Elastic co-tunnelling in SET's
- and other s-qubit issues -

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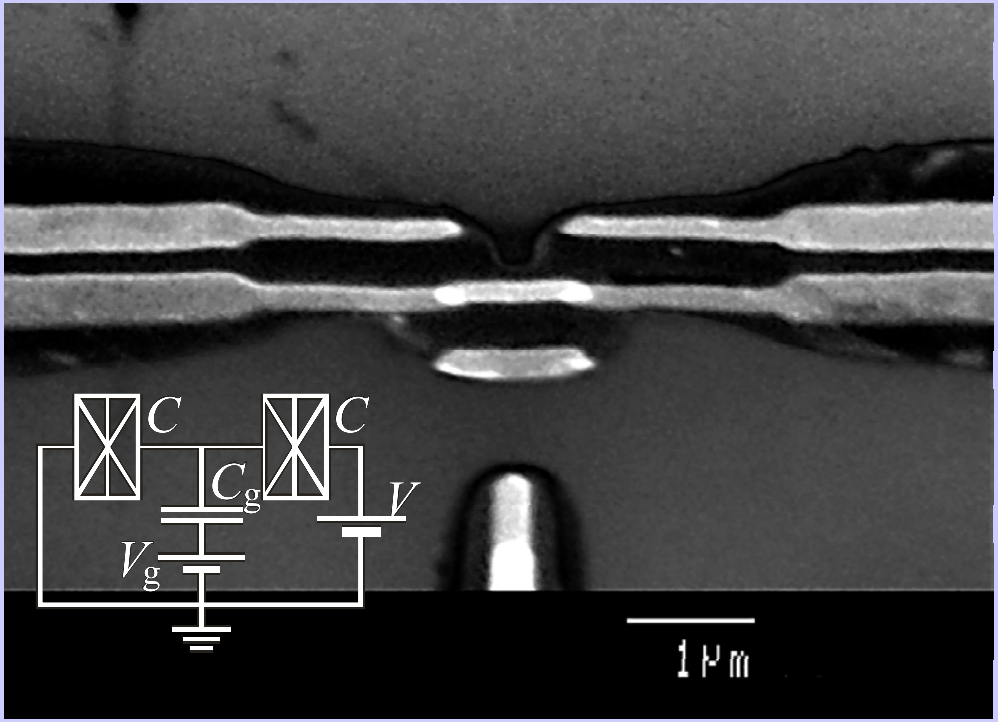
Nb as a construction material for nano-devices:

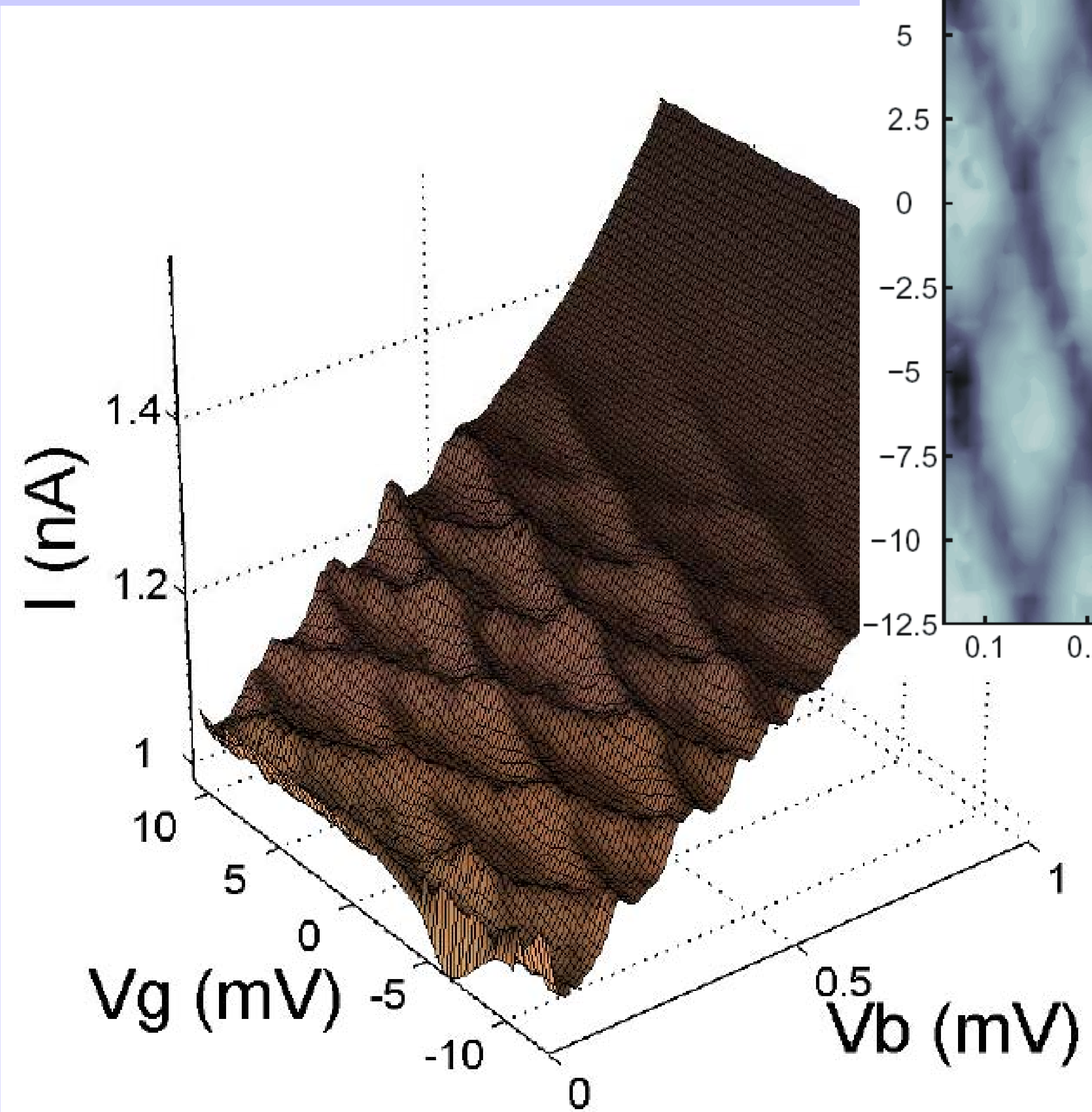
Bulk properties/Material	**Aluminum**	**Niobium**
Critical temperature	1.175 K (0.1 meV)	9.26 K (0.8 meV)
Superconducting gap	2.07 K (0.18 meV)	16.33 K (1.4 meV)
Critical magnetic field	0.0105 T	0.2060 T

