# Flow of He II due to an Oscillating Grid in the Low Temperature Limit


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**Lammi, January 7-11, 2004**

# Outlook

**Introduction** background, motivation

**Experiment** experimental setup

experimental results

**Current theoretical understanding**

**Summary**


- H.A.Nichol, L. Skrbek, P.C. Hendry, P.V.E. McClintock
Flow of He II due to an Oscillating Grid in the Low Temperature Limit, submitted to PRL

- H.A.Nichol, L. Skrbek, P.C. Hendry, P.V.E. McClintock
Experimental Investigation of the Macroscopic Flow of He II due to an Oscillating Grid in the Zero Temperature Limit, to be submitted to JLTP

- W.F. Vinen, L. Skrbek, H.A. Nichol
Nucleation of Superfluid Turbulence at very Low Temperatures by Flow through a Grid
submitted to Olli Lounasmaa Memorial issue of JLTP

# Cryogenic helium flow and turbulence

$^{4}$He

**Cryogenic He gas, He I**

**Classical turbulence:**
High Ra convection (Eugene, Grenoble, Trieste…)
High Re pipe flow (Eugene…)
Taylor – Green flow (Paris..)
Turbulent jet (CERN)

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**He II T>1K**

**Coflow** (Eugene, Paris, Bariloche, Tallahassee….)
**Counterflow** (……., Prague…)
**Anything in between** (e.g., Leiden…)
Two-fluid hydrodynamics, difficult to interpret

**He II low T**

**Pure superflow** (Lancaster, Regensburg, Paris, Osaka…)

**In the temperature range where He II contains an appreciable amount of the normal fluid, say above about 1.2 K, on increasing a suitably defined Re He II flow increasingly acquires classical character**

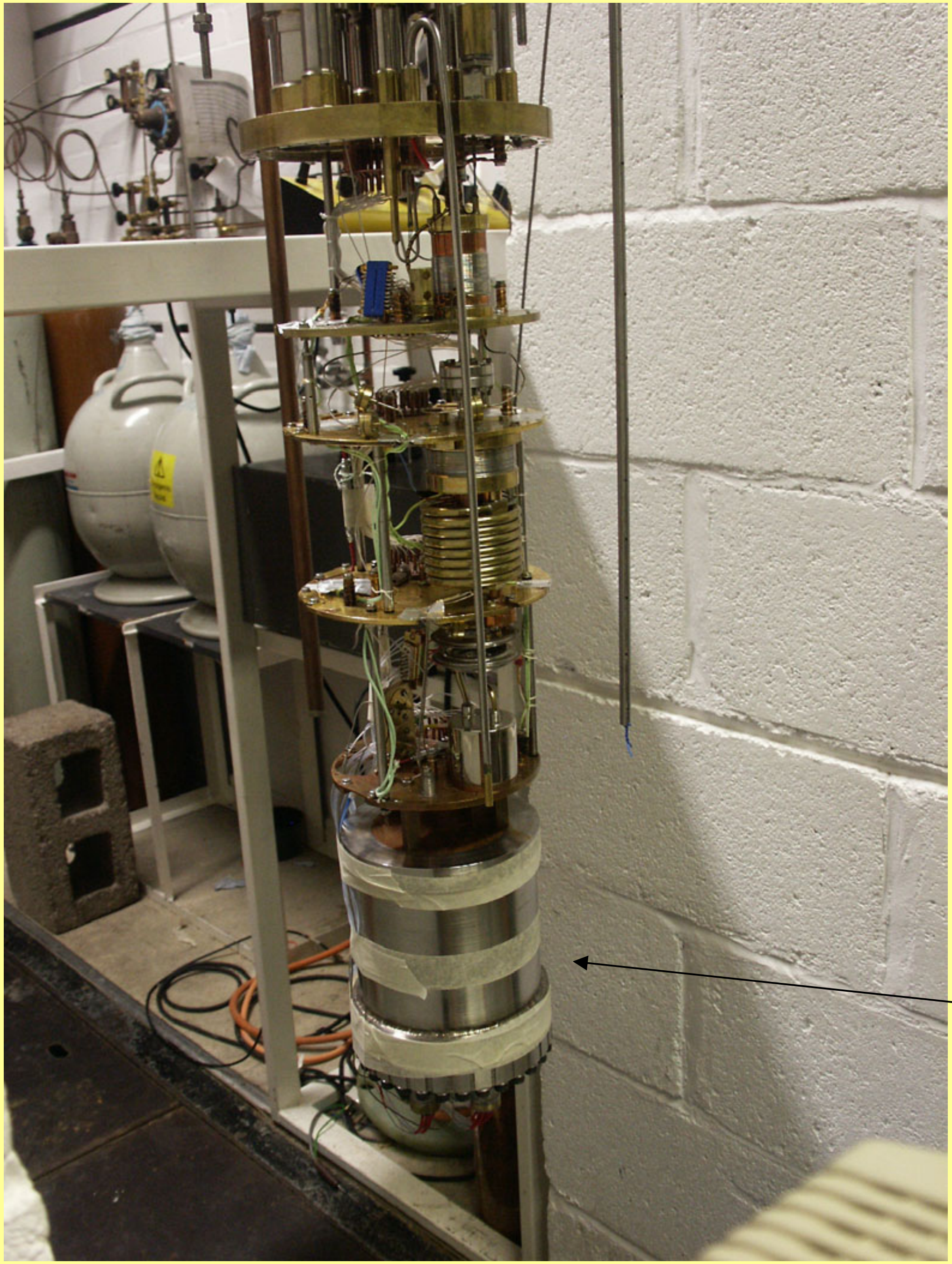
## Classical features of He II flow

- the surface of a rotating bucket forms nearly classical meniscus
- flow past the sphere displays both laminar and turbulent drag
- as well as the drag crisis
- pressure drop in high Re pipe flow hardly differs from classical
- energy spectrum of a turbulent He II involves an inertial range with classical K41 Kolmogorov roll-off exponent -5/3
- the decay of turbulence generated both by towing the grid through a stationary sample and in the counterflow displays classical character

Although these features are typically observed in a rather wide temperature range, it is generally not possible to exclude that this classical-like behaviour is caused by the presence of the viscous normal fluid.

**Clear call: study the macroscopic properties of the He II flow in a zero temperature limit !**

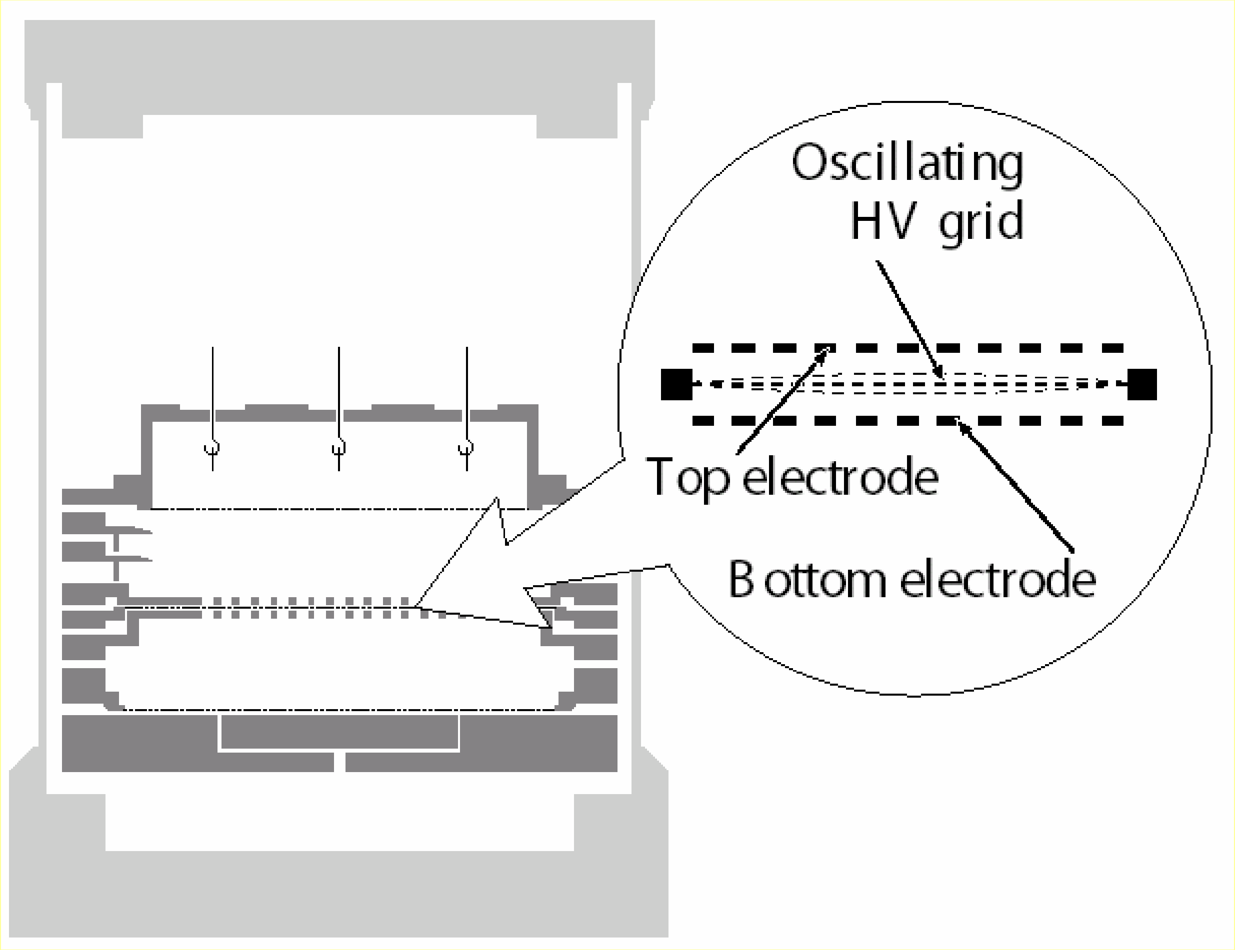
**( even more challenging task in view of experiments with a tiny sphere and a very thin vibrating wire displaying intriguing features, attributable to single vortices….)**



