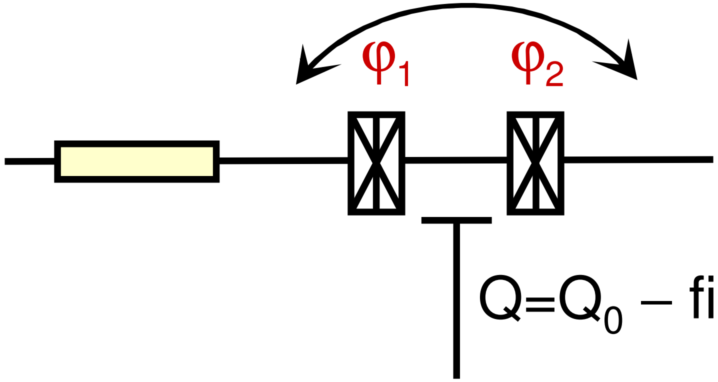
In a single junction: $E_{junc} = -E_J \cos\phi$ and $I_S = I_c \sin\phi$



In superconducting SET transistor:



ϕ - overall phase = $\varphi_1 + \varphi_2$ coupled to the bath



$Q=Q_0$ – fixed by gate, conjugate to $\varphi \equiv (\varphi_1 - \varphi_2)/2$

Variables ϕ and φ commute and at small C_0 variable φ is decoupled from the bath!

One can perform:

1. Quantum mechanics with variables φ–Q and get an effective Josephson energy of the transistor $E_{trans}(Q_0, \phi)$,
2. Perturbation theory for the outer variable ϕ (finding of Γ).

$$H = \boxed{H_{charge} + H_{Jos}} + H_{source+bath+interaction}(\phi),$$

$$H_{charge} = \frac{(Q+Q_0)^2}{e^2} E_c$$

$$H_{Jos} = -E_{J1}\cos\varphi_1 - E_{J2}\cos\varphi_2 = -(E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos\phi)^{\frac{1}{2}}\cos[\varphi + \gamma(\phi)],$$

$$\text{where} \quad \tan\gamma = \frac{E_{J1} - E_{J2}}{E_{J1} + E_{J2}}\tan\frac{\phi}{2},$$

$$[\varphi, Q] = 2ei$$

After diagonalization:

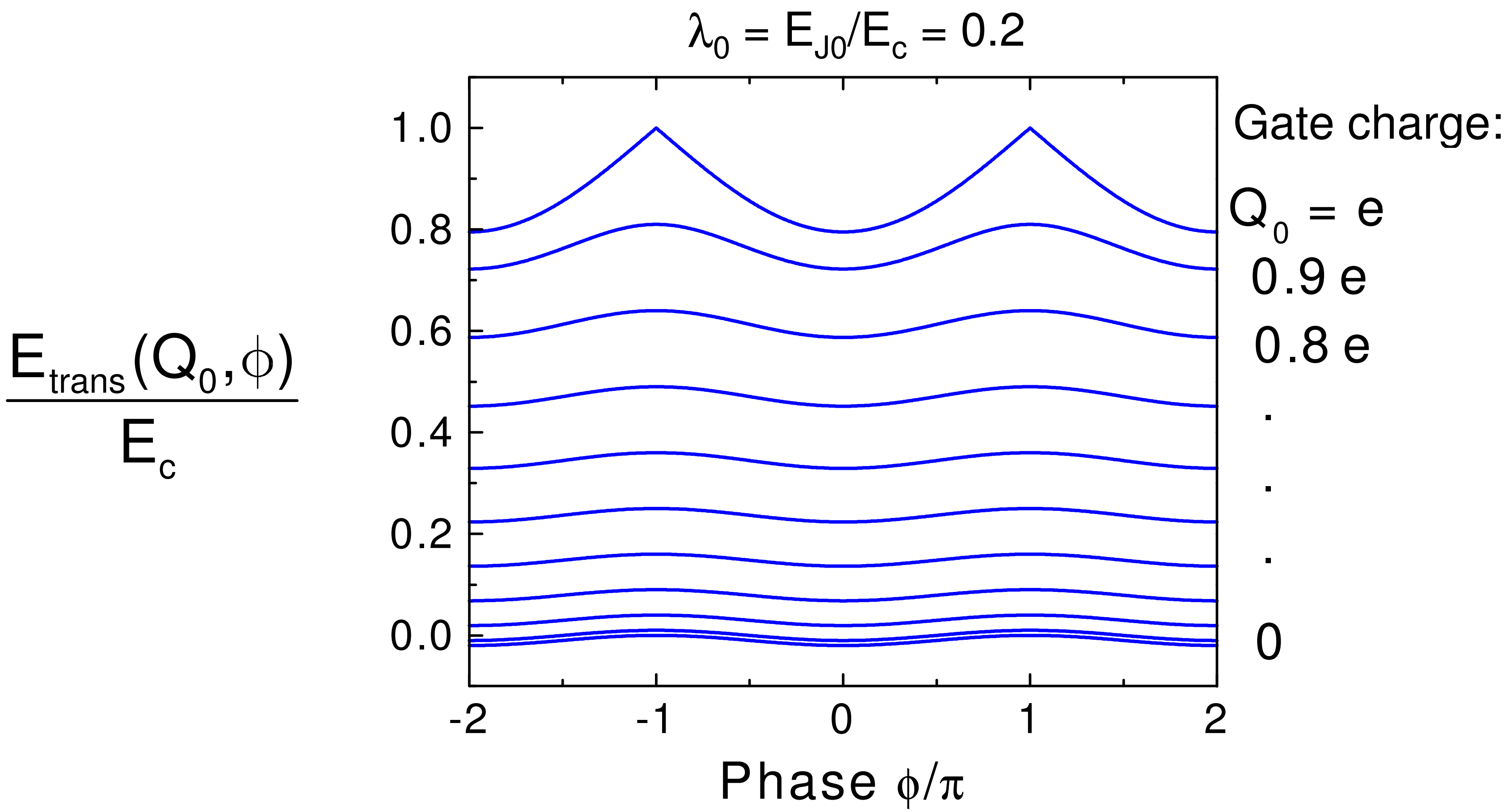
$$H_{charge} + H_{Jos} = H_{trans}(Q_0, n, \phi)$$