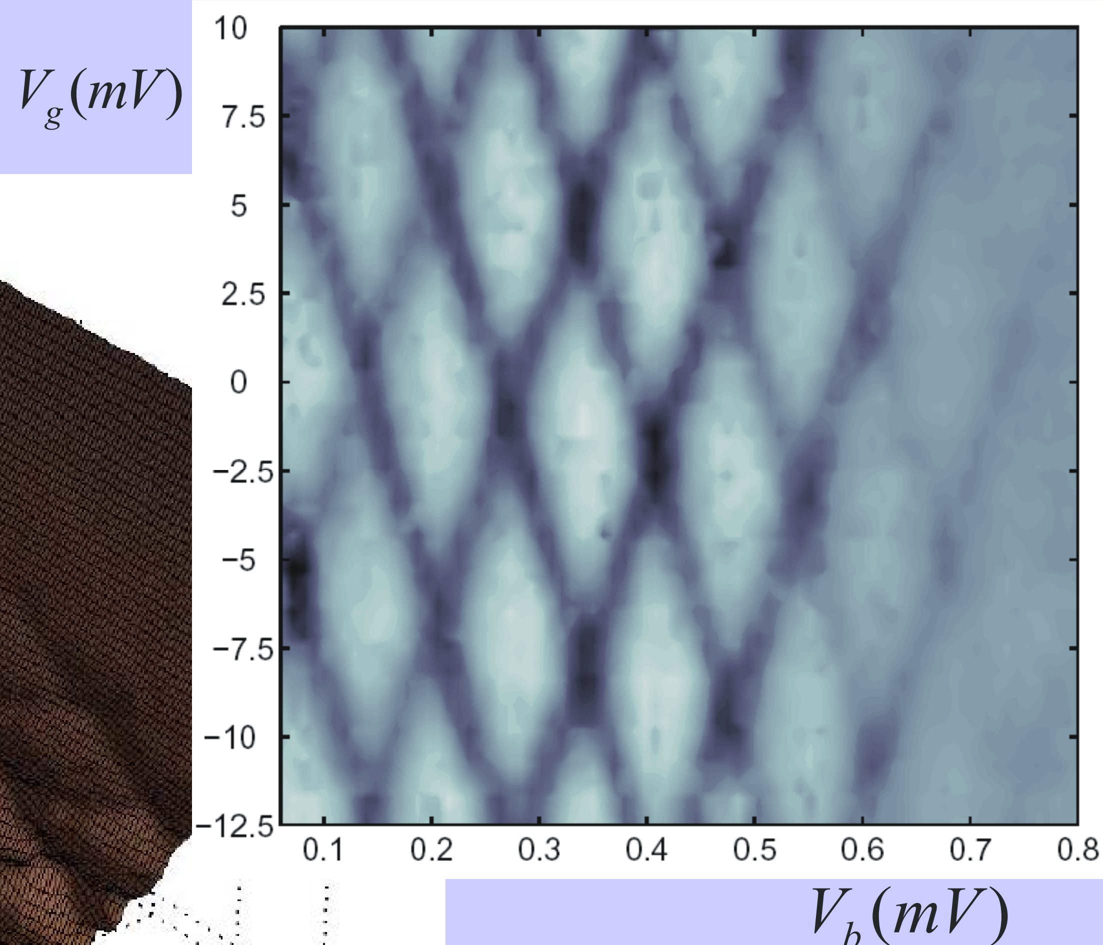
$$\mu_0 = 4\pi \times 10^{-7} H / m$$

Single-electron transitor Al/Nb/Al:



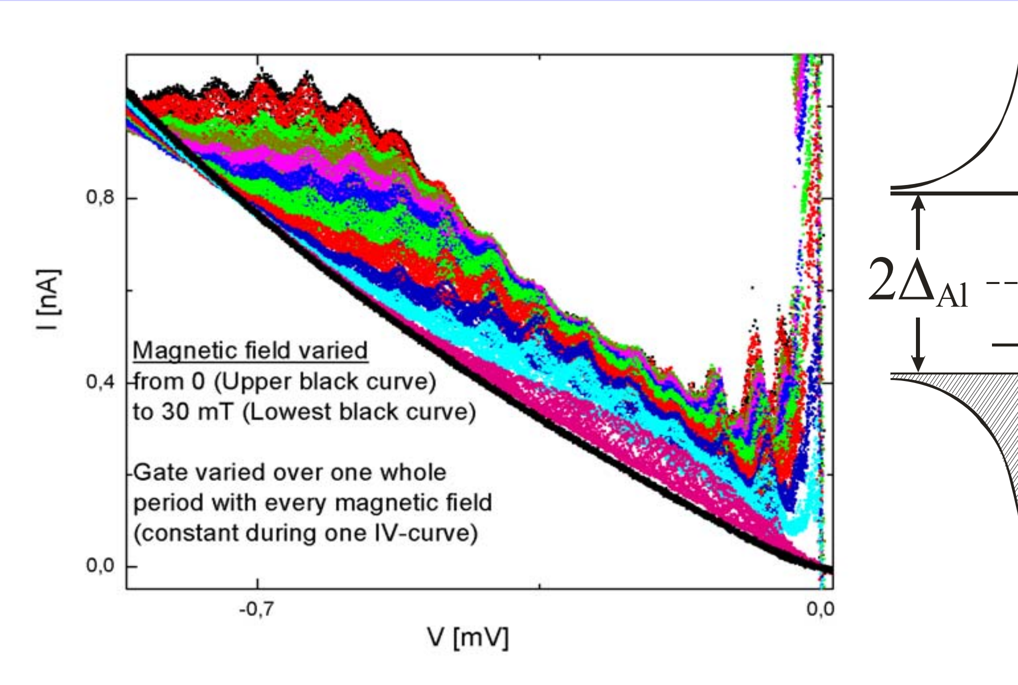
Cooper-pair resonances:

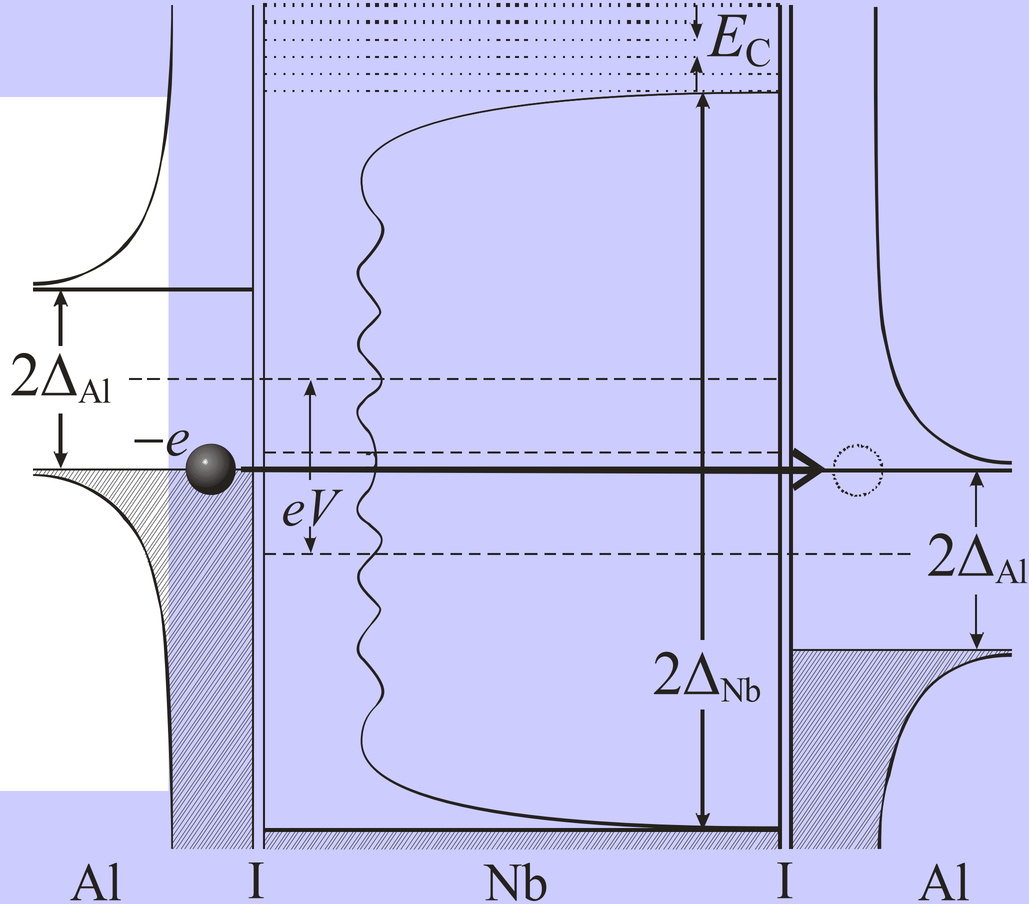




$$V = \frac{1}{C_\Sigma}\left(2e + 2C_{\rm g}V_{\rm g} + 2Q_0\right)$$

Measurements in magnetic field:





SIMPLEST INTERPRETATION:

assume that there are ”leaky” electronic states inside the gap of Nb.

