Helsinki University of Technology
Low Temperature Laboratory

HUT LTL

# OPTICAL STUDIES ON THE SHAPE OF $^3$He CRYSTALS

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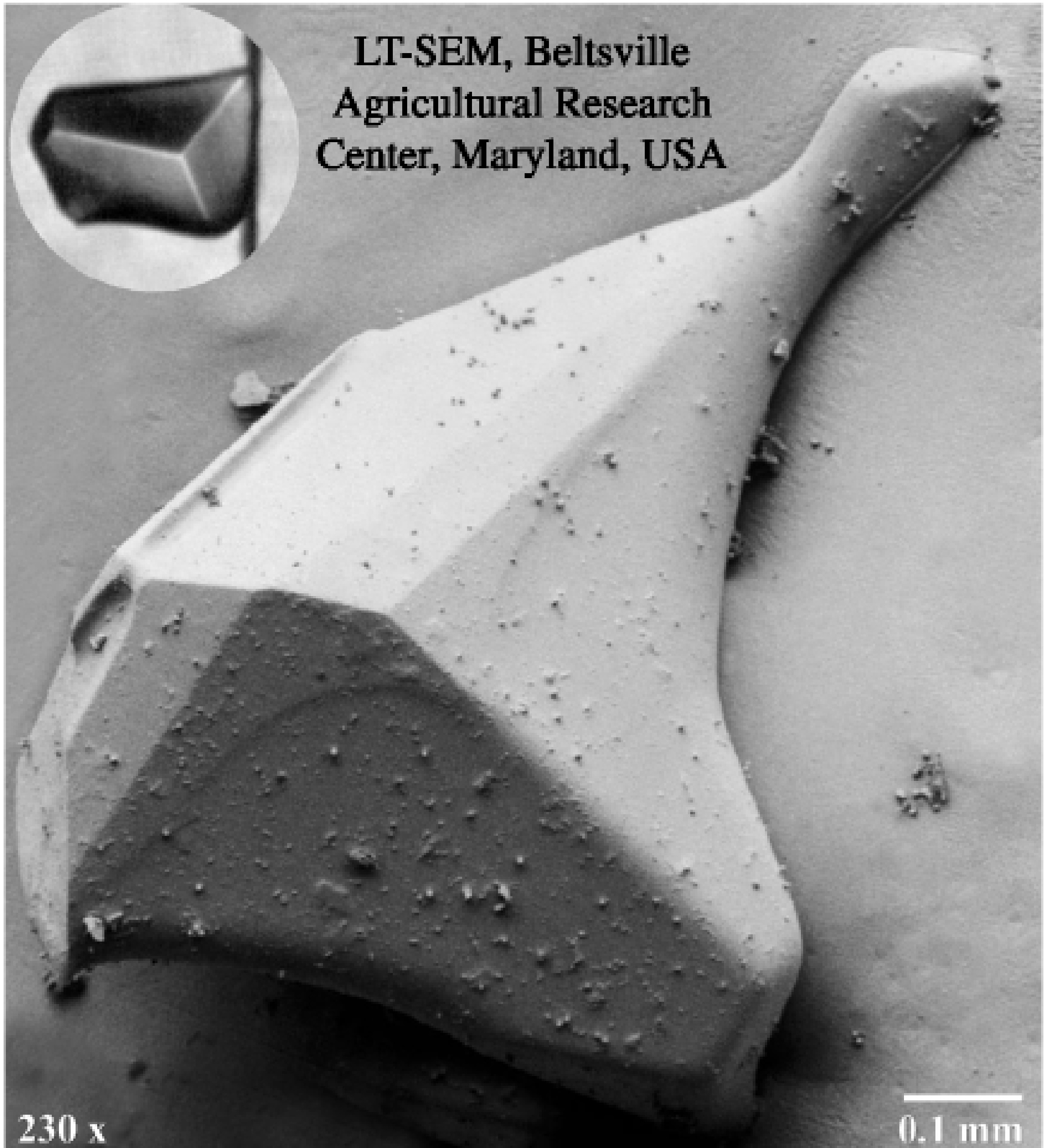
LT-SEM, Beltsville
Agricultural Research
Center, Maryland, USA
230 x
0.1 mm

# Why are there facets on the crystal surface ?

- FACETS are present on the crystal surface due to the anisotropy of the surface tension $\gamma$ which is provided by the underlying lattice

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## What is roughening transition ?