Bloch band index

Ground state: $H_{trans}(Q_0,0,\phi) = E_{trans}(Q_0,\phi)$

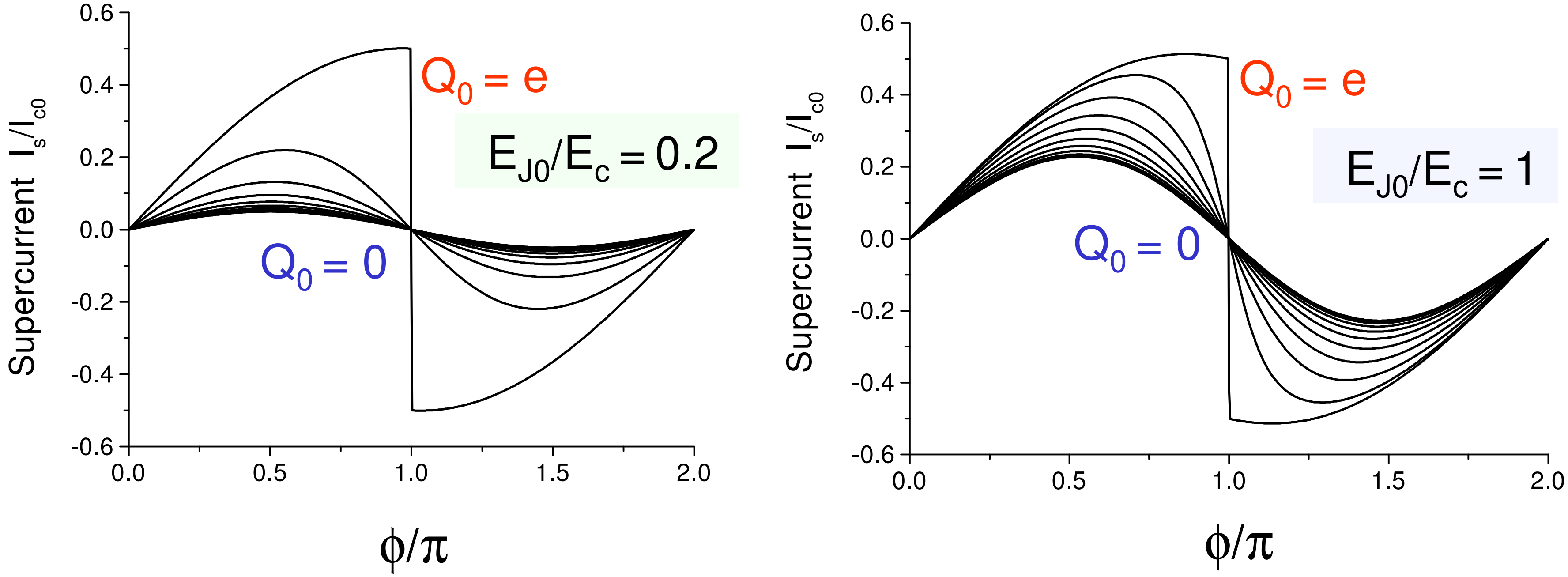
1. The phase dependence of E_{trans} is no longer harmonic,

