KT-transition in $2 + 1$ dimensions

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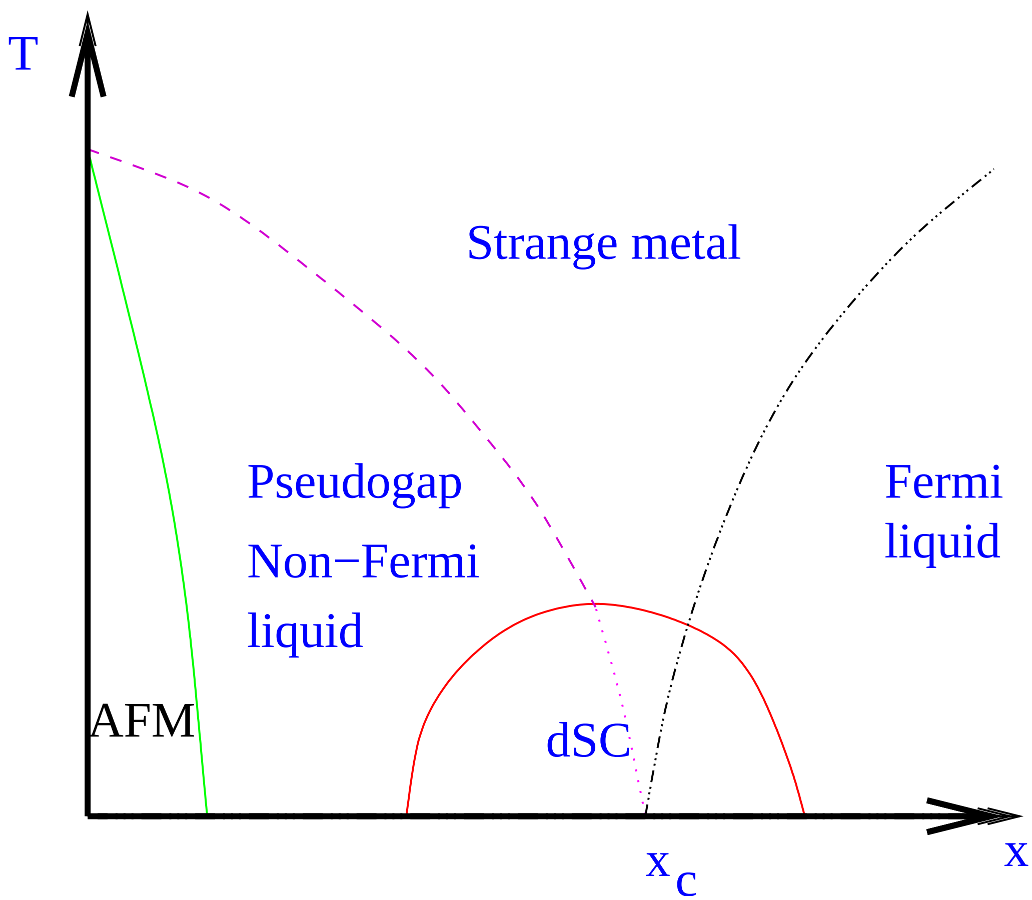
1. Ginzburg Landau gauge theory of strongly correlated fermions in $2 + 1$ dimensions
2. Kosterlitz-Thouless transition in $d = 2$
3. Matter-coupled gauge fields and Kosterlitz-Thouless transition in $d = 2 + 1$

Co-workers: Flavio S. Nogueira and Hagen Kleinert, Freie Universität Berlin

Phase diagram of high-T_c cuprates



The Hubbard- and $t-J$ model

Strongly correlated lattice electrons:

Hubbard model

$$H = -t \sum_{i,j,\sigma} c^{+}_{i,\sigma} c_{j,\sigma} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow}$$

$U \to \infty$: $t-J$ model

$$H = -t \sum_{i,j,\sigma} c^{+}_{i,\sigma} c_{j,\sigma} + J \sum_{i} \mathbf{S}_i \cdot \mathbf{S}_j$$

+ no-double occupancy constraint!

$c^{+}_{i,\sigma}$: creates *projected electron*. NOT a fermion!

