Spin-wave radiation from a superfluid ^{3}He Josephson junction

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http://boojum.hut.fi/research/theory/jos2.html

Outline

- superfluid ^{3}He
- Leggett equations
- spin-wave radiation
- explanation of experiments
- conclusion

Superfluid ^{3}He: triplet pairing

- spin state of $S = 1$ Cooper pairs ($S_z = 1, -1, 0$)

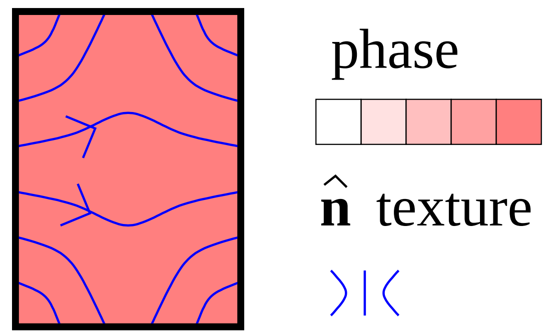
$$|\boldsymbol{d}\rangle = (-d_x + \mathrm{i}d_y)|\uparrow\uparrow\rangle + (d_x + \mathrm{i}d_y)|\downarrow\downarrow\rangle + d_z(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

- "order parameter" $\boldsymbol{d}(\hat{\boldsymbol{k}})$ is a vector
- p wave orbital state
  - B phase

$$\boldsymbol{d}(\hat{\boldsymbol{k}}) = e^{\mathrm{i}\phi} R(\hat{\boldsymbol{n}}, \theta_L)\hat{\boldsymbol{k}}$$

where $\theta_L = 104°$

- $\hat{\boldsymbol{n}}$ forms textures



Dynamics of $\boldsymbol{d}$: Leggett equations

$$F = \int \left[\frac{\gamma^2 \boldsymbol{S}^2}{2\chi} - \gamma \boldsymbol{H} \cdot \boldsymbol{S} \right] \mathrm{d}^3 r + F_{dip}[\boldsymbol{d}] + F_{grad}[\boldsymbol{d}] + F_{surf}[\boldsymbol{d}] + \cdots$$

$[S_i, d_j(\hat{\boldsymbol{k}})] = \mathrm{i}\hbar \epsilon_{ijk} d_k(\hat{\boldsymbol{k}})$ $\Rightarrow$

- equation for $\boldsymbol{d}$:
  $\dot{\boldsymbol{d}} = \gamma \boldsymbol{d} \times (\boldsymbol{H} - \gamma \boldsymbol{S}/\chi)$
- equation for $\boldsymbol{S}$:
  $\dot{\boldsymbol{S}} = \gamma \boldsymbol{S} \times \boldsymbol{H} + \boldsymbol{R}$
  torque $\boldsymbol{R} = -\langle \boldsymbol{d} \times (\delta F / \delta \boldsymbol{d}) \rangle_{\hat{\boldsymbol{k}}}$

"Internal Josephson effect"

- $\Delta \boldsymbol{\mu} = -\hbar(\partial F / \partial \boldsymbol{S}) = -\hbar \gamma (\boldsymbol{H} - \gamma \boldsymbol{S}/\chi)$

In NMR dynamics driven with $\boldsymbol{H}(t)$

- what if $\boldsymbol{H}(t) \equiv 0$?

Josephson coupling via a constriction

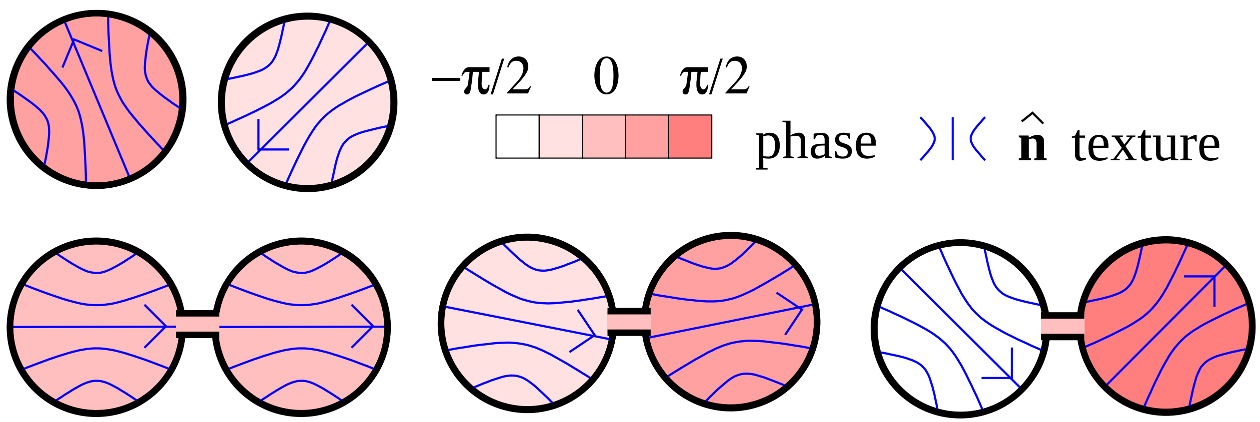
- coupling through a region $\sim \xi_0$ where Δ is suppressed
- phase and spin-orbit degrees of freedom coupled: "anisotextural effect"