2. The effective Josephson coupling energy depends on Q_0.



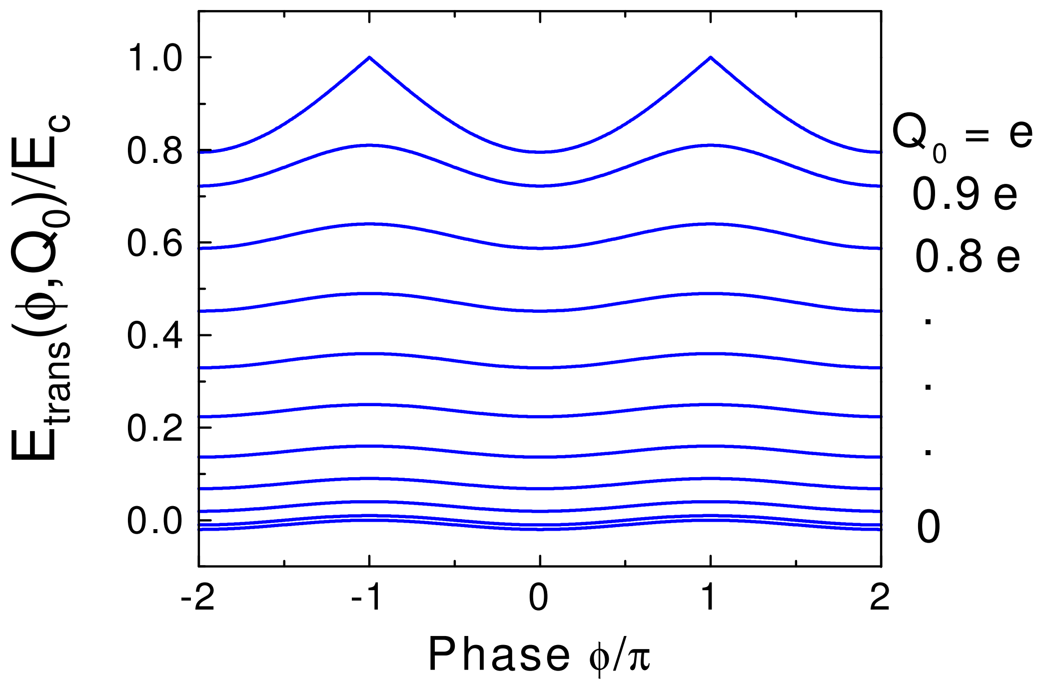
Josephson supercurrent, $I_s = \dfrac{\partial E_{trans}}{\partial \phi} \ldots$

Supercurrent across superconducting SET versus total phase ϕ



[A.Z. 1997]

$$\lambda_0 = \frac{E_{J0}}{E_c} = 0.2$$



Away from points $Q_0 = \pm e$:

$$E_{trans}(Q_0,\phi) \approx -E_J(Q_0)\cos\phi\ , \quad \text{with} \quad E_J(Q_0) = \frac{\lambda_0^2}{4}\left(\frac{\Phi_0}{2\pi}\right)E_c\left[1-\left(\frac{Q_0}{e}\right)^2\right]^{-1} \propto \lambda_0^2,$$

that is due to the wave function of the island of type:

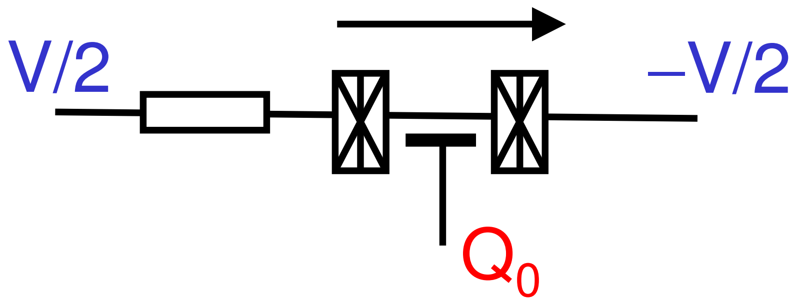
$$\Psi(Q_0) \approx e^{ik\varphi} + \frac{\lambda_0}{4(1+2k)}e^{i(k+1)\varphi} + \frac{\lambda_0}{4(1-2k)}e^{i(k-1)\varphi}, \quad k = \frac{Q_0}{2e}$$

[Likharev & A.Z., 1985]

Rate of cotunneling away from the points $Q_0 = \pm e$

(note, in experiment normally $-e/2 \le Q_0 \le e/2$):

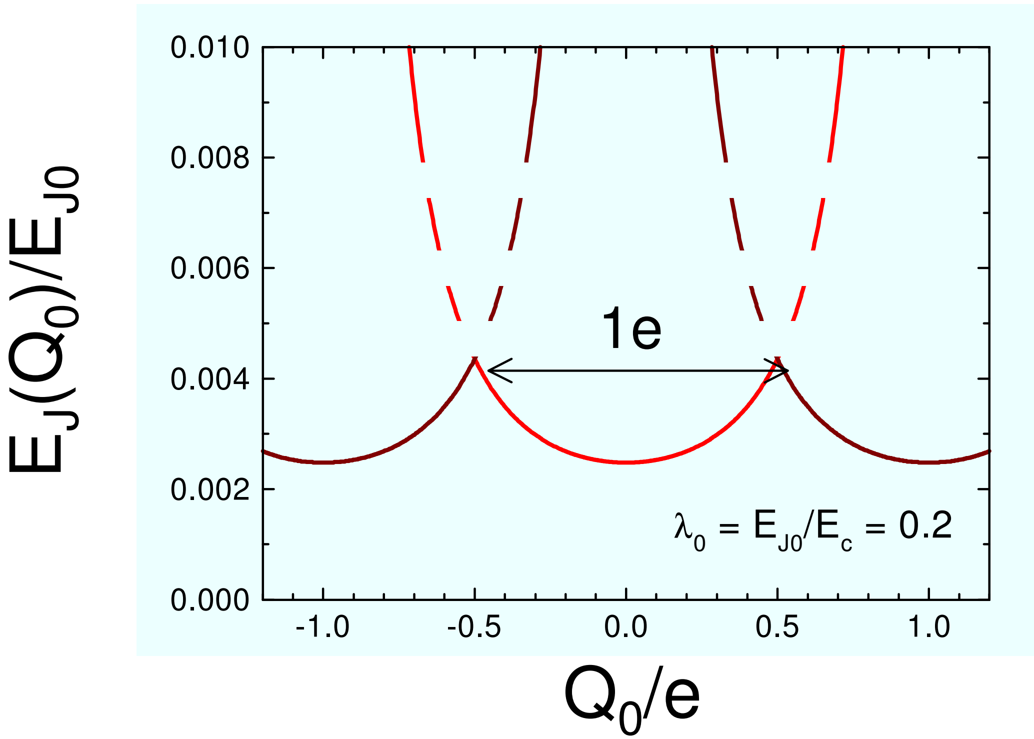
$$\Gamma_{cot}(V) = \frac{\pi}{2\hbar} E_J^2(Q_0)\, P(2eV) \propto \lambda_0^4$$