# Oxford Instruments dilution refrigerator

Experimental cell

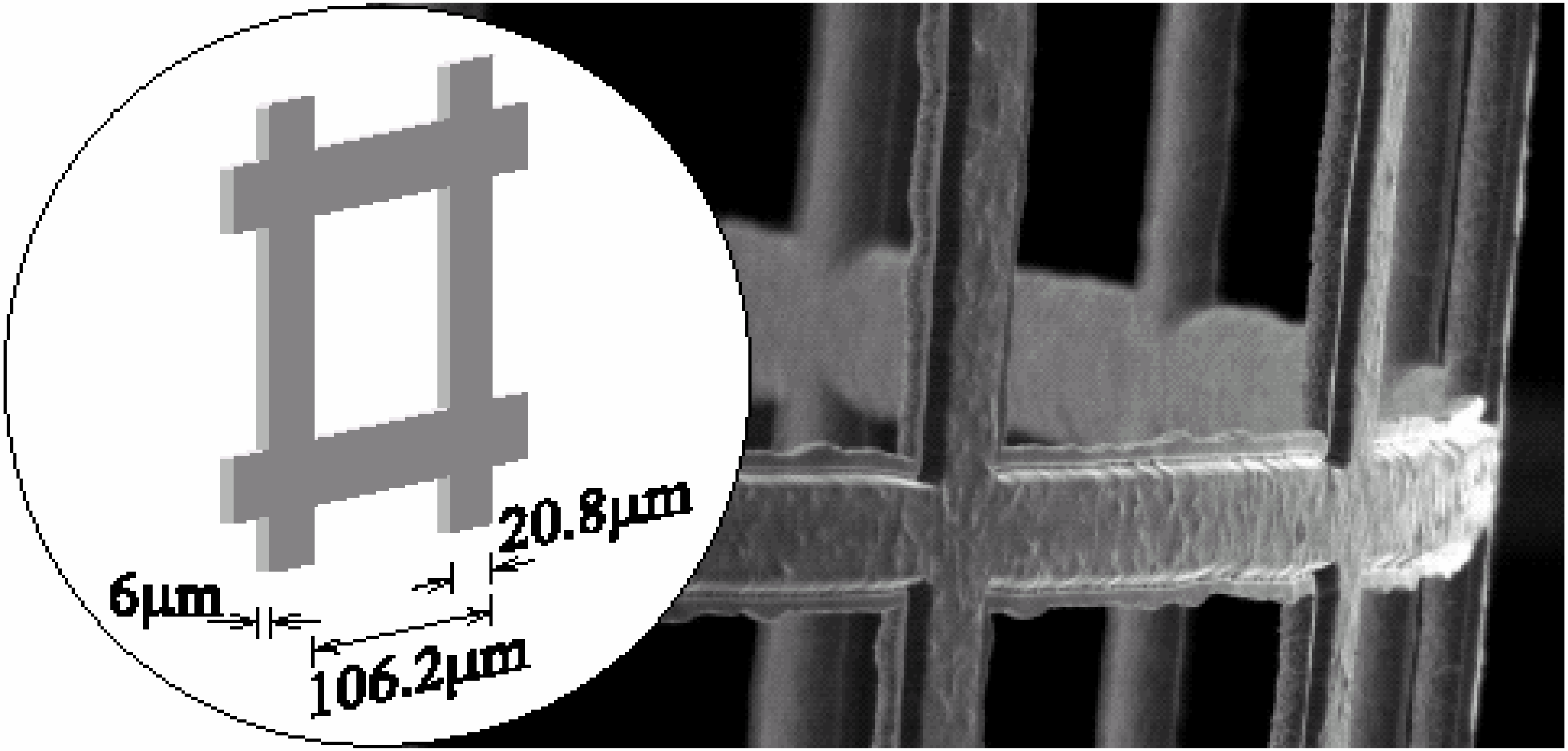
# Experimental cell containing 1.5 litre of spectrally pure He-4, attached to the dilution refrigerator (base T about 10 mK)

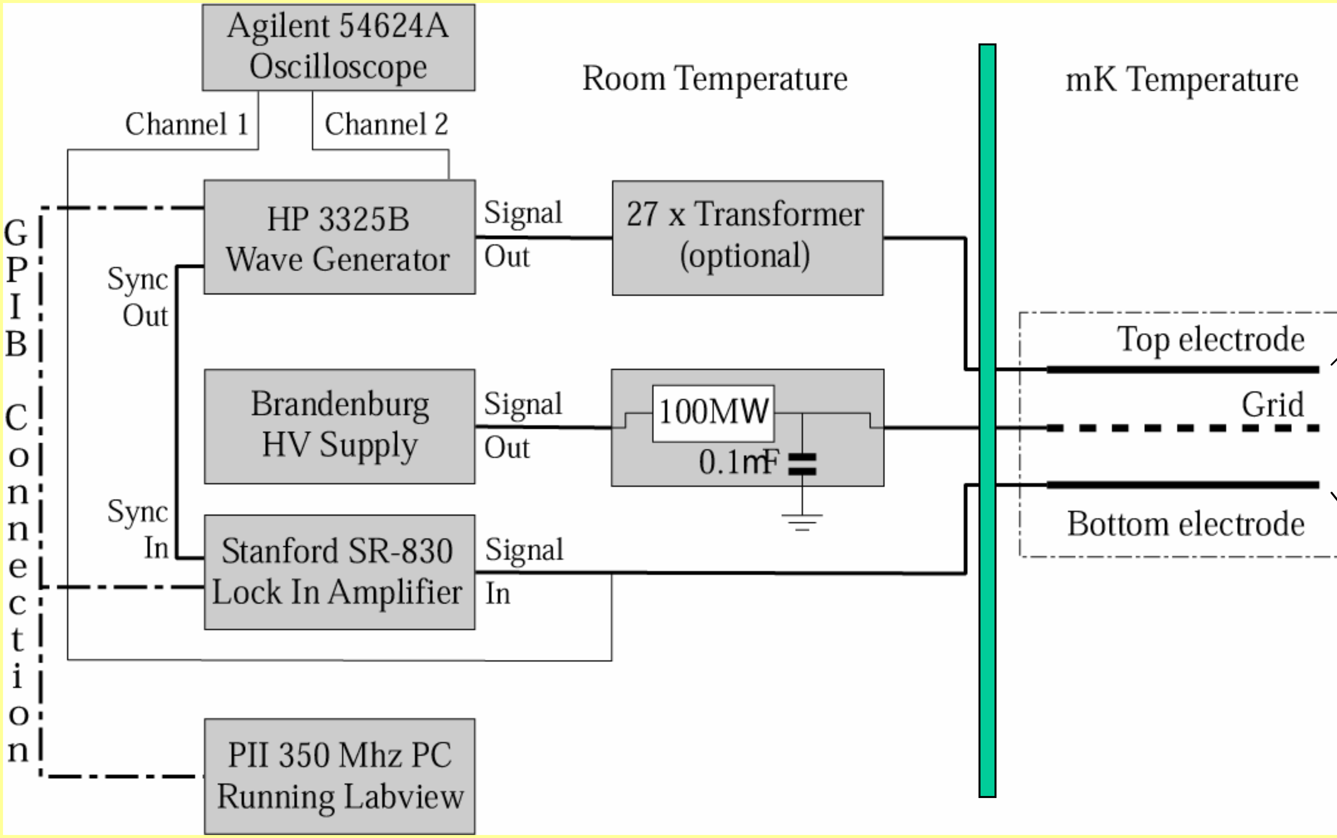



$T < 130$ mK
$0 < p < 25$ bar

Earlier McClintock's group showed (using the ion technique) that turbulence can be created and decays