Express *projected* electrons as composite particle:

$$\begin{aligned} c^{\dagger}_{i,\sigma} &= b_i\ f^{\dagger}_{i,\sigma}; \quad c_{i,\sigma} = b^{\dagger}_i\ f_{i,\sigma} \\ 1 &= b^{+}_i\ b_i + \sum_{\sigma} f^{\dagger}_{i,\sigma} f_{i,\sigma} \end{aligned}$$

b-fields: Spinless bosons.

f-fields: Chargeless fermions.

Gauge theory of $t - J$ model:I

$$
\begin{aligned}
H &= -t \sum_{<i,j>,\sigma} b_i^\dagger b_j f_{i,\sigma} f_{j,\sigma}^+ \\
&- \frac{J}{2} \sum_{<i,j>} \sum_{\sigma} f_{j,\sigma}^+ f_{i,\sigma} \sum_{\sigma'} f_{i,\sigma'}^+ f_{j,\sigma'} \\
Q_i &\equiv b_i^+ \, b_i + \sum_{\sigma} f_{i,\sigma}^\dagger f_{i,\sigma} = 1
\end{aligned}
$$

Enforce constraints by fluctuating Gauge Fields.

Strong correlations $\Rightarrow$ emergent gauge fields

$$
\int_{-\pi}^{\pi} \frac{d\lambda_i}{2\pi} e^{i\lambda_i (Q_i - 1)} = \delta_{Q_i - 1, 0}
$$

Decouple f^4-term by using

$$
\begin{aligned}
\chi_{ij} &= e^{i a_{ij}} \\
< \chi_{ij} > &= < \sum_{\sigma} f_{i,\sigma}^\dagger f_{j,\sigma} + \frac{8t}{3J} b_j^\dagger b_i >
\end{aligned}
$$

Gauge theory of $t - J$ model:II

Partition function

$$
\begin{aligned}
Z &= \int D[b^\dagger]D[b]D[f^\dagger]D[f]D[\lambda]D[\chi] \;\; e^{[S]} \\
S &= \int_0^\beta d\tau[-\sum_i b_i^\dagger \frac{\partial b_i}{\partial \tau} - \sum_{i,\sigma} f_{i,\sigma}^\dagger \frac{\partial f_{i,\sigma}}{\partial \tau} - H] \\
H &= \frac{3J}{8} \sum_{<i,j>} |\chi_{ij}|^2 \\
&- \sum_{<i,j>} [\chi_{ij}(\frac{3J}{8}\sum_\sigma f_{j,\sigma}^\dagger f_{i,\sigma} + t b_j^\dagger b_i) + h.c.] \\
&+ \frac{8t^2}{3J} \sum_{<i,j>} b_j^\dagger b_i b_i^\dagger b_j + i\lambda_i (Q_i - 1)
\end{aligned}
$$

Frozen gauge fields $\Rightarrow$ b- and f-dynamics is independent! Spin-charge separation.
Wild fluctuations in gauge-fields $\Rightarrow$ b- and f-dynamics not independent Spin-charge confinement.

Spin-charge separation and confinement-deconfinement transition

- Matter fields

$$b_i = e^{i\theta_i}$$

They have topological defects: Vortex loops!

- Gauge-fields are angular variables

$$\lambda_i = e^{i\phi_i}; \chi_{ij} = e^{ia_{ij}}$$

They have topological defects: Monopoles!

- Monopole-proliferation: Confined phase. Monopoles bound in dipoles: Deconfined phase.
- Confinement phase $\Leftrightarrow$ Electrons intact
- Deconfined phase $\Leftrightarrow$ Electrons splintered
- Enough to study dynamics of gauge fields!! f-fields can be integrated out exactly.