I-V curve: $I \approx I_{cot} = 2e\,[\Gamma_{cot}(V) - \Gamma_{cot}(-V)]$,

because sequential tunneling is exponentially suppressed: $I_{seq} \propto \exp(-2E_c/k_BT)$.

Infrequent qp tunneling usually returns the values of Q_0 to the interval (-e/2,e/2)



$$I \propto E_J^2(Q_0) \propto \left[1 - \left(\frac{Q_0}{e}\right)^2\right]^{-2}$$

Expected depth of the dc current modulation:

$$\frac{I_{max}}{I_{min}} = \frac{(I_{cot})_{max}}{(I_{cot})_{min}} = \frac{E_J^2(\pm e/2)}{E_J^2(0)} = \frac{16}{9} \approx 1.78$$

For <u>ohmic</u> environment, the analytic expression for function $P(E)$

is available [Ingold et al. (1994)]; using it we get:

$$I_{cot}(Q_0, V) = \frac{\pi^2}{\Phi_0} \underbrace{\frac{E_J^2(Q_0)}{E_c}}_{\text{Gate dependence}} e^{-2\gamma\rho} \rho^{2\rho} \left(\frac{2E_c}{\pi^2 k_B T} \right)^{1-2\rho} \underbrace{\frac{\left| \Gamma\left[\rho - i \frac{eV}{\pi k_B T} \right] \right|^2}{\Gamma(2\rho)} \sinh\left(\frac{eV}{k_B T} \right)}_{\text{Transport voltage dependence}}.$$

Here Γ is Gamma function, $\gamma = 0.577...$ Euler's constant

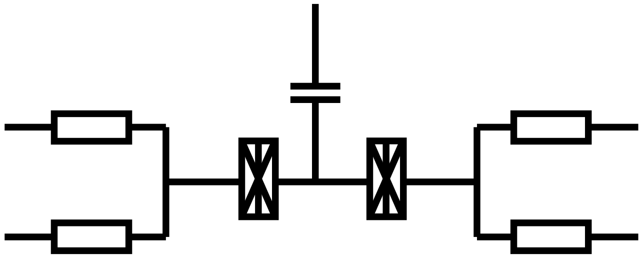
This formula valid for $I_{cot} \ll \rho I_c^{trans}$, where $\rho = R/R_Q$

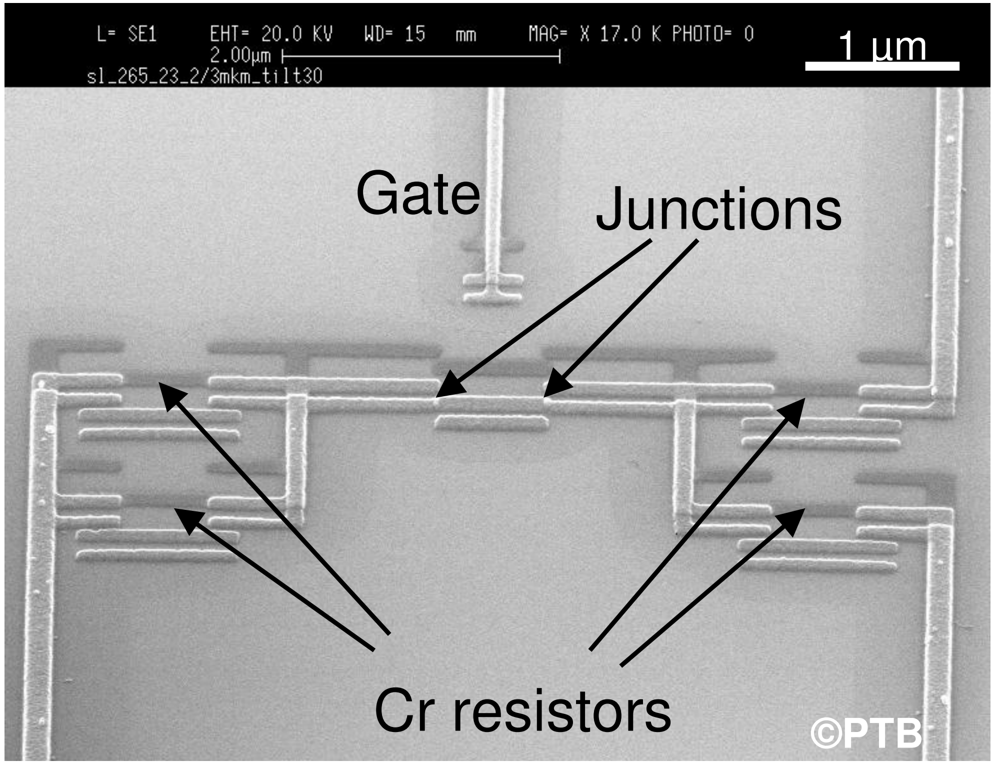
$$eV \ll E_c$$

experiment...