# Schematic drawing and an electron microscope picture of the Ni electroformed grid






$V_1 = V_{10} cos\omega t$

**R=4 cm** $V = 500$ volts

$f_d = 2\varepsilon_0\varepsilon_r\pi\ R^2 V V_1/d^2$

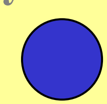
$V_2 = V\Delta D/(2d)$

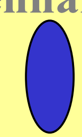
**Oscillation amplitude**

- **In vacuum the grid – tightly stretched membrane oscillates (0,1) axisymmetric mode** $f_0 = (1117.2 \pm 0.05)$ Hz

- **In He II at low amplitude the grid mass becomes hydrodynamically enhanced by**

$\Delta M_H = \beta \varrho_{He}(p) M/\varrho_G$ $\beta = 3.01 \pm 0.05$

 $\beta = 1$

 $\beta = 3$

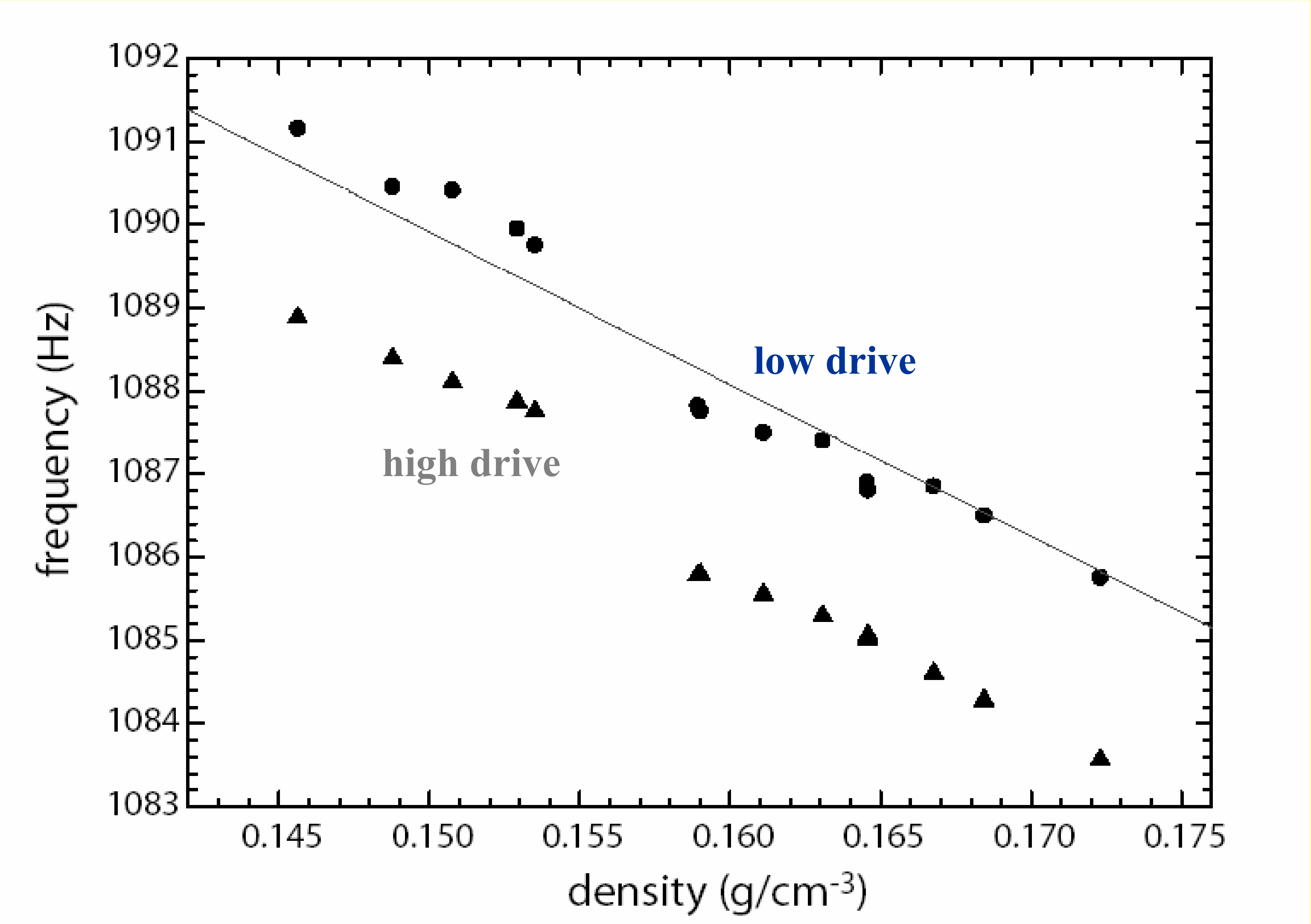
**and its resonant frequency shifts down, depending on pressure, by about 30 Hz**

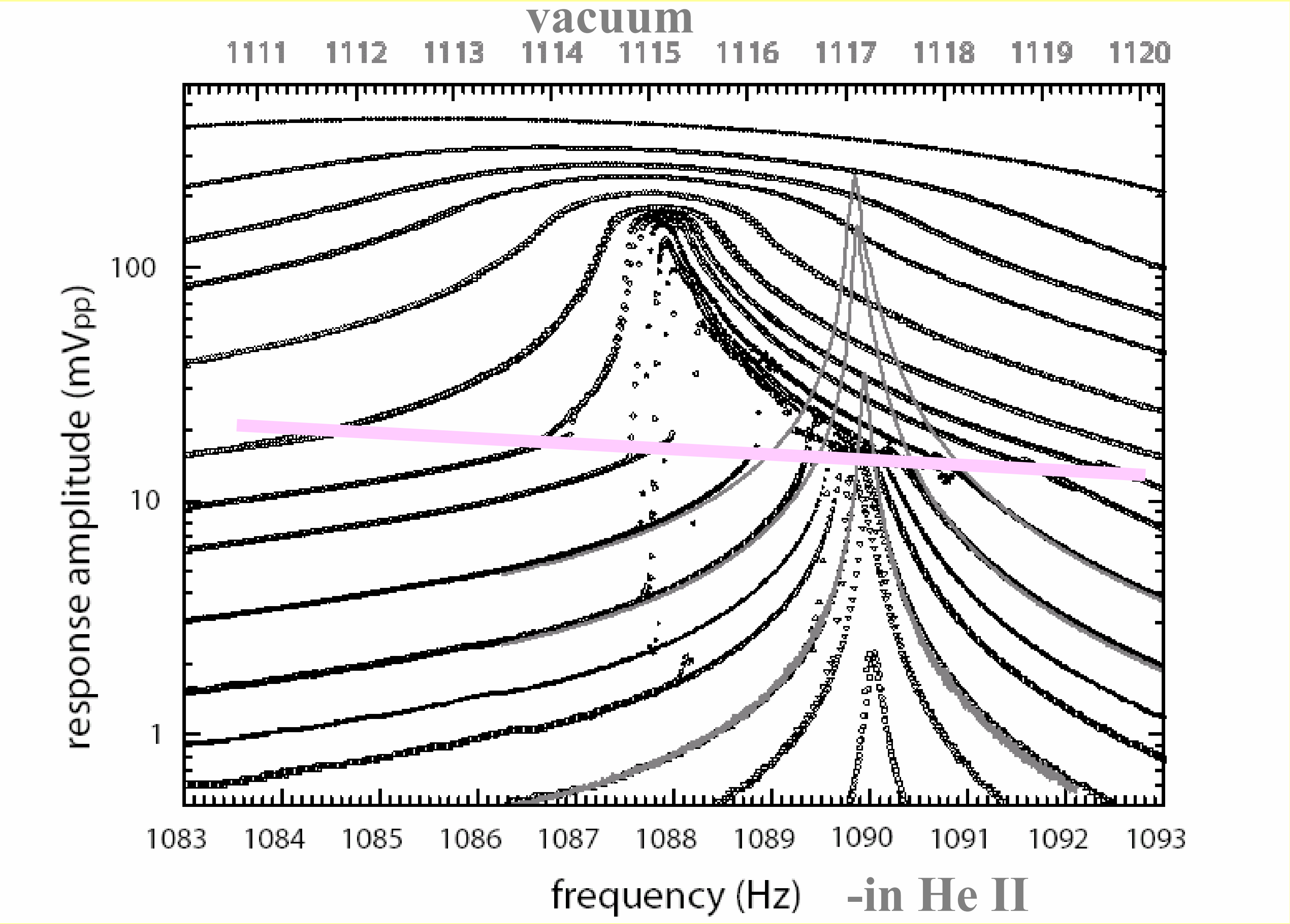
# “Virgin” versus “regular” behaviour

- “virgin” behaviour – obtained immediately after the desired pressure in the cell is established.
- Irreproducible effects observed, such as changes in resonant frequency and linewidth.