Gauge theory of $t - J$ model:III

Effective gauge-theory for strongly interacting lattice fermions: ϕ_i, a_{ij} combined into one gauge-field A

$$
\begin{aligned}
Z &= \int_{-\pi}^{\pi} \left[\prod_{j=1}^{N} \frac{d\theta_j}{2\pi} \right] \int_{-\pi}^{\pi} \left[\prod_{j,\mu} \frac{dA_{j\mu}}{2\pi} \right] \exp\left[S\right] \\
S &= \beta \sum_{j,\mu} [1 - \cos(\Phi_{\mu j})] + \kappa \sum_{\mathrm{P}} [1 - \cos(\mathcal{A}_{\mu j})]
\end{aligned}
$$

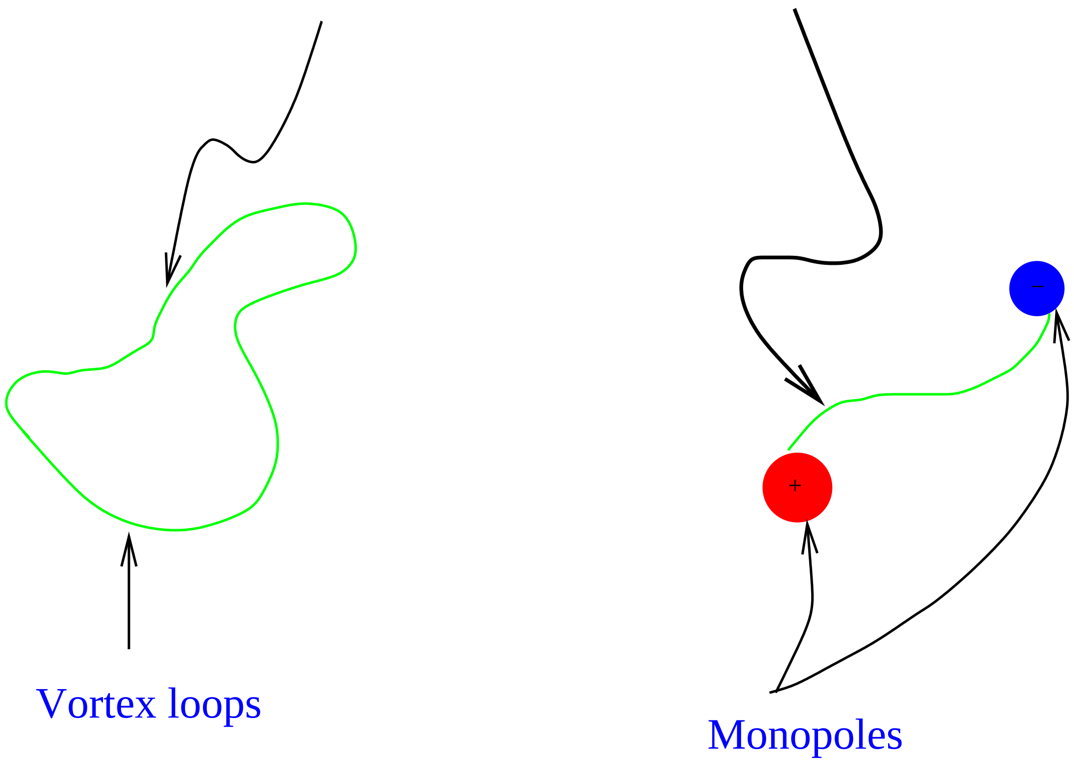
N is number of lattice sites, $\sum_{\mathrm{P}}$ runs over the plaquettes of the lattice,

$$
\begin{aligned}
\Phi_{\mu j} &\equiv \Delta_\mu \theta_j - q A_{j\mu} \\
\mathcal{A}_{\mu j} &\equiv \varepsilon_{\mu\nu\lambda} \Delta_\nu A_{j\lambda}.
\end{aligned}
$$

Compact Higgs model

in $d = 2 + 1$

$$S = \beta \sum_{j,\mu} [1 - \cos(\Phi_{\mu j})] + \kappa \sum_{\mathrm{P}} [1 - \cos(\mathcal{A}_{\mu j})]$$

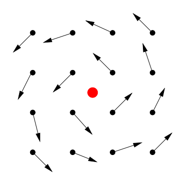


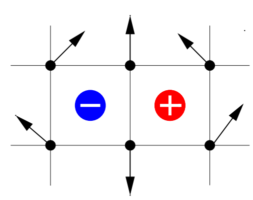
Kosterlitz-Thouless in $d = 2$: I

$2D$ planar spin model (XY model).

$$S = K \sum_{j,\mu} [1 - \cos(\Delta_\mu \theta_j)]$$

HMW theorem: No LRO, $T > 0$!