Sample layout

$C_g \approx 9$ aF





<u>Typical parameters</u>

Junctions: Al/AlO_x/Al; $C_\Sigma \approx 500$ aF, $E_c = e^2/2C_\Sigma \approx 150$ µeV; $\Delta_{Al} \approx 200$ µeV;

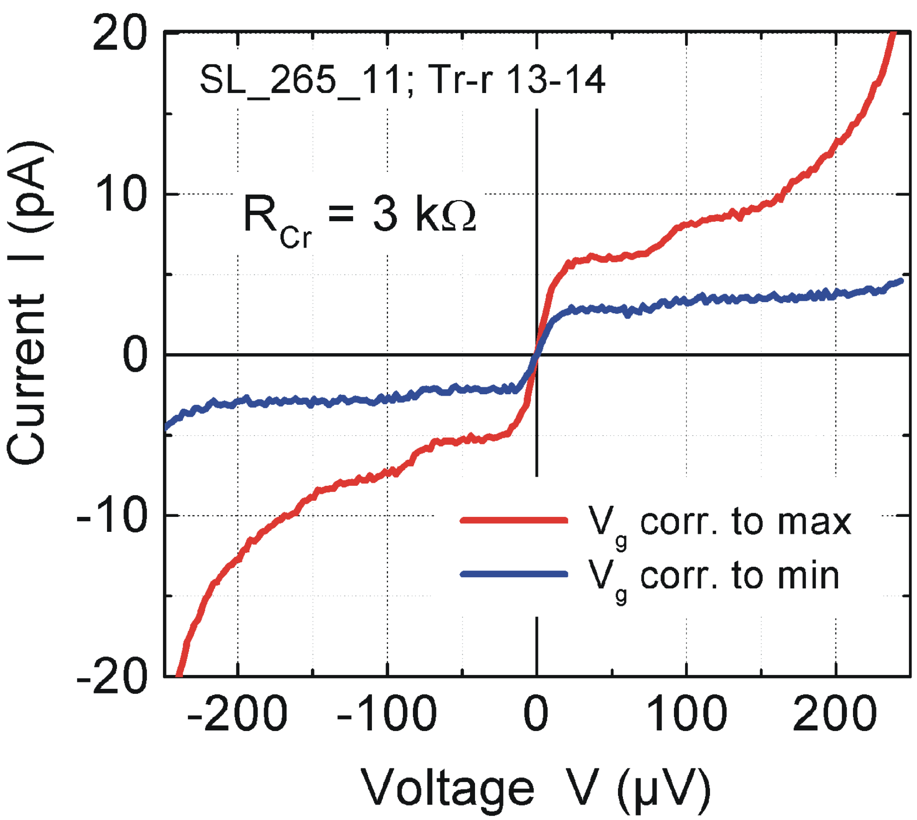
$E_{J1,J2} = E_{J0} = (\Phi_0/2\pi)I_{c0} \approx 30$ µeV; $I_{c0} \approx 16$ nA;

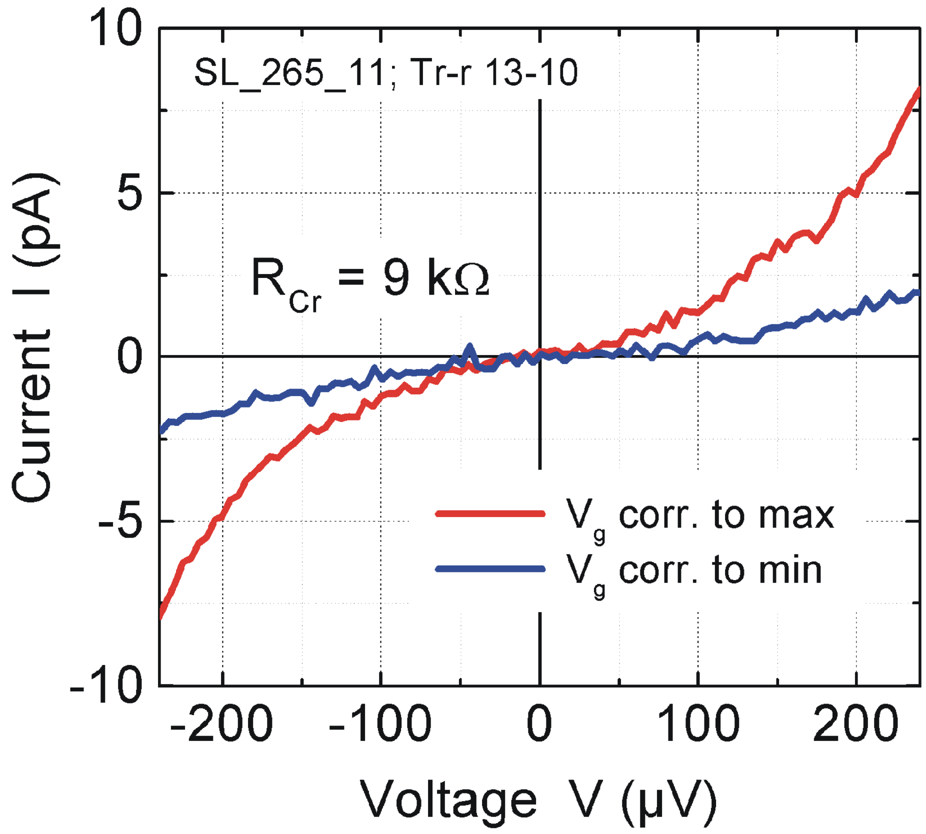
$\mathbf{E_J/E_c \approx 0.2}$

Resistors: **R = 2–20 kΩ**; w = 100 nm, h = 7 nm, l = 0.3–3 µm; material - Cr;