- “regular” – reproducible behaviour of the grid, after applying the highest available drive 10Vpp.
- After applying this “cleaning” procedure, all* observed effects found stable within time scale of days.

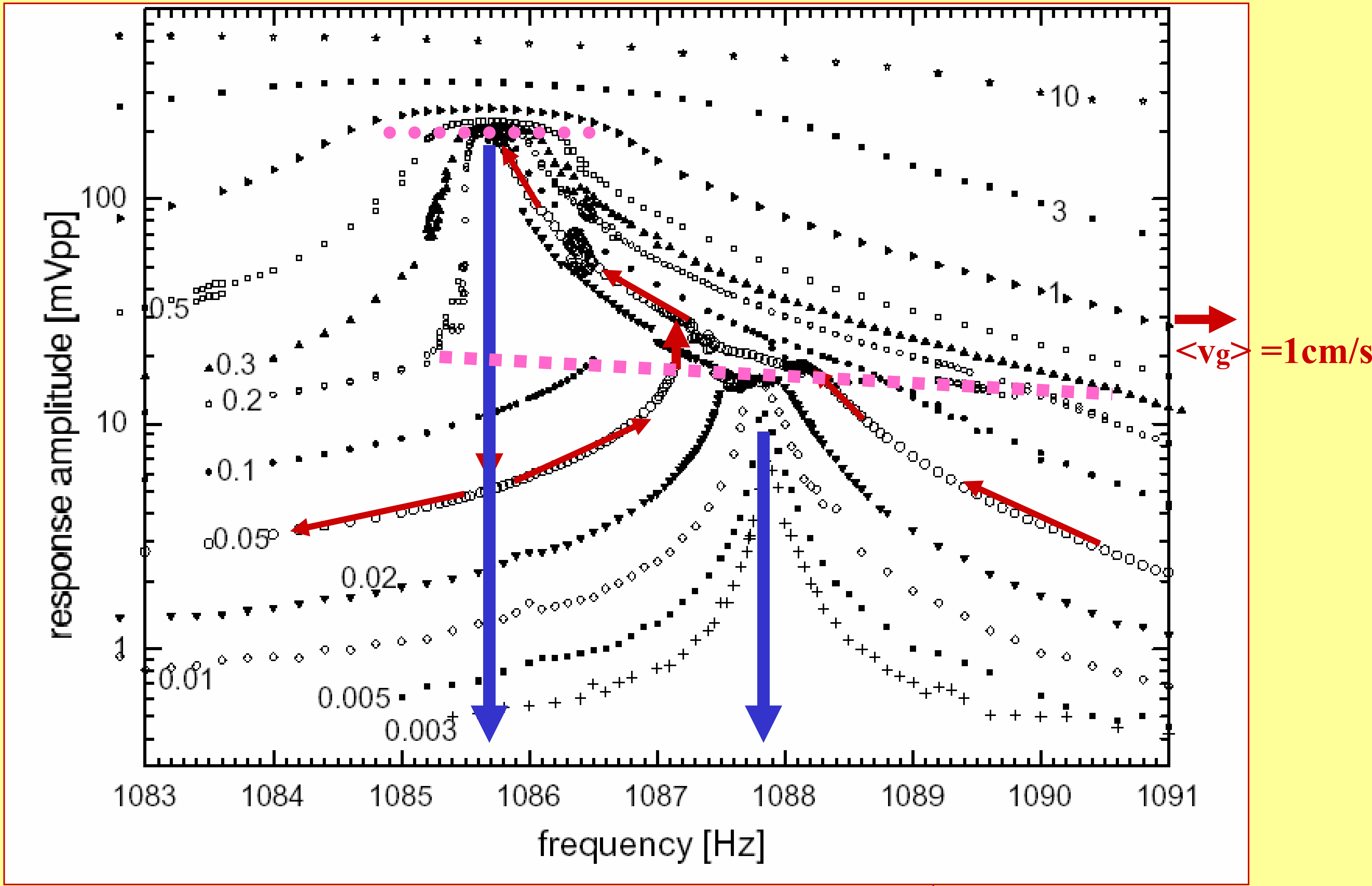
# Resonant frequency – low and high drive level





vacuum
1111
1112
1113
1114
1115
1116
1117
1118
1119
1120
100
10
1
response amplitude (mVpp)
1083
1084
1085
1086
1087
1088
1089
1090
1091
1092
1093
frequency (Hz)
-in He II

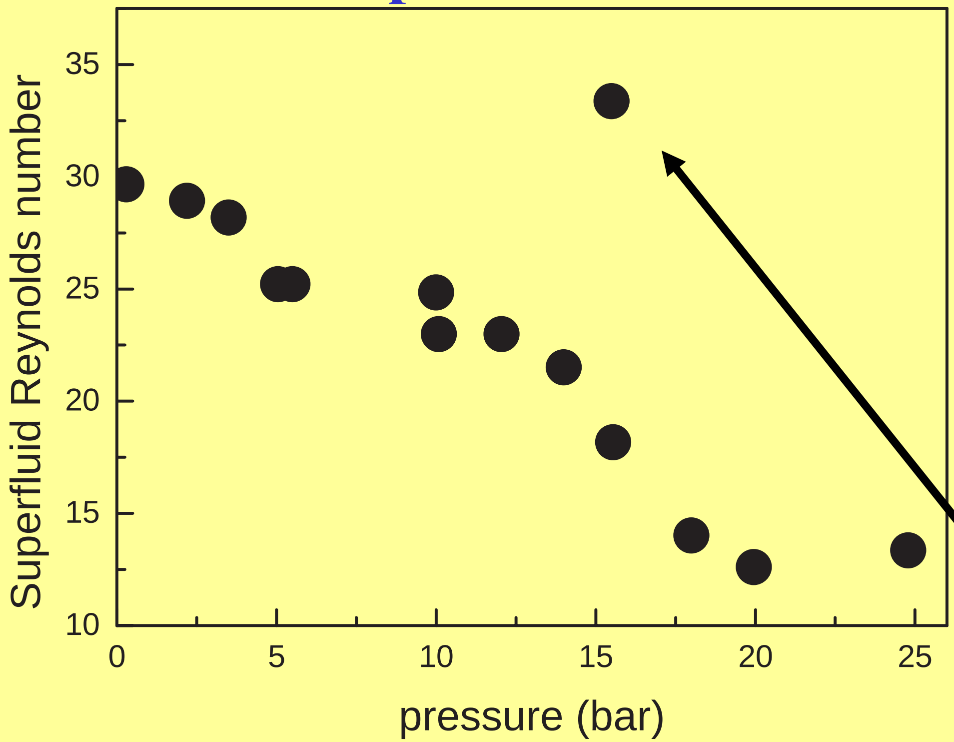
# Resonance response of the oscillating grid




**Two thresholds** 1st at ~ 1cm/s 2nd at ~ 10 cm/s

**decreases with pressure by factor of two** **app. stable**

# Pressure dependence of the first threshold




$$\mathrm{Re}_s = \frac{\mathrm{v}_{flow}^{peak}\, G}{\kappa}$$

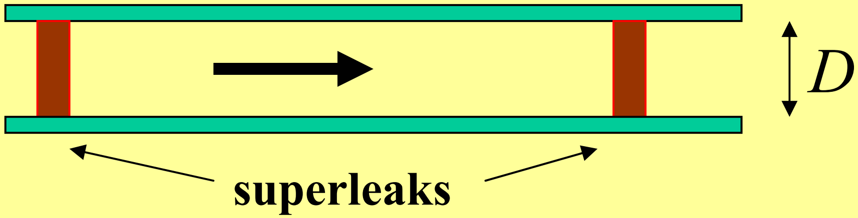
$$\mathrm{v}_{flow}^{peak} \cong 1.43 \times 2.3 \times \mathrm{v}_g^{peak}$$