Coupling energy:

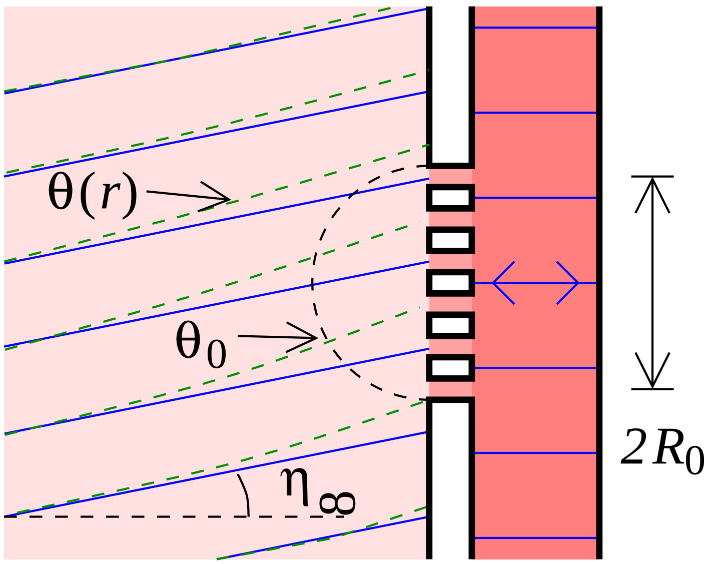
$$F_J \approx -E_J(\psi_{ij})\cos(\phi)$$

$$\phi = \phi^r - \phi^l$$

$$\psi_{ij} = R^l_{\mu i} R^r_{\mu j}$$



Textural dynamics: spin waves



Pressure-biased junction
and texture in half-space;
$\boldsymbol{H} = 0, \ \omega_J = (2m/\rho\hbar)P$

$$F[\theta] \approx -(E_J^\infty - A\theta_0)\cos(\phi) + \frac{K}{2}\int d^3r|\boldsymbol{\nabla}\theta|^2$$

Leggett equations ⇒

- $\ddot{\theta} = c^2\nabla^2\theta$, $c = \sqrt{\gamma^2 K/\chi}$
- $2\pi R_c^2 K\theta'(R_0) = I_{sp}(\phi)$

  $I_{sp}(\phi) = \partial F_J/\partial\theta_0 = A(\eta_\infty)\cos(\phi)$

$\phi(t) = \omega_J t$ ⇒ I_{sp} drives oscillations of $\boldsymbol{S}$, which radiate out as *spin waves*:

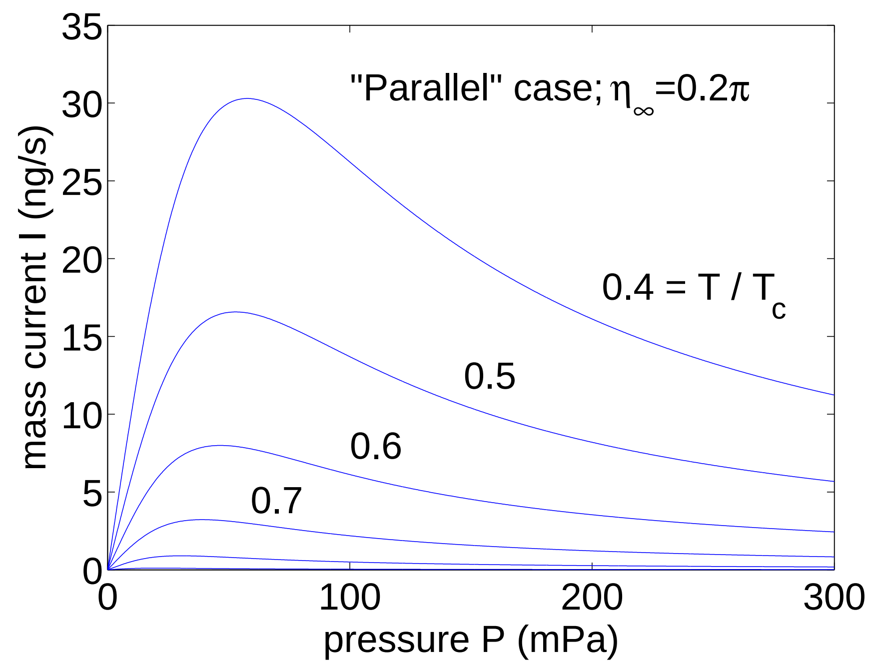
- $\theta(r,t) = \mathrm{Re}[\frac{C}{r}e^{+i(kr-\omega_J t)}], \ \ k = \omega_J/c$

DC current from anisotextural coupling

- time average of supercurrent
  $I_S(\phi) = (2m/\hbar)(\partial F_J/\partial\phi)$

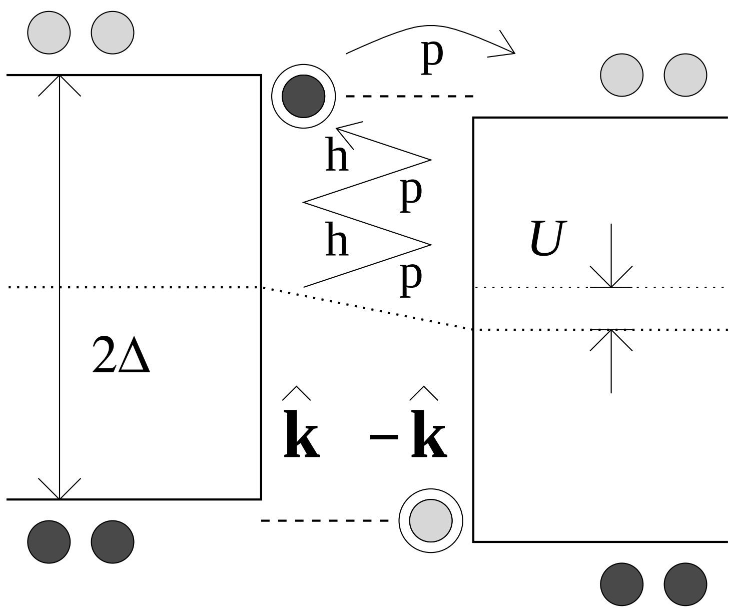
$$I_{s,ave}(P) = \frac{2m}{\hbar}\frac{[A(\eta_\infty)]^2}{2\pi K R_0}\frac{\omega_J(P)\tau}{1+(\omega_J(P)\tau)^2}$$

- $\tau = R_0/c = R_0/\sqrt{\gamma^2 K/\chi}$
- DC current depends on textural configuration through $A(\eta_\infty)$