$R_{square} = 0.55 - 0.7$ kΩ

Effect of the transistor gate on the I-V curve

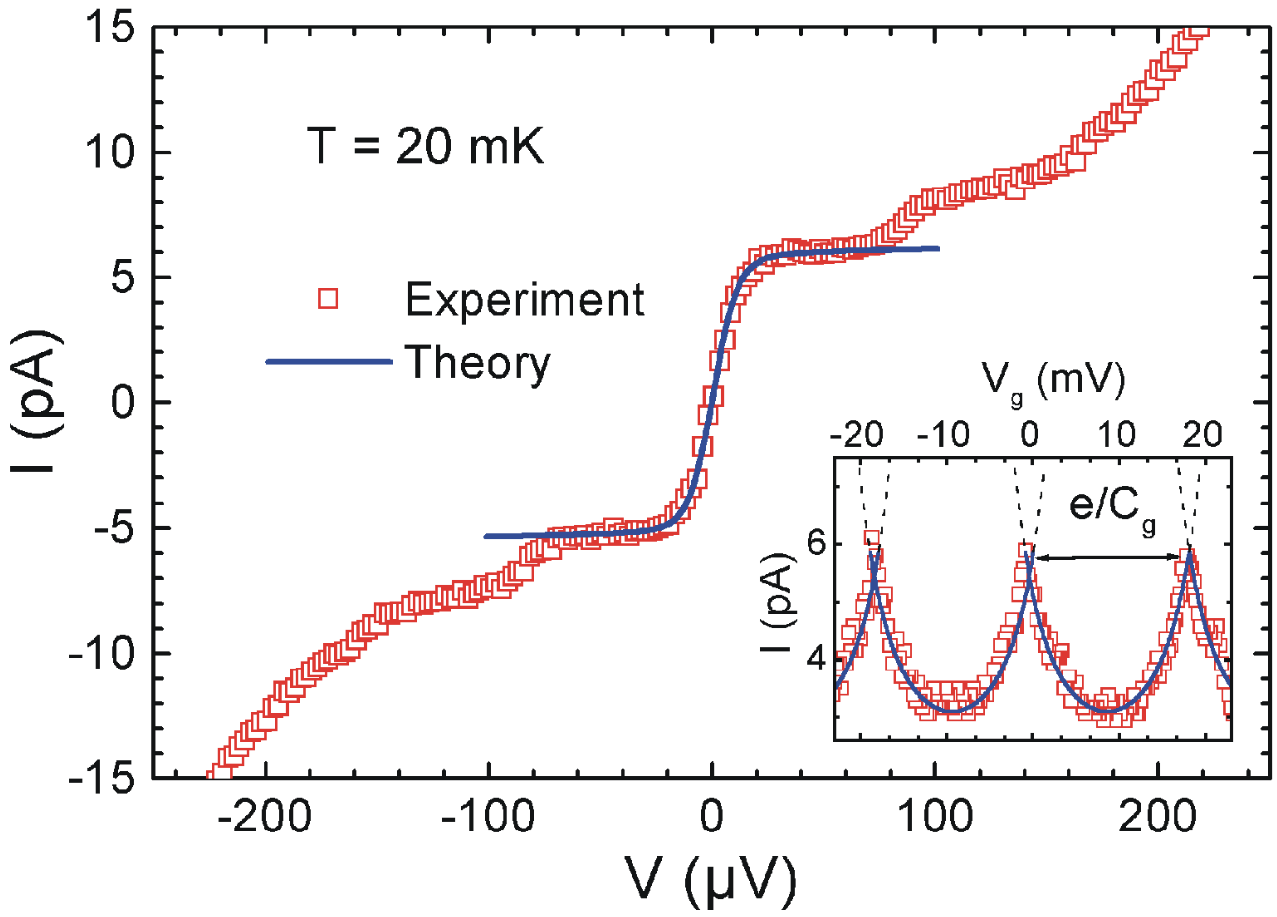




Period of modulation = 1e, i.e. qp poisoning took place!