**1.43 – due to 70% transparency**

**2.3 – due to Bessel function – like profile**

These values of Res compare well with T independent values observed in He II pipe flow above 1K [Baer *et al*, PRL 51 (1983) 2295]

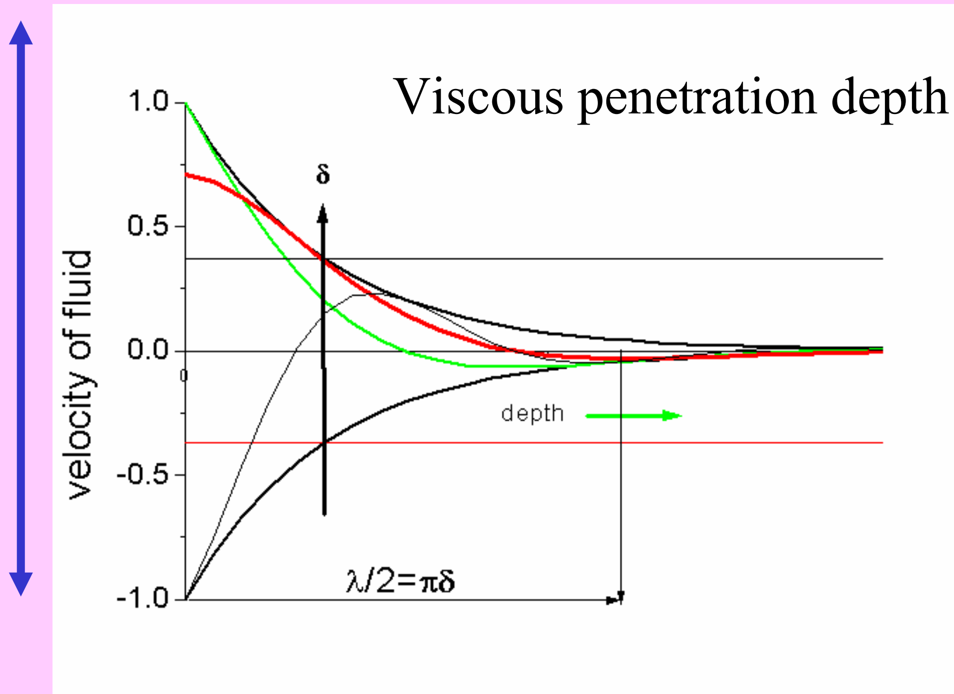
$$\mathrm{Re}_s = \frac{\mathrm{v}\, D}{\kappa} \cong 20$$




**Second threshold does not depend on pressure - Res about 200**

## **Diversion** Classical oscillatory boundary layer flow

**•Simplest case – a plane oscillating along itself in a viscous fluid**




$$\delta = \sqrt{\frac{2\nu}{\omega}}$$

**There is a viscous wave with rapidly decreasing amplitude inside the fluid**

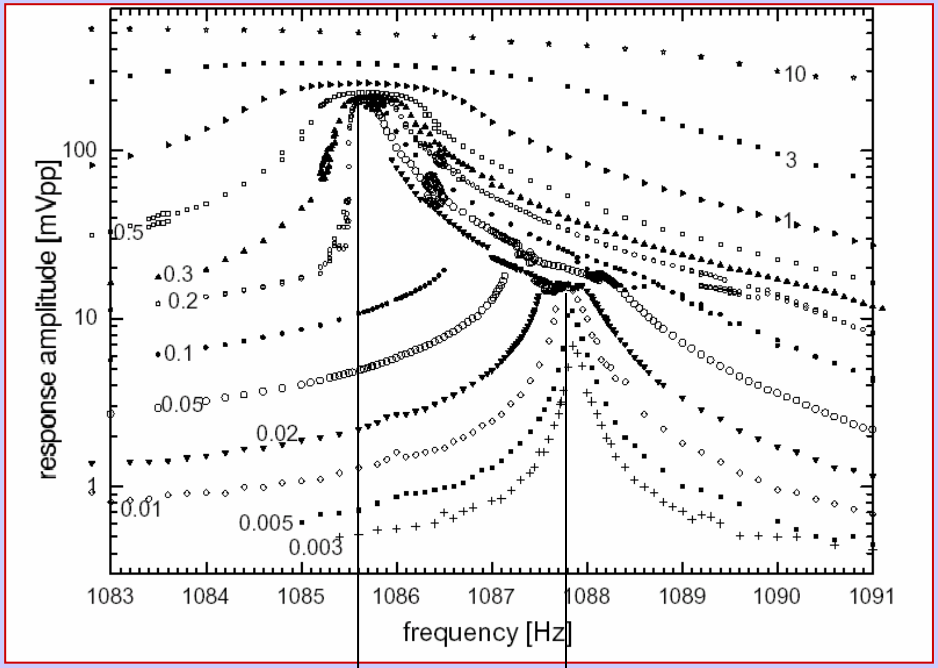
$$\mathbf{v}_y = \mathbf{v}_0 \exp\left\{-\frac{x}{\delta}\right\}\exp\{i[\frac{x}{\delta} - \omega t]\}$$

**•Oscillatory motion of a submerged body**

Viscous force ~ (1+i) , i.e., shifted by 45 degrees in phase with respect to velocity, acting on an oscillating submerged body

**Damping term causes broadening of the resonance and downshift in resonant frequency**

**Hydrodynamic enhancement of the effective mass**




**Assume that the frequency shift above 1st thresholds is a result of additional mass enhancement due to an effective boundary layer dragged with the grid**

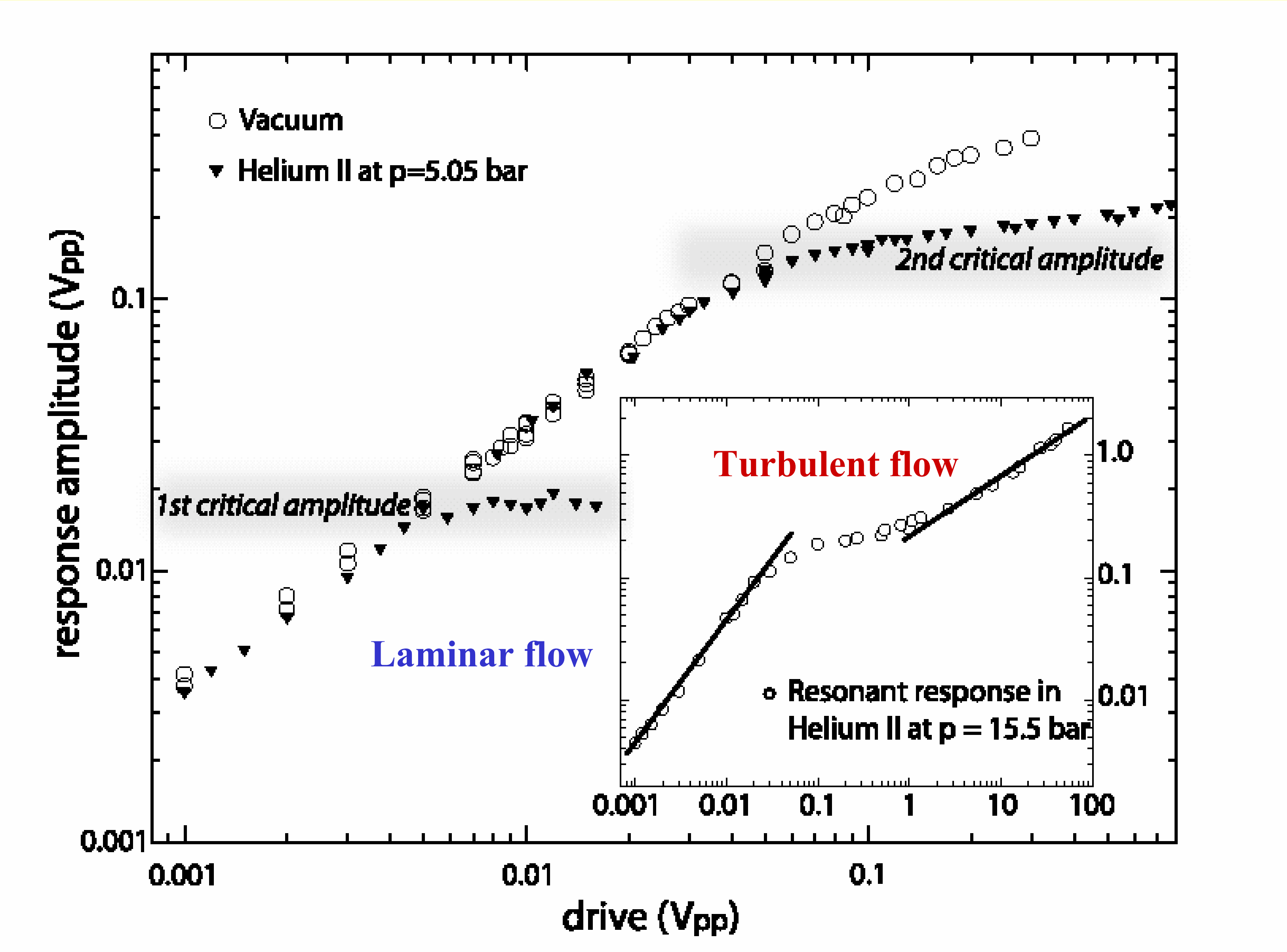
$$\Delta M_\delta \cong A\delta\rho_s$$

$A$ – surface area of the grid

$$\delta = \frac{M + \Delta M_H}{A\rho_s}\left(\frac{f_{res}^2}{f_{sh}^2} - 1\right) \cong 0.5\mu m$$