DC current from MAR

- Multiple Andreev Reflections



- consider relaxation-limited case

  $U = (m/\rho)P \ll \Gamma_0$ = relaxation rate

  $I_0(P) = G_n(\Delta/\Gamma_0)g(T)P$

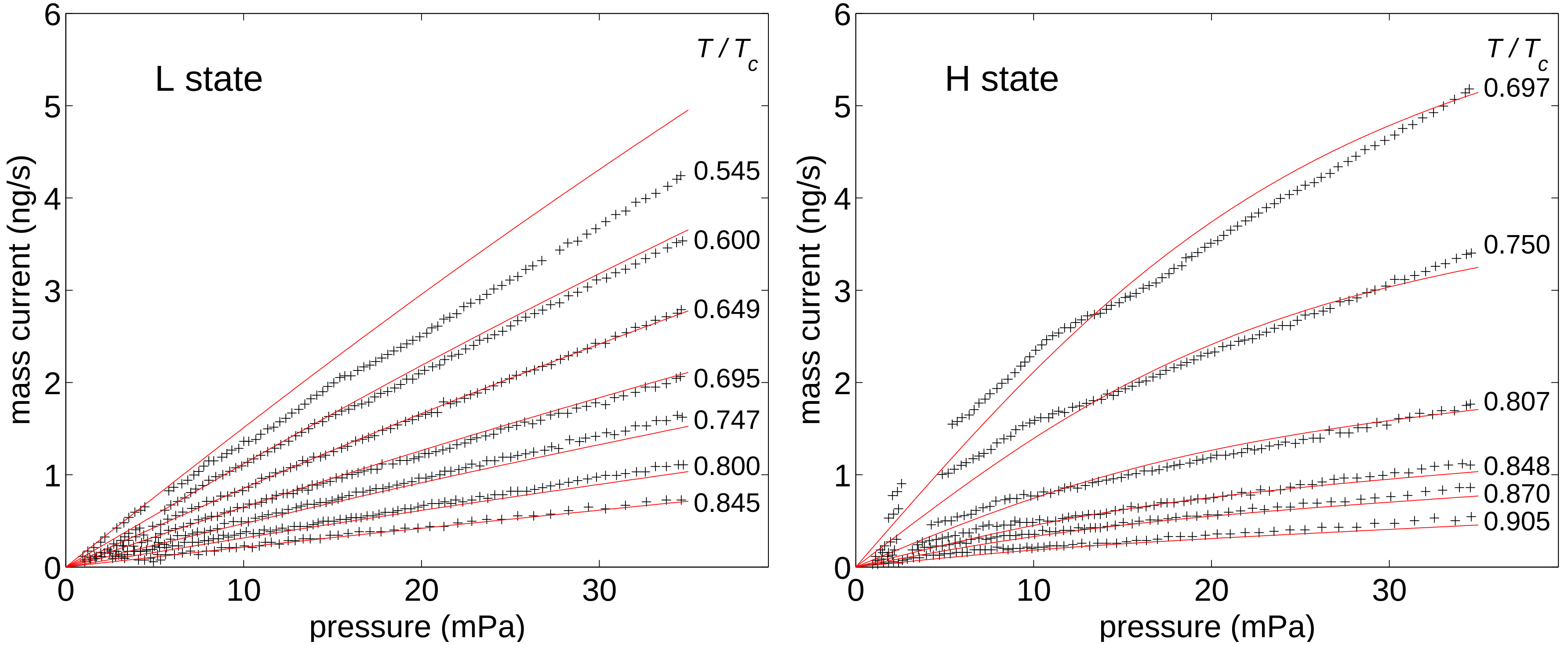
  - linear also with gap suppression
  - independent of texture

Total DC current:

$$I_{dc}(P) = I_0(P) + I_{s,ave}(P)$$

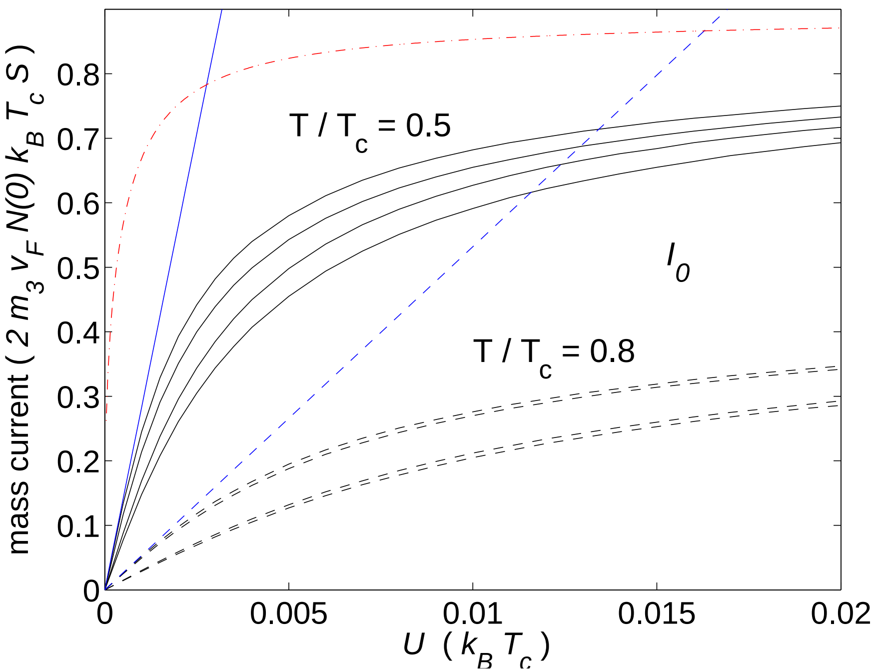
Comparison to experiment

- fit of $I_{dc} = I_0 + I_{s,ave}$ to Berkeley data [PRL **84**,6062 (2000)]