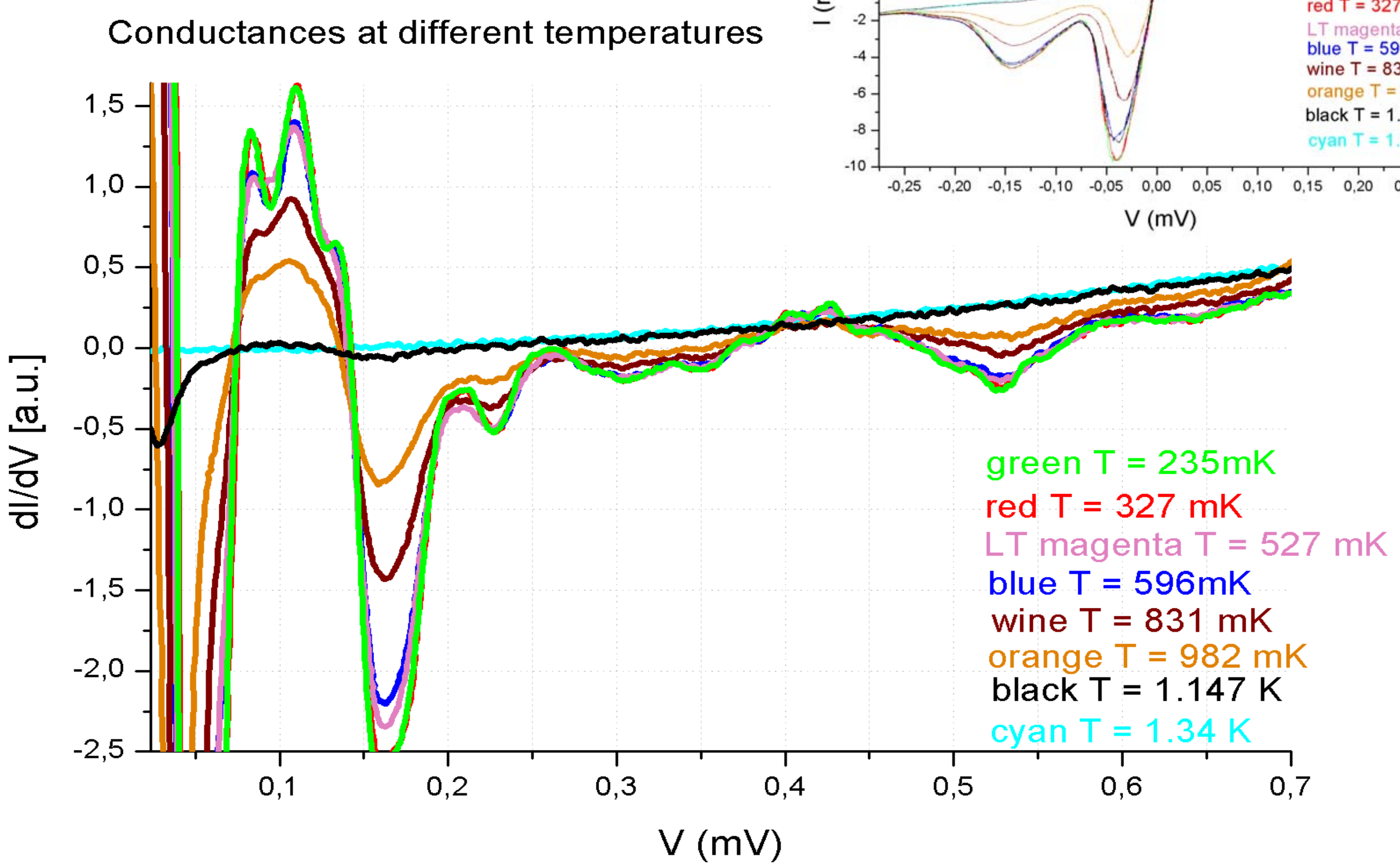
Then the background current would be given by sequential tunneling through these states.

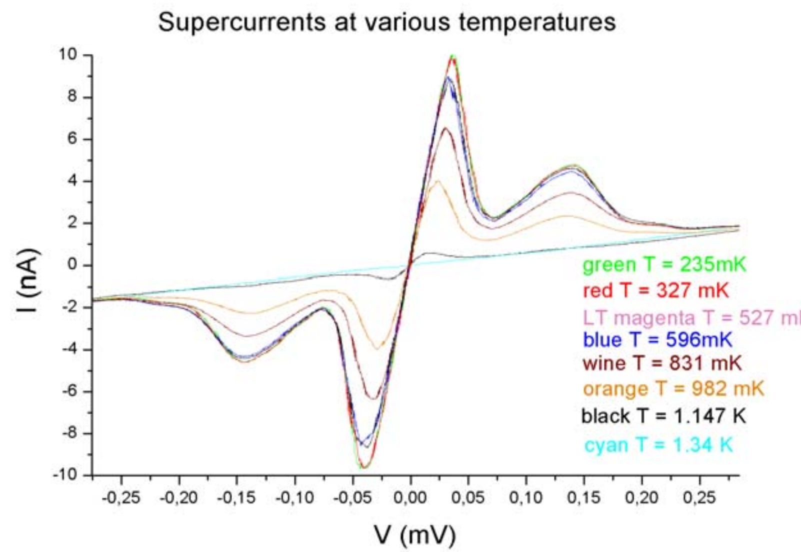
$$\Gamma_{LI}(n) = \frac{1}{e^2 R_{LI}} \int\int d\varepsilon d\varepsilon' N_L(\varepsilon) N_I(\varepsilon') f_L(\varepsilon)\left[1 - f_L(\varepsilon')\right]\delta\left[E_{CH}(n) + \varepsilon' - \varepsilon\right]$$

$$N_L(\varepsilon) = \Theta\left(|\varepsilon| - \Delta_{Al}\right)\frac{|\varepsilon|}{\sqrt{\varepsilon^2 - \Delta_{Al}^2}}$$

then use the ”orthodox” theory, etc.

"Leak" currents in a single AlNb junction:



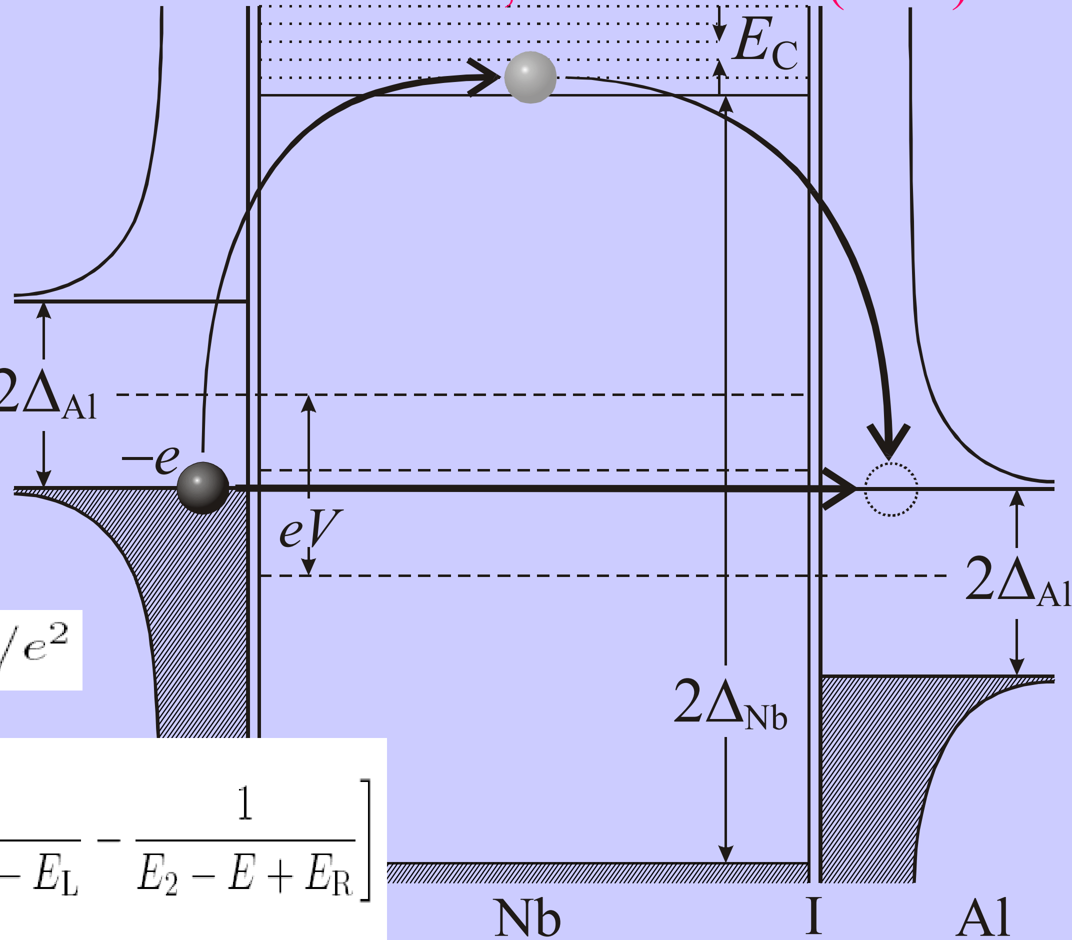


The theory of elastic co-tunneling:

1. Normal-metal SET's: elastic co-tunneling through the Coulomb barrier.

D. V. Averin and Yu. V. Nazarov, PRL 65 2446 (1990)

2. Superconducting SET's: elastic co-tunneling through the gap of the island and the Coulomb barrier



$$E_1 = \frac{e}{C_\Sigma}\left[e\left(n+\frac{1}{2}\right)-V\left(C_2+\frac{C_g}{2}\right)+C_gV_g\right]$$

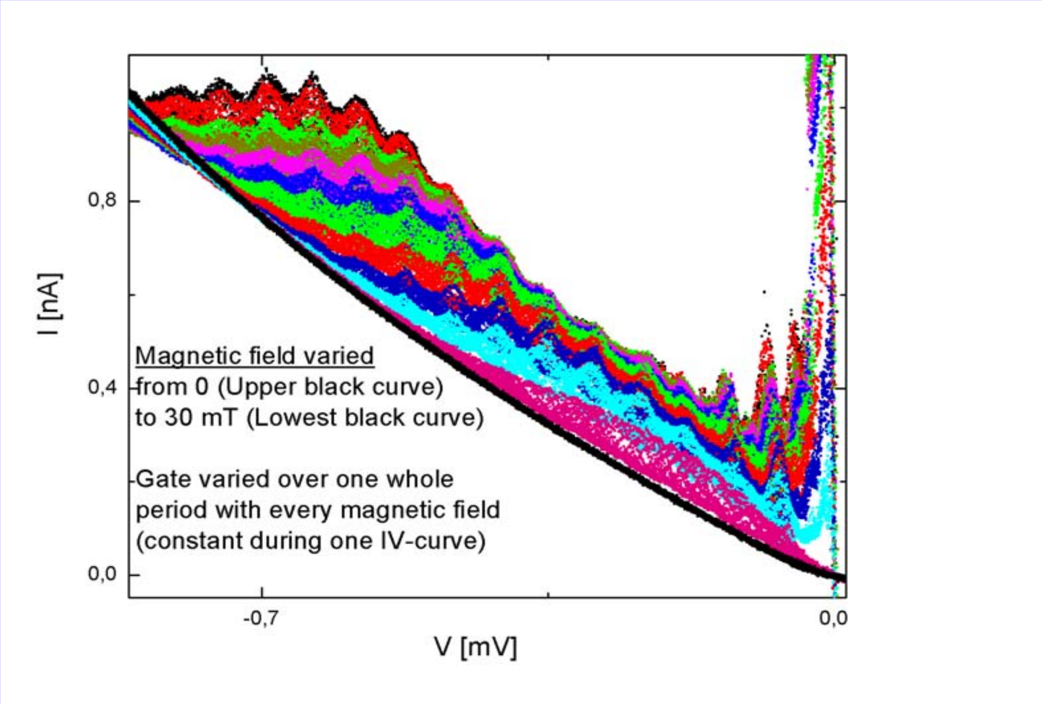
$$E_2 = \frac{e}{C_\Sigma}\left[-e\left(n-\frac{1}{2}\right)-V\left(C_1+\frac{C_g}{2}\right)-C_gV_g\right]$$