Vortices: Topological defects causing phase transition. Interact logarithmically in $d = 2$ $\leftrightarrow$ 2-component Coulomb-plasma in $d = 2$, $q_i = \pm 1$

$$S = -2\pi K \left[\sum_{i,j} q_i q_j \ln(|r_i - r_j|) + \sum_i q_i^2 \ln\left(\frac{r_0}{a}\right) \right]$$

Kosterlitz-Thouless in $d = 2$: II

Bare stiffness: K

Bare fugacity: $\zeta = e^{-2\beta\pi J \ln(r_0/a)}$

K and ζ are renormalized

$$K \to K(l); \zeta \to y(l); l \equiv \ln(r/r_0)$$

Renormalization group equations for $K(l)$, $y(l)$ (J. M. Kosterlitz and D. J. Thouless, (1973)):

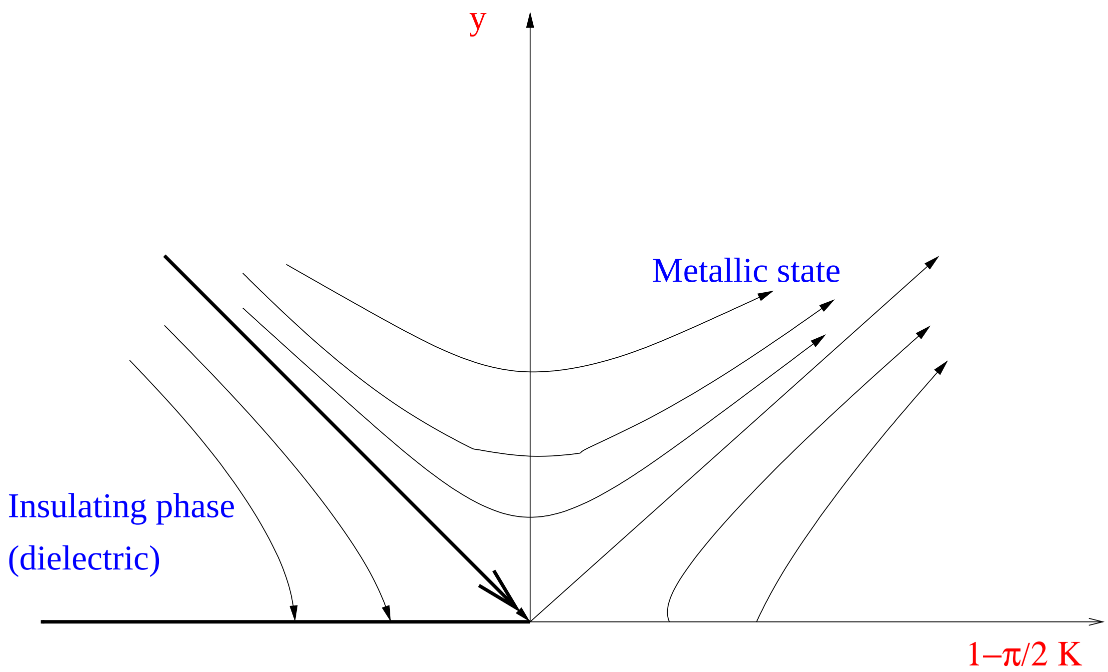
$$\frac{dK^{-1}(l)}{dl} = y^2$$
$$\frac{dy(l)}{dl} = [2 - \pi K(l)]\, y(l)$$

Fixed-point solution: $y^* = 0, K^* = \frac{2}{\pi}$

- In $d = 2$, Coulomb-gas at low T is an insulator (dielectric)
- At high T it is a plasma (metallic).
- Due to logarithmic interaction in $d = 2$ the system undergoes a metal-insulator transition