Full calculation of MAR in point contact

$$I(P,t) = I_0(P) + \sum_{n=1}^{\infty}[I_n^c(P)\cos(n\omega_J t) + I_n^s(P)\sin(n\omega_J t)]$$

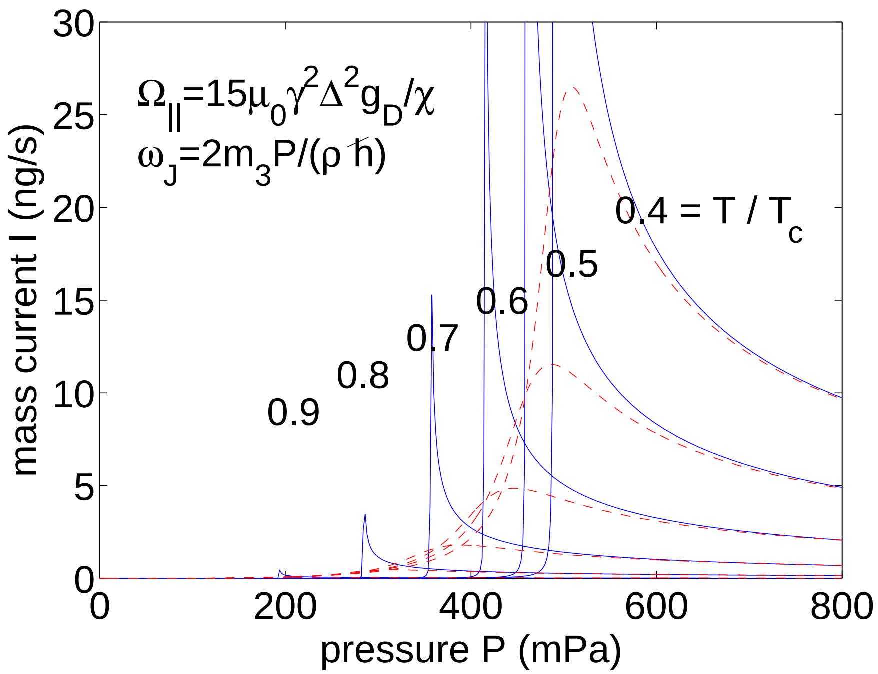


Prediction: resonance with longitudinal mode

- example: $-\hat{\boldsymbol{n}}^l = \hat{\boldsymbol{n}}^r = \hat{\boldsymbol{z}}$
- spin-orbit angle distorted from $\theta_L \approx 104^\circ$
- DC current

$$I_{s,ave}(P) \propto \mathrm{Im}\left[1/\sqrt{1-(\omega_J/\Omega_{||})^2}\right]$$

- resonance at $\omega_J = \Omega_{||}$
- also at $\omega_J = \Omega_{||}/n$?



Summary

- MAR not sufficient to explain experiments of dissipative currents in array-type junctions
- spin dynamics may be driven with a Josephson junction
- spin-wave radiation from the junction explains the additional, texture-dependent dissipation
- the following should be measurable:
  - oscillating spin polarization (with a pickup coil)
  - longitudinal resonance (with biases < 1 Pa)

Mass and spin currents: hydrostatics

- in bulk regions Δ = constant

mass current

$$\boldsymbol{j}_s = \rho_s \boldsymbol{v}_s$$

$$\boldsymbol{v}_s = (\hbar/2m)\boldsymbol{\nabla}\phi$$

spin current

$$j^{sp}_{\alpha i} = \rho^{sp}_{\alpha\beta,ij} v^{sp}_{\beta j}$$

$$\boldsymbol{v}^{sp}_{\alpha} = -(\hbar/4m)\epsilon_{\alpha\beta\gamma} R_{\beta j} \boldsymbol{\nabla} R_{\gamma j}$$

- gradient energy

$$F_{grad} = \frac{1}{2}\int \mathrm{d}^3 r [\rho_s \boldsymbol{v}_s^2 + \rho^{sp}_{\alpha\beta,ij} v^{sp}_{\alpha i} v^{sp}_{\beta j}]$$

- phase and spin-orbit degrees of freedom decoupled