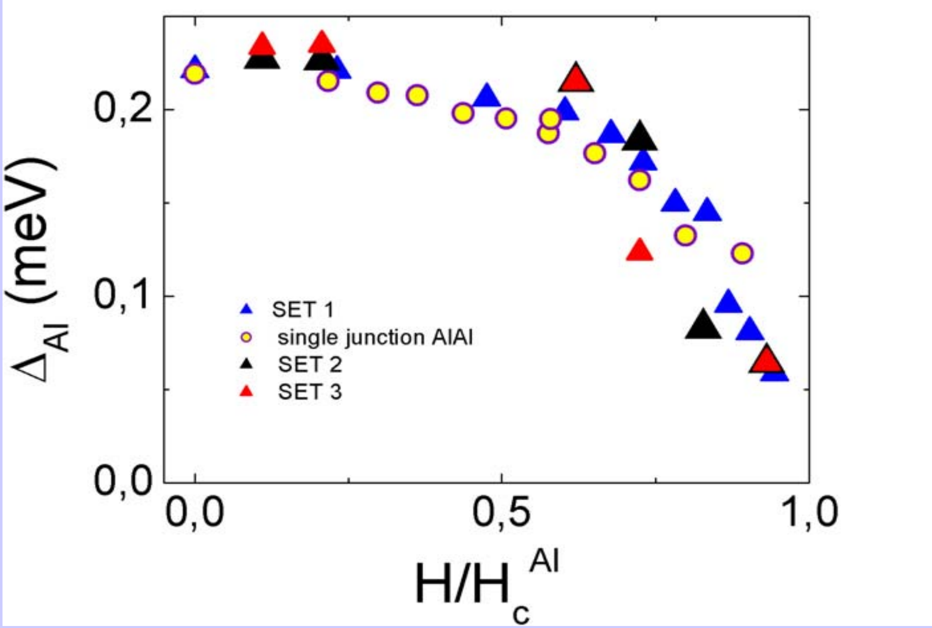
$$R_{\rm eff} = R_{\rm T,1}R_{\rm T,2}/R_{\rm K} \qquad R_{\rm K} = h/e^2$$

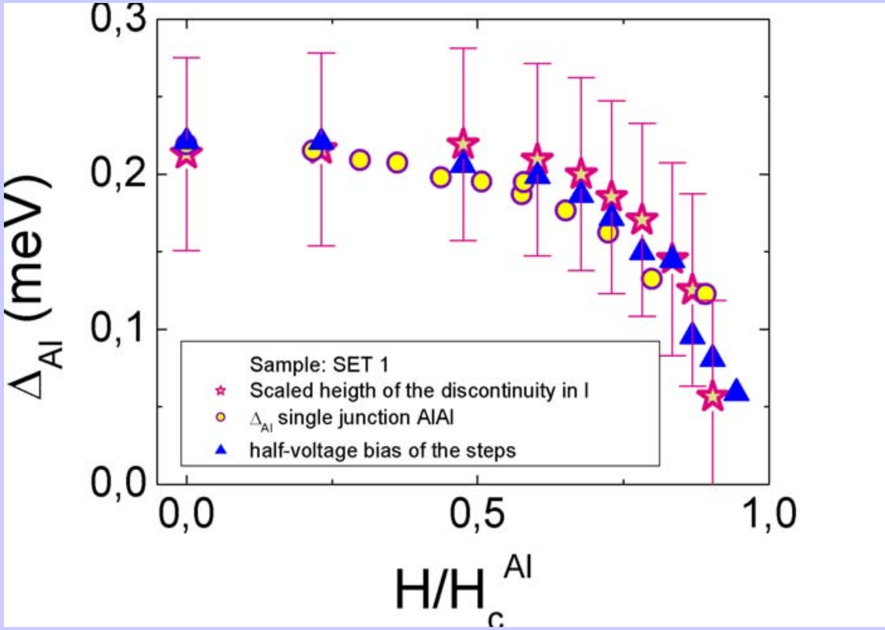
$$T_{\rm eff}(E_{\rm L}, E_{\rm R}) = 2\pi\int_{\Delta_{\rm Nb}}^{\infty} dE \frac{E}{\sqrt{E^2-\Delta_{\rm Nb}^2}}\left[\frac{1}{E_1+E-E_{\rm L}} - \frac{1}{E_2-E+E_{\rm R}}\right]$$

$$I_{\rm el} = \frac{1}{eR_{\rm eff}}\int_{\Delta_{\rm Al}}^{\infty} dE_{\rm L}\int_{\Delta_{\rm Al}}^{\infty} dE_{\rm R}\frac{E_{\rm L}E_{\rm R}}{\sqrt{E_{\rm L}^2-\Delta_{\rm Al}^2}\sqrt{E_{\rm R}^2-\Delta_{\rm Al}^2}}T_{\rm eff}^2(E_{\rm L}, E_{\rm R})\delta(eV-E_{\rm L}-E_{\rm R})$$

Background current:
-behaviour in magnetic field
-comparison with the elastic co-tunnelling theory