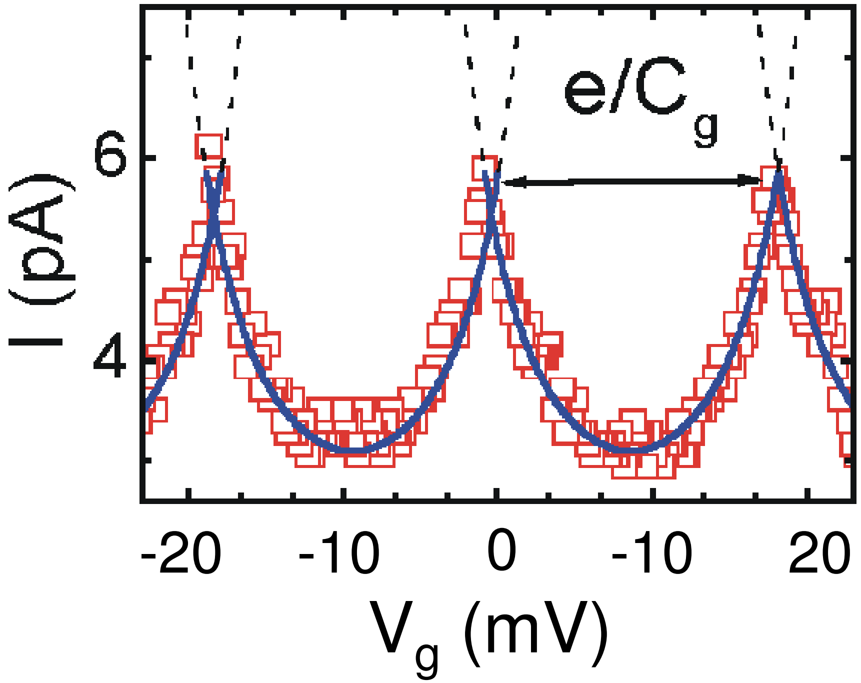
Transport voltage dependence

	Exp.	Fit
R_{Cr}[kΩ]	3	3.3
E_{J0}[μeV]	30	26
T [mK]	20	90



Gate dependence of cotunneling current

$E_J/E_c \sim 0.2$; $R = 3.3\ k\Omega$

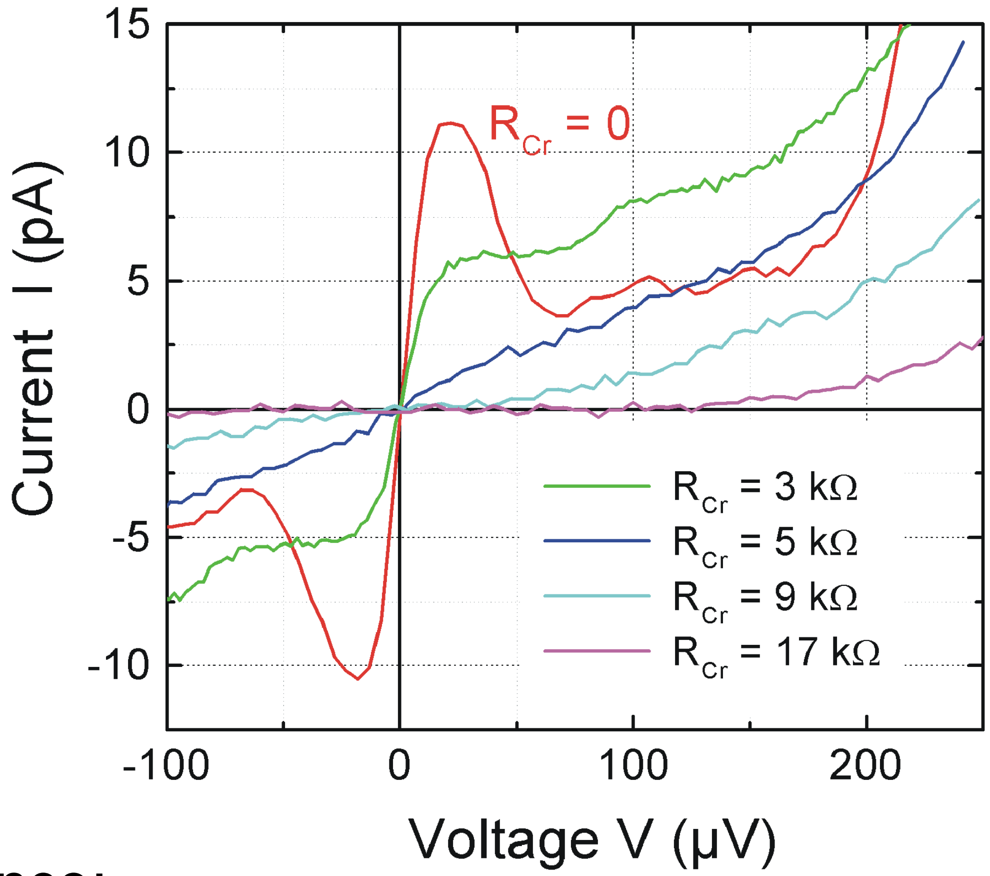


- exp. data
- theory, accessible range
- theory, inaccessible range

$I_{max}/I_{min} \sim 2$, in fairly good agreement with theory (= 1.78)

Suppression of CP current at larger values of R

Gate voltage values correspond to maximum current (presumably, $C_gV_g = Q_0 = \pm e/2$)