Flowdiagram for $y(l), K(l)$

in $d = 2$ Coulomb gas



Low-temperature phase: $2 - \pi K(l) < 0$

$\lim_{l\to\infty} y(l) \to 0$

$\lim_{l\to\infty} K(l) \to \frac{2}{\pi}$

High-temperature phase: $2 - \pi K(l) > 0$

$\lim_{l\to\infty} y(l) \to \infty$

$\lim_{l\to\infty} K(l) \to 0$

Kosterlitz-Thouless in d dimensions: I

Coulomb potential in d-dimensions:

$$V(r) = \frac{\Gamma\left(\frac{(d-2)}{2}\right)}{(4\pi)^{d/2}} \left[\left(\frac{|r|}{r_0} \right)^{2-d} - 1 \right]$$

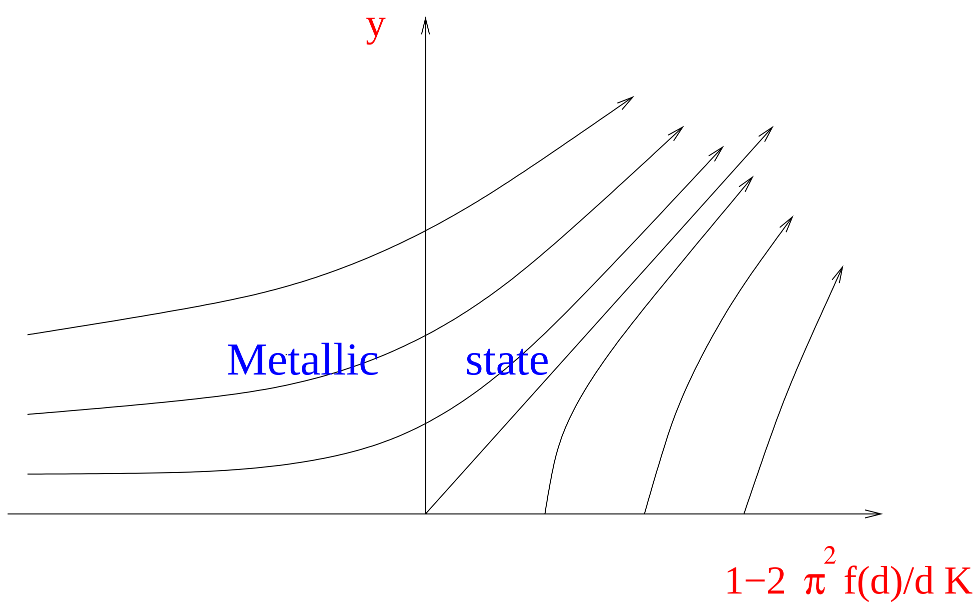
Renormalization group equations for $K(l)$, $y(l)$ [J. M. Kosterlitz (1977), D. R. Nelson (1982)]:

$$\begin{aligned} \frac{dK^{-1}(l)}{dl} &= y^2 - (2-d)K^{-1}(l) \\ \frac{dy(l)}{dl} &= \left[d - 2\pi^2 f(d) K(l) \right] y(l) \\ f(d) &= \frac{(d-2)\Gamma((d-2)/2)}{(4\pi)^{d/2}} \end{aligned}$$

No solution $y^* = 0, K^* \neq 0$ when $d > 2$!!
Coulomb-plasma $d > 2$ is always in the metallic state

Flowdiagram for $y(l), K(l)$

in $d > 2$ Coulomb gas



Low-temperature phase: Does not exist

High-temperature phase: $d - 2\pi^2 f(d) K(l) > -\infty$

$\lim_{l\to\infty} y(l) \to \infty$

$\lim_{l\to\infty} K(l) \to 0$

Corresponds to permanent confinement of Polyakov

Renormalized test charge potential

Gauge-field propagator (interaction between test charges) is renormalized by matter fluctuations

$$< A(x)A(y) >\sim \frac{1}{|x-y|^{d-2+\eta_A}}$$

Coupling between matter- and gauge-fields:

$$\Phi_{\mu j} \quad \equiv \quad \Delta_\mu \theta_j - qA_{j\mu}$$

Gauge-invariance

$$\begin{aligned} q_0^2 A_0^2 &= q^2 A^2; \quad A_0^2 = Z_A\ A^2 \\ q_0^2 &= (q_0^0)^2 l^{d-4} \\ \frac{\partial q^2}{\partial \ln l} &= \left[d - 4 + \underbrace{\frac{\partial \ln(Z_A)}{\partial \ln l}}_{\equiv \eta_A} \right] q^2 \end{aligned}$$

Charged fixed point $q^* \neq 0$: $\eta_A = 4 - d$! Charge FP exists! (J. Hove, A.S., PRL, **84**, 3426 (2000).)