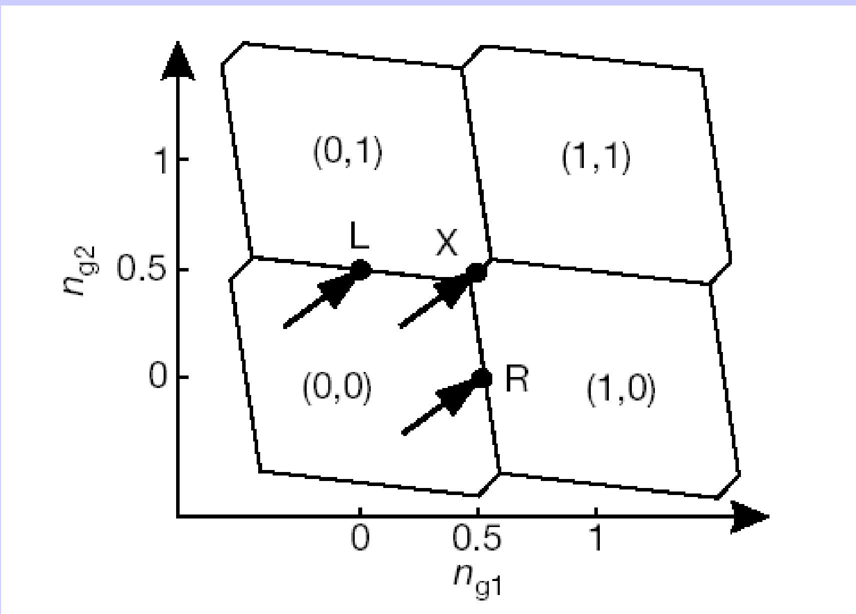


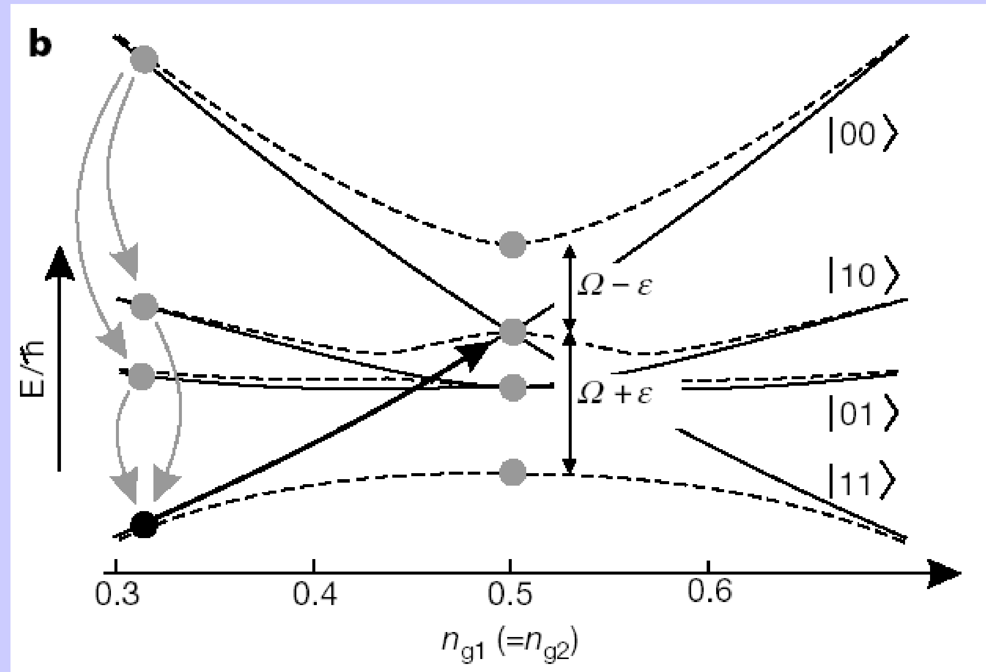
TWO SQUBITs ISSUES:-

quantum manipulations with r.f. pulses -

Generic form for the 2-squbit Hamiltonian (with interaction):

$$H = \begin{bmatrix} E_{00} & -\frac{1}{2}E_{J1} & -\frac{1}{2}E_{J2} & 0 \\ -\frac{1}{2}E_{J1} & E_{10} & 0 & -\frac{1}{2}E_{J2} \\ -\frac{1}{2}E_{J2} & 0 & E_{01} & -\frac{1}{2}E_{J1} \\ 0 & -\frac{1}{2}E_{J2} & -\frac{1}{2}E_{J1} & E_{11} \end{bmatrix} + \text{measurement part}$$





Yu. A. Pashkin *et. al.*, Nature 421, 823(2003).

$$\varepsilon = \frac{1}{2\hbar}\sqrt{\left(E_{J2} - E_{J1}\right)^2 + \left(\frac{E_m}{2}\right)^2}$$

$$\Omega = \frac{1}{2\hbar}\sqrt{\left(E_{J1} + E_{J2}\right)^2 + \left(\frac{E_m}{2}\right)^2}$$

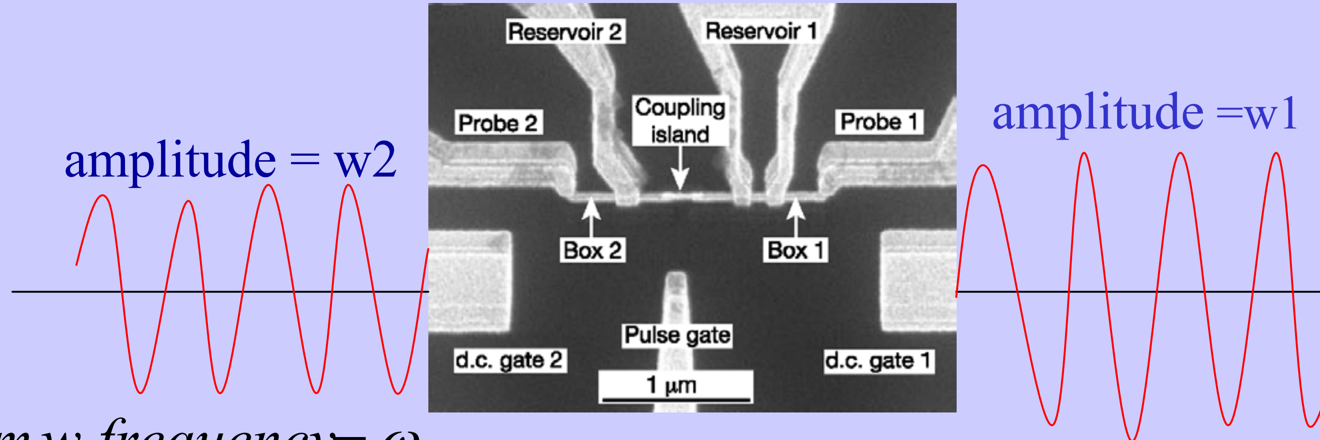
2-qubit Rabi pulses:

Detunings:

$$\delta = \omega - (\Omega - \varepsilon)$$

$$\Delta = \omega - (\Omega + \varepsilon)$$

R.W.A. in a four-level system!