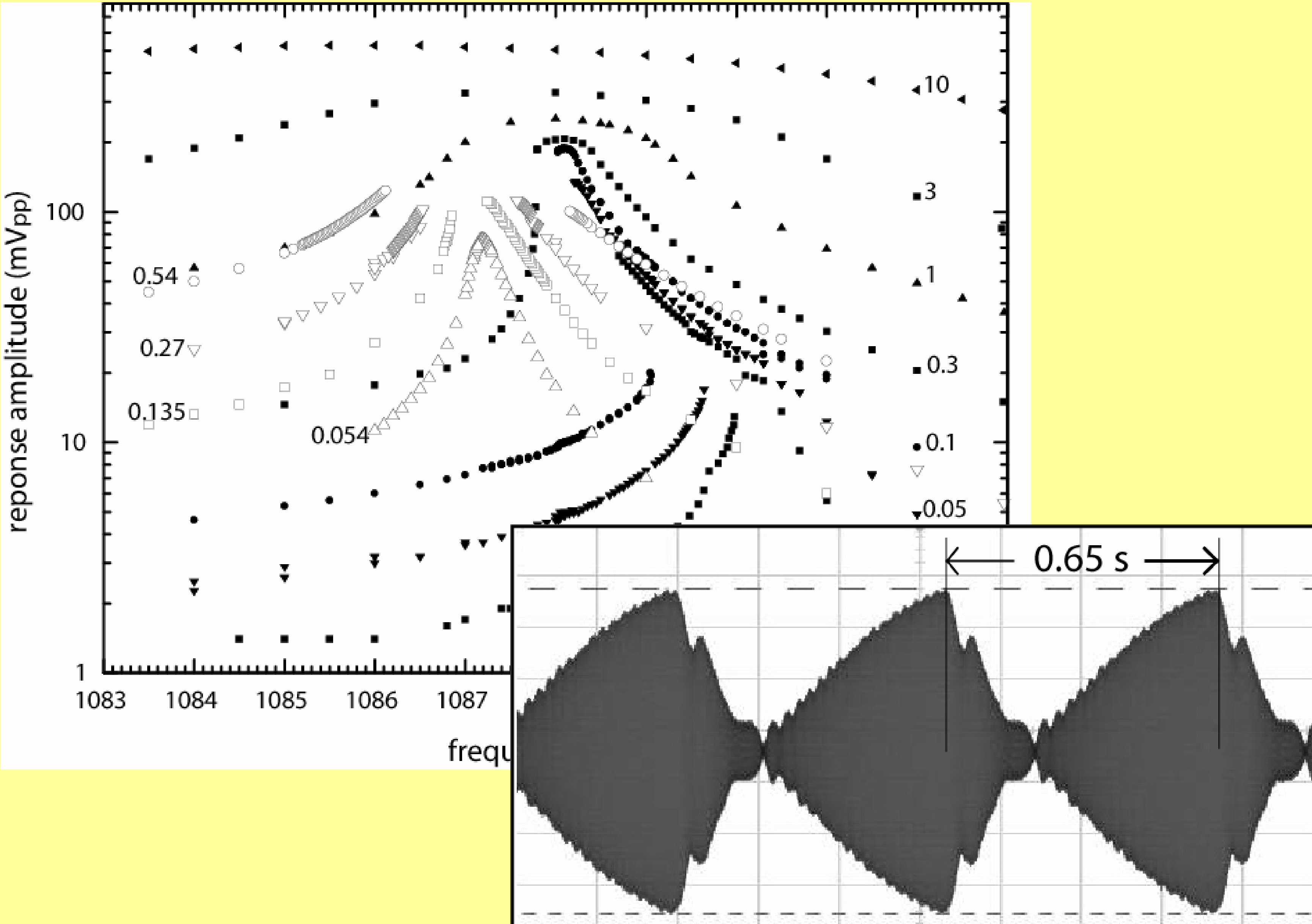
$$\delta = \sqrt{2\nu_{eff}/\omega}$$

**Formally, we can define an effective kinematic viscosity ~$10^{-5}$ cm$^2$/s,**
**But !!! there is no appreciable increase in damping above the first threshold, until the second threshold is reached !!!**

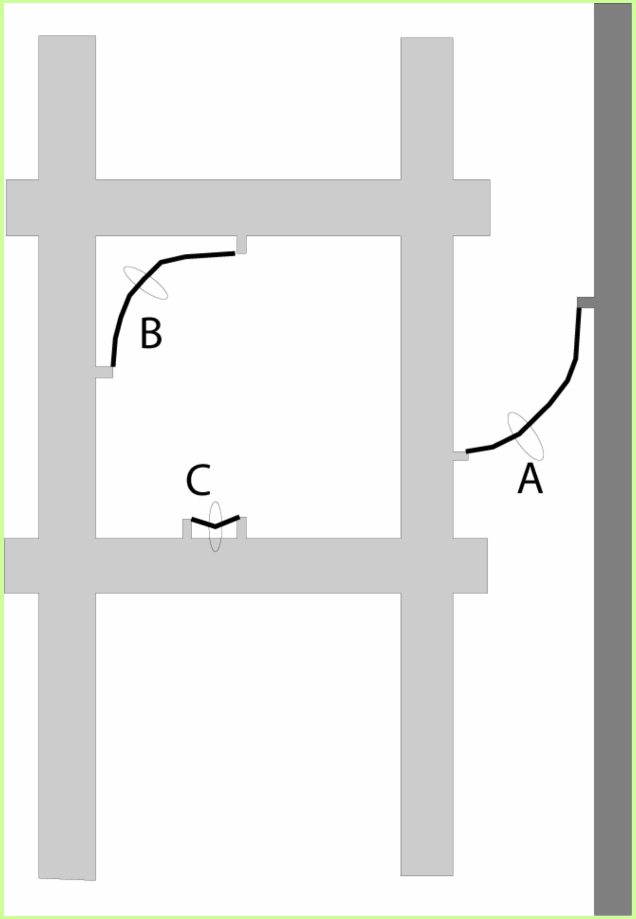



**With increasing the superfluid Re number, in zero temperature limit He II flow changes character from pure superflow via viscous-like flow and turbulence**

# “virgin” behaviour of the oscillating grid



•Assumption: the observed non-linear grid behaviour is caused by the presence of quantized vortices




**Vortex (Kelvin) waves with dispersion relation**

$$\omega(k) \cong \frac{\kappa k^2}{4\pi} \ln\left(\frac{1}{ka}\right)$$

$$\omega \cong 6000 \rightarrow \ell \approx 13\,\mu m$$

•If vortex loop is significantly shorter, its response to the flow will be adiabatic, i.e., it will always take the equilibrium position in the flow – hardly any dissipation