η_A and monopole potential

Test charge potential:

$$V(R) \sim \frac{1}{R^{d-2+\eta_A}} = \frac{q^2(R)}{R^{d-2}}$$
$$q^2(R) \sim R^{-\eta_A}$$

Dirac condition:

$$q_m(R)\; q(R) \sim 1$$

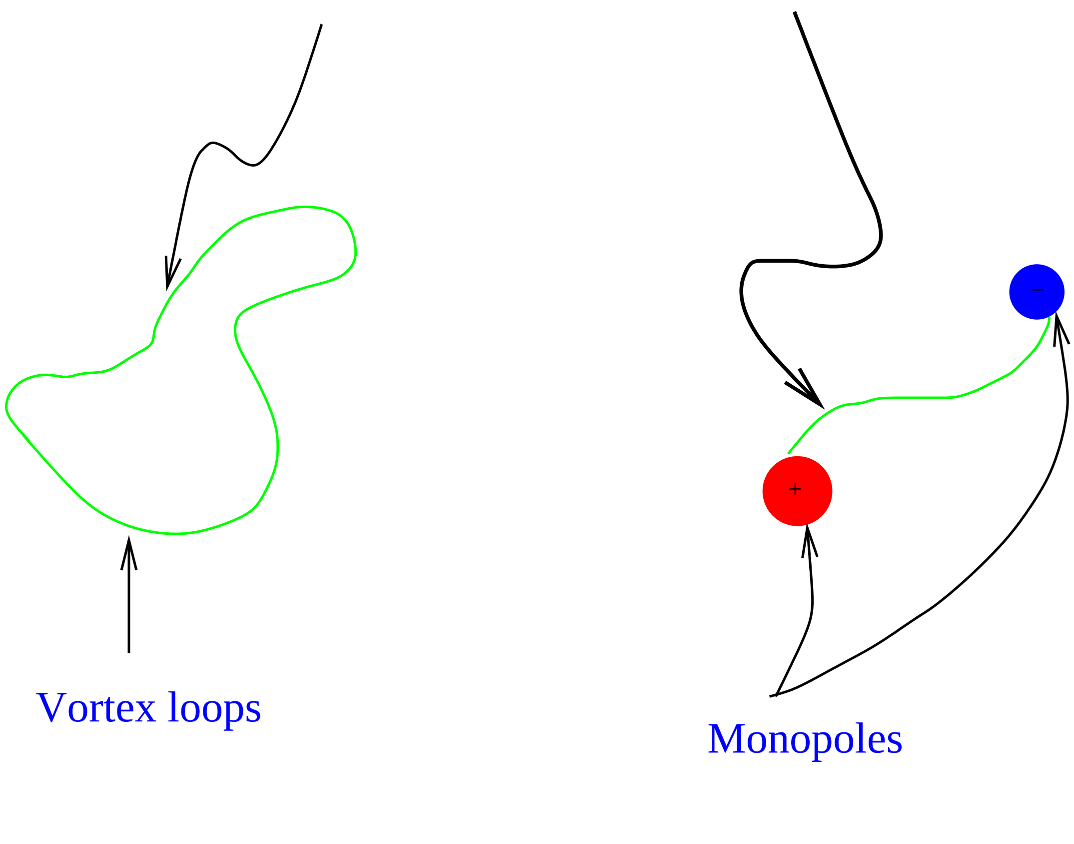
Monopole potential:

$$V_m(R) \sim \frac{q_m^2(R)}{R^{d-2}} = \frac{1}{R^{d-2-\eta_A}}$$

Compact Higgs model

in $d = 2 + 1$

$$S = \beta \sum_{j,\mu} [1 - \cos(\Phi_{\mu j})] + \kappa \sum_{\mathrm{P}} [1 - \cos(\mathcal{A}_{\mu j})]$$