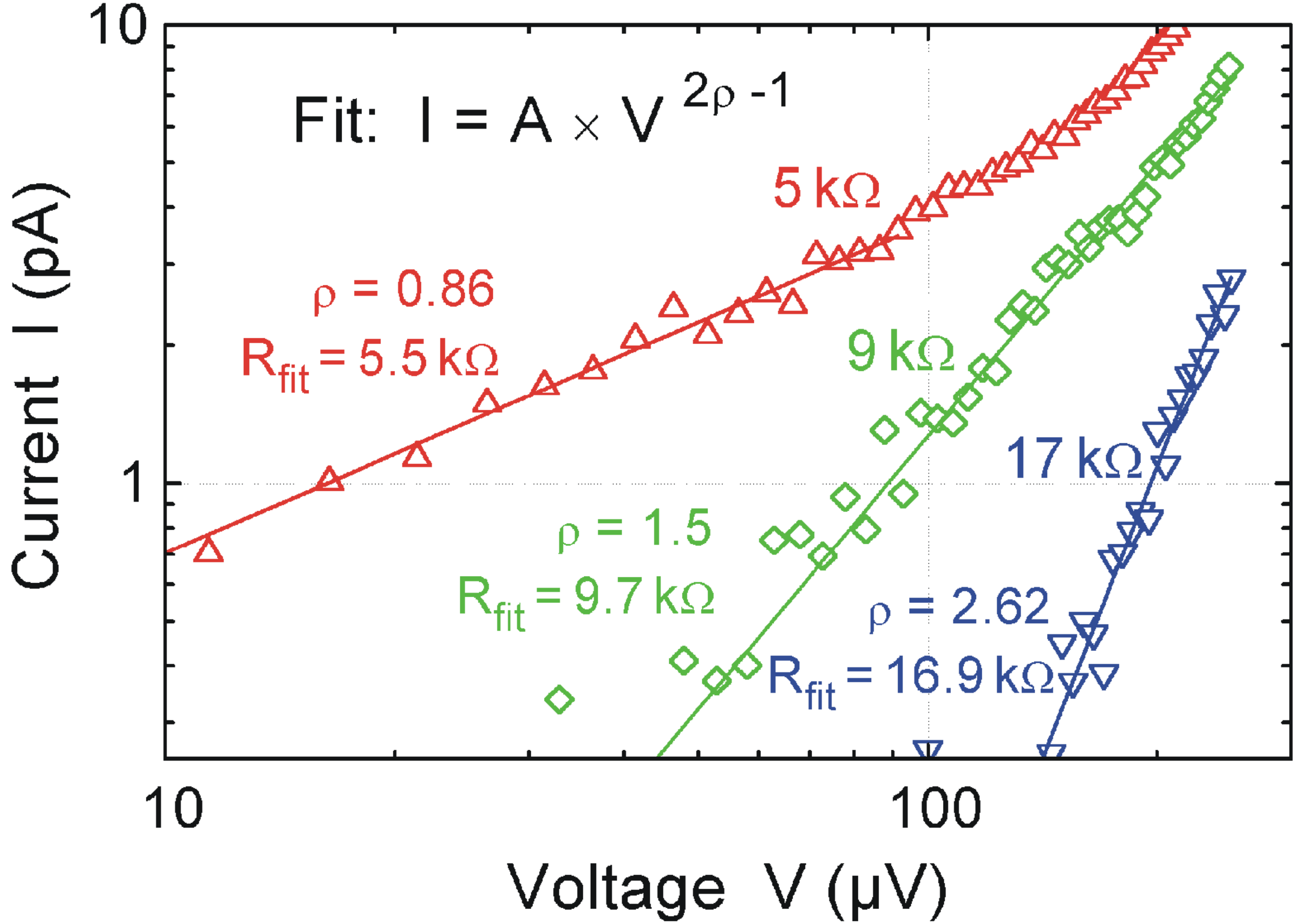


For ohmic environmental impedance:

$$I_{cot} = I_0 \left(\frac{V}{V_0} \right)^{2\rho - 1}, \quad \text{where} \quad I_0 = \frac{\pi\rho\Phi_0 e^{-2\gamma}(I_c^0)^2}{32\Gamma(2\rho)E_c} \quad \text{and} \quad V_0 = \frac{2e}{\pi\rho C}.$$

$\rho = R/R_Q, \quad T = 0.$

Comparison with experimental data

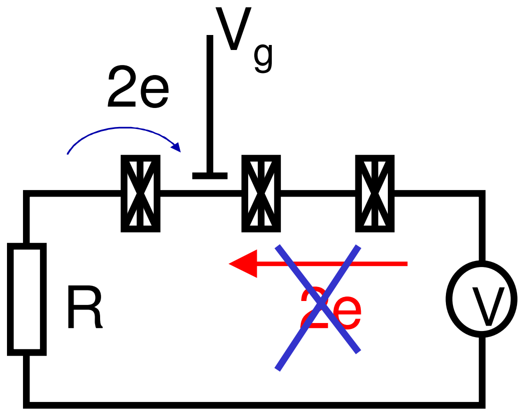


Possible application of the effect of suppression of Cooper pair cotunneling

Cooper pair pump

effective damping

$$\rho = \left(\frac{1}{3}\right)^2 \frac{R}{R_Q} \approx 0.11 \frac{R}{R_Q}$$



effective damping

$$\rho = \left(\frac{2}{3}\right)^2 \frac{R}{R_Q} \approx 0.44 \frac{R}{R_Q}$$

$R \geq 4 \div 5\ R_Q$ should efficiently suppress CP cotunneling.

Conclusion

1. Cotunneling of CP in superconducting SET can be described by the model of “effective Josephson element”.

2. Cotunneling in CP pump can be significantly suppressed by a local resistor.

Reference: S.V. Lotkhov *et al*. PRL 91, 197002 (3 Nov. 2003)