•Long loops – vortex waves – Kelvin wave cascade-reconnections - dissipation

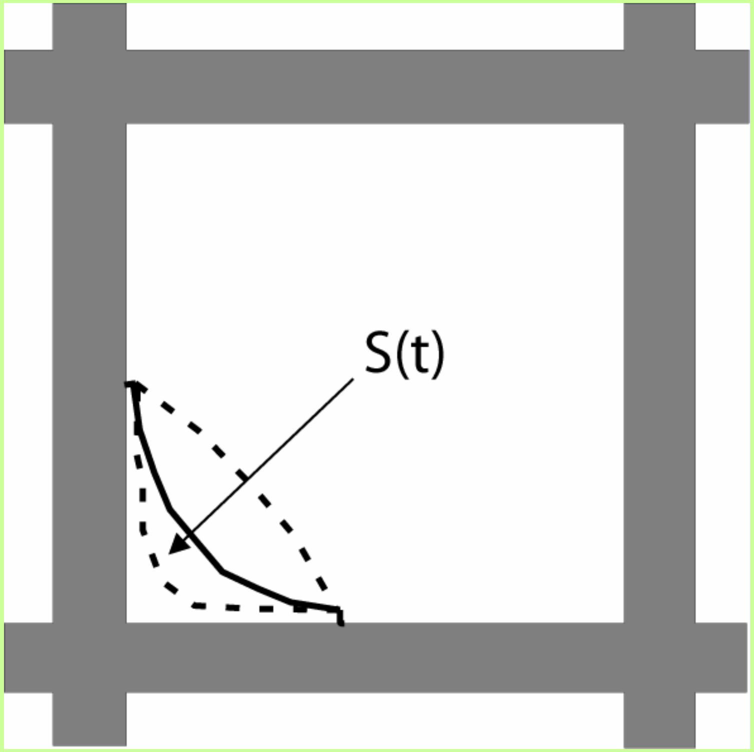
**Remnant vortices always present**

•Type A length > 1 mm

•Type B length ~ 0.1 mm

•Type C roughness length scale ~ micron

## How does the movement of the vortex loop affect effective mass ?




**The impulse to create the vortex loop**

$$\rho_s \kappa S$$

**The rate of change of momentum for each aperture of the grid**

$$M_{eff} \frac{d\,\mathrm{v}}{dt} = M \frac{d\,\mathrm{v}}{dt} + M_{pot} \frac{d\,\mathrm{v}}{dt} + \rho_s \kappa \frac{dS}{dt}$$

**If the loop responds adiabatically**

$$\frac{dS}{dt} = \frac{dS}{d\,\mathrm{v}} \frac{d\,\mathrm{v}}{dt}$$

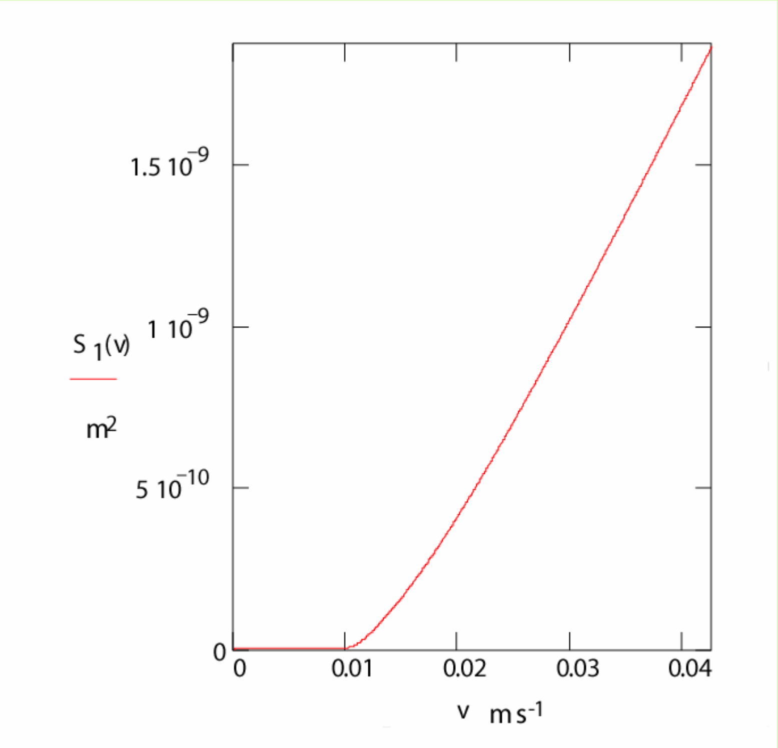
**and therefore**

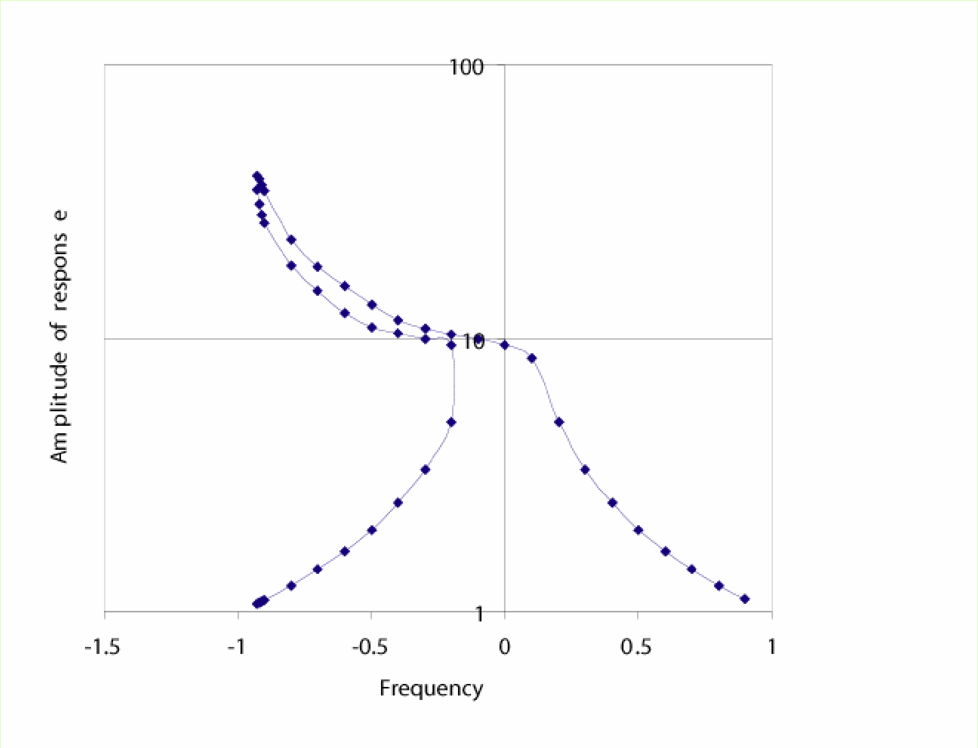
$$M_{eff} = M + M_{pot} + \rho_s \kappa \frac{dS}{d\,\mathrm{v}}$$

**The quantity S(v) must be obtained from simulations – a challenging task to be done**

**Let work backwards –what kind of non-linearity reproduces the experiment?**

**Assume S(v)=So(v)+S1(v), where So(v) is linear with velocity**



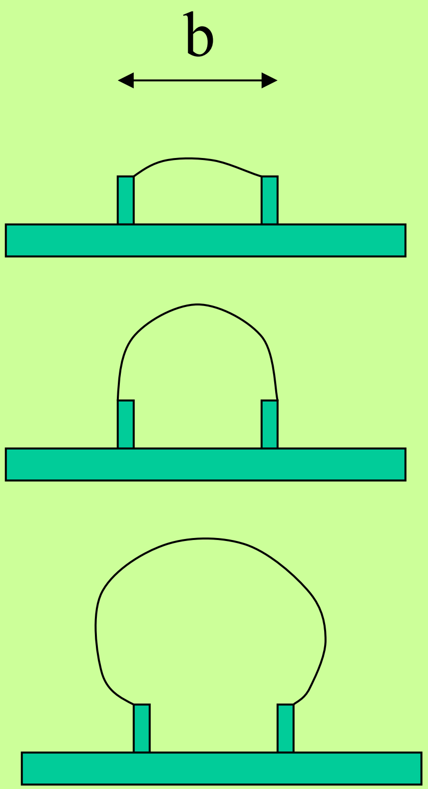




**To get the response similar to that observed in the experiment, we need S1(v) about 2500 square microns per aperture at the second threshold**

**Assuming only one loop per aperture, it will be of the length** $b \approx \sqrt{S1(\mathrm{v})} \approx 50\mu m$

- **it cannot respond adiabatically – there must be more of them ( at least about 15)**
- **therefore at least some of them must be of the type C**

# The evolution of a precursor vortex into an unstable structure




Consider a loop of type C – in the flow it bows out, its radius of curvature decreases, becomes minimal (semicircle) and then grows again – instability

It sets in when flow velocity exceeds $\approx \kappa / b$

If we identify this instability with the upper threshold (1D approx.) 5 cm/s – b=4 microns

Taking into account 70% transparency and the drumskin-like velocity profile, the typical length becomes of order 1 micron.