Kosterlitz-Thouless in d dimensions: II

Generalized potential in d-dimensions [H. K., F. S. N., A. S, PRL, **88**, 232001 (2002); Nucl. Phys. **B 666**, 361 (2003)]:

$$V(r) = \frac{2^{-\eta_A}\Gamma\left(\frac{(d-2-\eta_A)}{2}\right)}{(4\pi)^{d/2}\Gamma((2+\eta_A)/2)}\left[\left(\frac{|r|}{r_0}\right)^{2-d+\eta_A} - 1\right]$$

RG eqs. for $K(l)$, $y(l)$ (hep-th/0209132)

$$\begin{aligned}
\frac{dK^{-1}(l)}{dl} &= y^2 - (2-d+\eta_A)K^{-1}(l) \\
\frac{dy(l)}{dl} &= \left[d - \eta_y - 2\pi^2\tilde{f}(d)K(l)\right]y(l) \\
\tilde{f}(d) &= \frac{(d-2+\eta_A)\Gamma((d-2+\eta_A)/2)}{2^{\eta_A}(4\pi)^{d/2}\Gamma((2+\eta_A)/2)}
\end{aligned}$$

Solution $y^* = 0, K^* = \frac{d-\eta_y}{2\pi^2\tilde{f}(d)}$ if $2-d+\eta_A = 0$

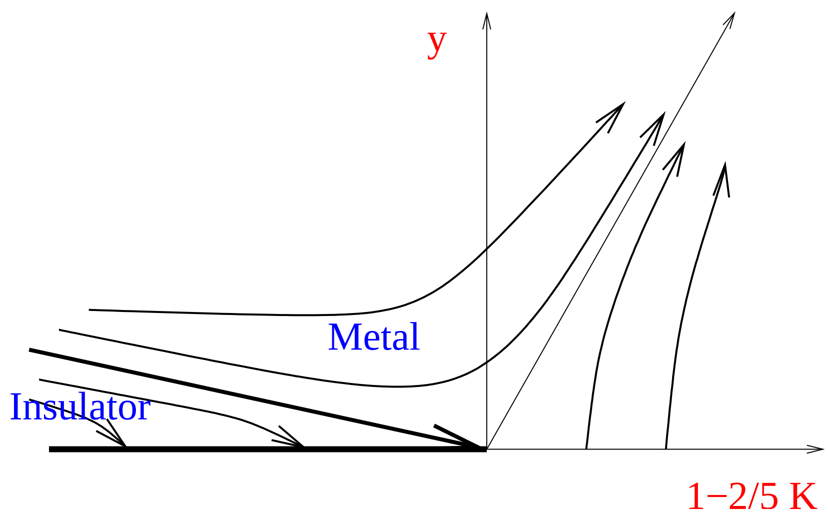
$\eta_y = \eta_A/2$

$\eta_A = 4-d$ [J. Hove, A.S., PRL, **84**, 3426 (2000)]

$3d$ ln-plasma has KT metal-insulator transition!

Flowdiagram for $y(l), K(l)$

in $d = 3$ ln-plasma. $\eta_A = 1$!!



Low-temperature phase: $\frac{5}{2} - K(l) < 0$

$\lim_{l \to \infty} y(l) \to 0$

$\lim_{l \to \infty} K(l) \to \frac{5}{2}$

High-temperature phase: $\frac{5}{2} - K(l) > 0$

$\lim_{l \to \infty} y(l) \to \infty$

$\lim_{l \to \infty} K(l) \to 0$

Corresponds to destruction of Polyakov permanent confinement

Summary

- Pure gauge theories with compact $U(1)$ gauge fields. Permanently confined in $d = 2 + 1$.
- Matter fields in fundamental representation *do* matter: Destroy permanent confinement $\Rightarrow$ Confinement-deconfinement transition.
- Possible application: $(2 + 1)$-dimensional Compact Higgs model = effective field theory of spin-charge separation in strongly correlated fermion systems in $d = 2$. (H. Kleinert, F. S. Nogueira, A. Sudbø, Phys. Rev. Lett., **88**,232001, (2002); Nucl. Phys. **B 666**, 361, 2003). See also `hep-th/0303265`, `cond-mat/0311524`.