$m.w.\ frequency = \omega$

$$\theta = -\arctan\frac{E_m}{E_{J1}+E_{J2}}$$

$$\chi = -\arctan\frac{E_m}{E_{J2}-E_{J1}}$$

$$H = \begin{bmatrix} \frac{\hbar}{2}(\delta+\Delta) & M_{1+} & M_{2+} & 0 \\ M_{1+} & \frac{\hbar}{2}(-\delta+\Delta) & 0 & M_{2-} \\ M_{2+} & 0 & \frac{\hbar}{2}(\delta-\Delta) & M_{1-} \\ 0 & M_{2-} & M_{1-} & \frac{\hbar}{2}(-\delta-\Delta) \end{bmatrix}$$

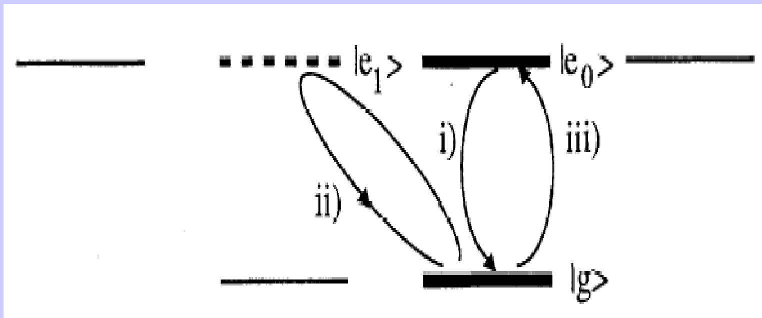
2-qubit operations:
the "almost" theorem of
S. Lloyd, PRL 75, 346 (1995).

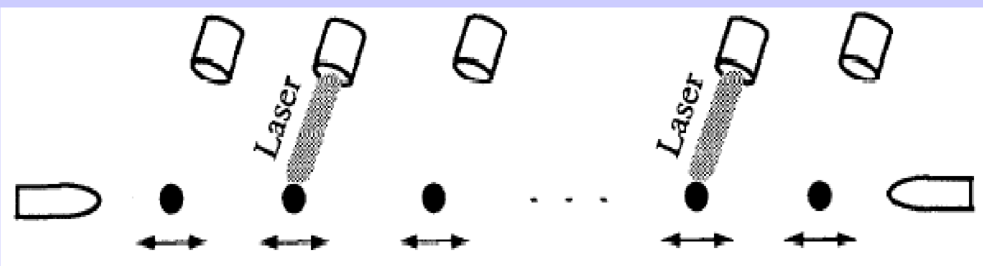
$$M_{1\pm} = \frac{E_{c1}w_1}{2}\cos\frac{\theta-\chi}{2} \pm \frac{E_{c2}w_2}{2}\sin\frac{\theta+\chi}{2}$$

$$M_{2\pm} = \frac{E_{c2}w_2}{2}\cos\frac{\theta+\chi}{2} \pm \frac{E_{c1}w_1}{2}\sin\frac{\theta-\chi}{2}$$

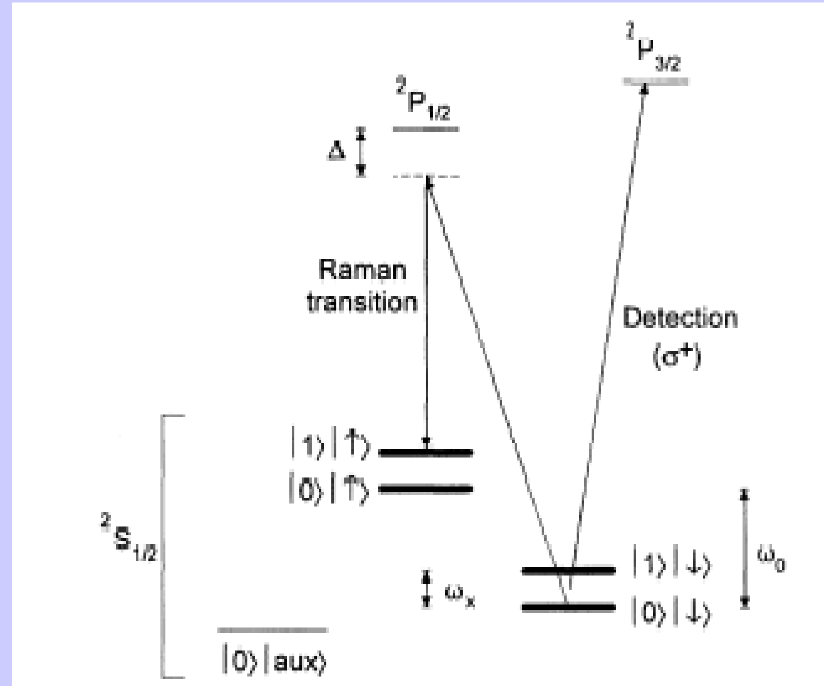
Comparison: trapped ions as qubits

Theory – Cirac&Zoller: PRL 74, 4091 (1995)





Experiment - NIST: PRL 75, 4714 (1995)



$$\hat{H}_{n,q} = \frac{\eta}{\sqrt{N}}\frac{\Omega}{2}\left[|e_q\rangle_n\langle g|a e^{-i\phi} + |g\rangle_n\langle e_q|a^\dagger e^{i\phi}\right]$$

$\eta = [\hbar k_\theta^2/(2M\nu_x)]^{1/2}$ = Lamb-Dicke parameter

$$H = \frac{\eta\Omega}{2\sqrt{N}}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \exp(-i\phi) & 0 \\ 0 & \exp(i\phi) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$