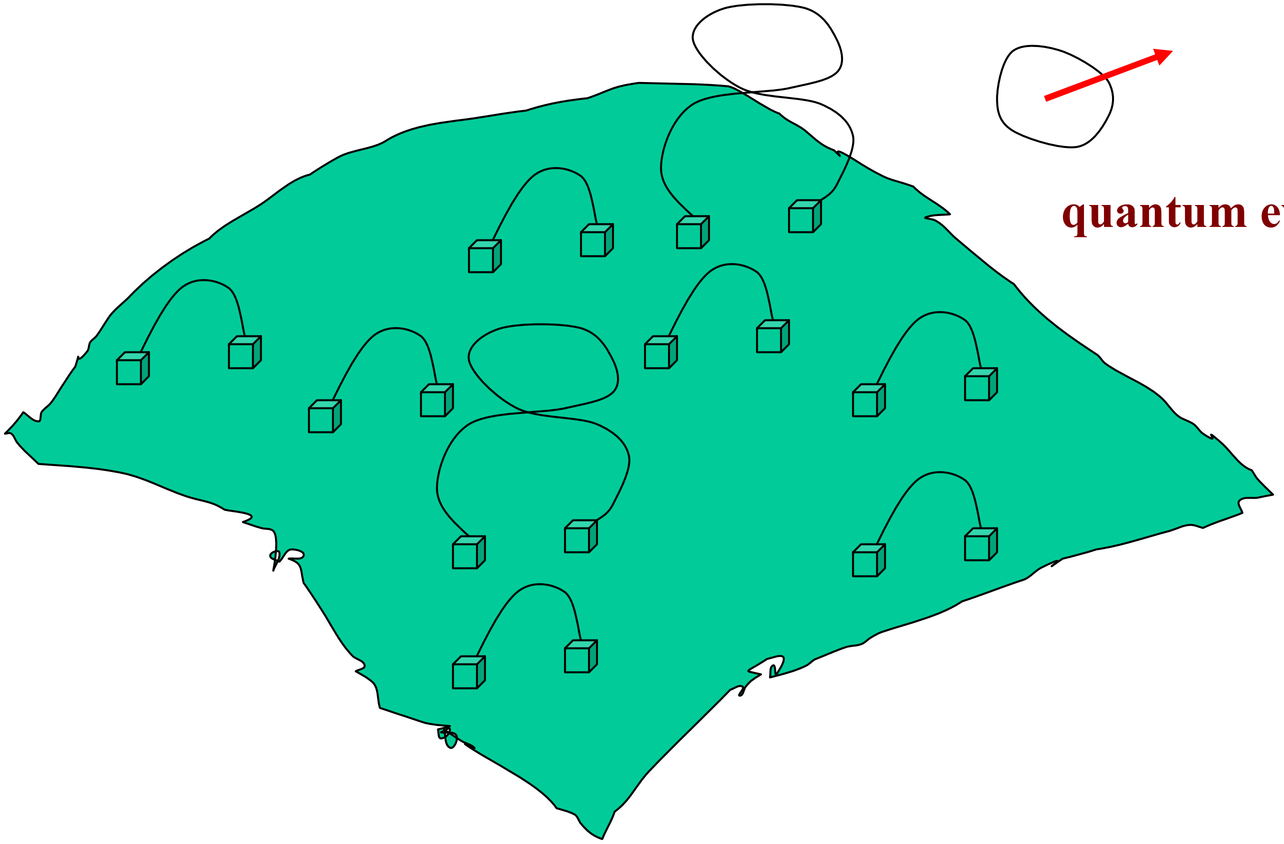
There must therefore be of order 1000 of vortex loops per aperture-

an effective boundary layer constituent of vortex loops, of thickness less than a micron, in accord with classical boundary layer thickness estimated above

It is tempting to suppose that formation of this boundary layer is intrinsic to superflow over any solid boundary

However, formation of the boundary layer can be a result of the “cleaning”procedure Big loops of the type A and B are removed (i.e., less dissipation, resonant frequency moves up and linewidth decreases)

# Effective boundary layer constituent of vortex loops



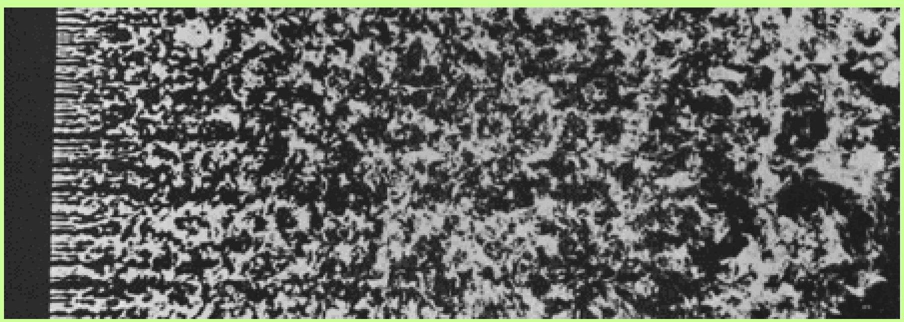

**Second threshold – instability against balooning out, vortex wave of the drive frequency becomes excited, the loop twists, reconnects and the free vortex ring flies away – dissipation**

## Dissipation can occur without reconnections

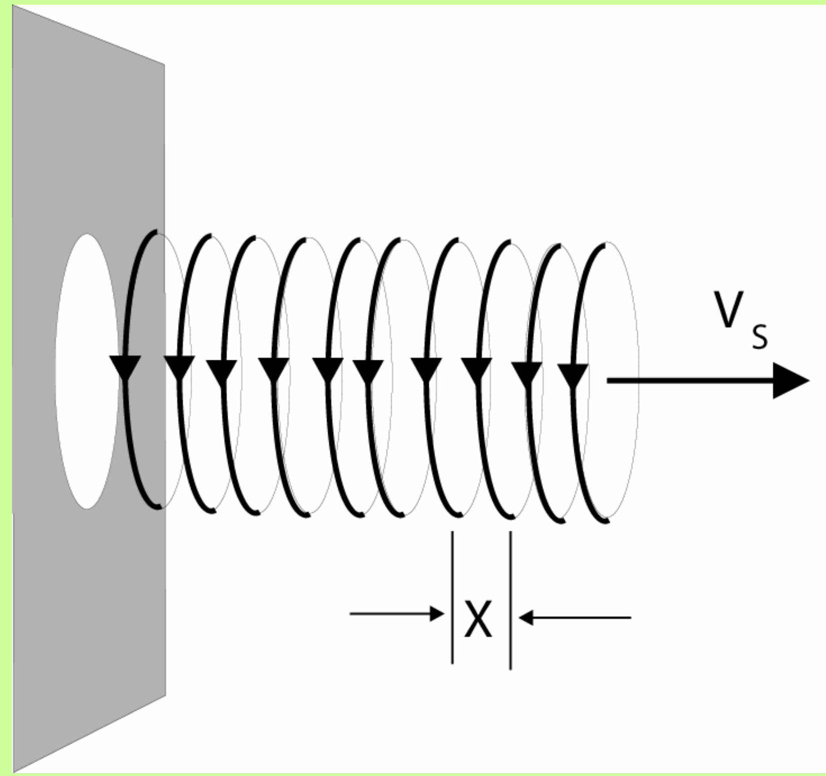
- **According to Schwarz** [PRL 57(1996)1448]**, even at zero temperature, dissipation occurs – by ends of vortex loops sliding the rough surface**
- **Dissipation due to sound emission – Kolmogorov cascade on the long enough vortex loop**

**On increasing the drive level, quantum turbulence becomes generated, eventually displaying the square root velocity vs drive dependence, characteristic for classical fully developed turbulence**

grid



**Visualization of a classical flow through the grid**




**A "jet" formed from vortex rings produced by steady flow through a circular aperture**

# Summary

- Macroscopic flow of He II due to an electrostatically oscillating grid in the low temperature limit has been experimentally investigated and analysed

- "virgin" versus "regular" behaviour (after "cleaning")

- In a low velocity limit, the "regular" response of the grid is similar to that in vacuum, shifted down in frequency due to hydrodynamic enhancement of the eff. mass of the grid

- Above the first threshold the response becomes strongly non-linear. The resonant frequency shifts down without appreciable increase in damping. This behaviour is attributable to a dynamic response of small vortex loops constituent an effective boundary layer, further incresing the effective mass of the grid but not the damping

- Upon reaching the second threshold (marking the onset of superfluid turbulence), the resonant frequency reaches a stable value, the response amplitude almost stops growing, but damping increases.

- At highest drive levels, the turbulent flow displays classical velocity vs drive dependence

*Many open questions*

Is this behaviour universal? Is there an analog to a classical boundary layer flow? Other quantum fluids? Do these studies lead to a better undestanding of turbulence in general?

Further experiments, analysis and simulations are under progress…