- **Burton, Cabrera and Frank** (1949, 1951); the crystal surface should undergo a transition from the faceted (smooth) to the rough (rounded) state, the facets should disappear from the crystal surface at the so called ROUGHENING TRANSITION; they used the 2D Ising model to predict the transition temperature $T_R$; later it was shown that their model is only a rough approximation and the critical behaviors near $T_R$ are actually different

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- **Fisher and Weeks** (1983) have found the so called universal relation between $T_R$ and $\gamma$ :

$$k_B T_R = \frac{2}{\pi} \gamma d^2 \quad ,$$

where $d$ is the crystal periodicity along the corresponding orientation

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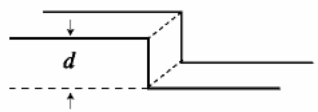
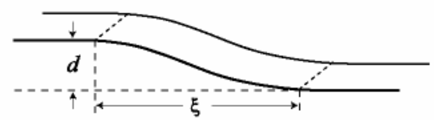
- **Nozières and Gallet** (1987) developed further the renormalization group theory of roughening; the roughening transition is known to belong to the Kosterlitz-Thouless class of transitions which are of "infinite order"

# What is an elementary step on a facet ?

| | Strong coupling | Weak coupling |
|---|---|---|
| Profile of an ELEMENTARY STEP at $T = 0$ |   |   |
| Step energy | $\beta_0 \sim \gamma_0 d$ <br> for well-separated steps, i.e., for higher order facets with large Miller indices, with elastic interaction between steps <br> $\beta_0 \propto d^4$ | $\beta_0 << \gamma_0 d$ |

# What is an elementary step on a facet ?

Profile of an
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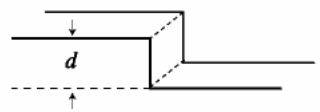
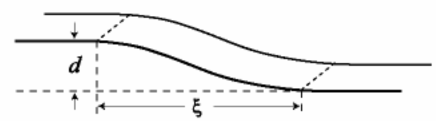
Strong coupling




Weak coupling




Step energy

$\beta_0 \sim \gamma_0 d$

for well-separated steps, i.e., for higher order facets with large Miller indices, with elastic interaction between steps

$\beta_0 \propto d^4$

$\beta_0 << \gamma_0 d$

# How to locate the roughening transition temperature ?

The step free energy β decreases exponentially to zero as $T$ approaches $T_R$ from below; above $T_R$, $\beta = 0$

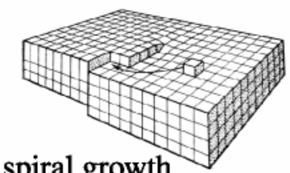
The only practical way to locate $T_R$ is to measure $\beta(T)$ and extrapolate it to zero !

$$\beta = \frac{4d}{\pi}\sqrt{\gamma V} \; ,$$

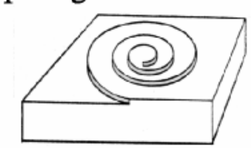
where V is the pinning potential of the crystal lattice

# How facets grow - spiral growth of facets

- Rough surfaces grow easily and their growth rate is proportional to the applied overpressure, $v \propto \delta p$



spiral growth



- The facets have to grow layer by layer and a continuous growth is possible in the presence of continuous sources of the elementary steps like screw dislocations, this is a SPIRAL GROWTH; the screw dislocations appear when dislocations terminate on the crystal surface

- The growth velocity of a facet $v$ (on a $^3$He crystal):

$$v = \frac{\mu d^3}{19\,\beta} \frac{(\rho_s - \rho_1)^2}{\rho_1^2} (\delta p)^2 ,$$

where $\mu$ is the step mobility defined as

$$\mu = \frac{v_{step}}{d} \frac{\rho_1}{(\rho_s - \rho_1)} \frac{1}{\delta p}$$

- When the step velocity $v_{step}$ reaches some critical velocity $v_{cr}$, the growth velocity of a facet $v$ becomes linear with respect to the applied overpressure $\delta p$ :

$$v = \frac{v_{cr}\, d^2}{2\pi\,\beta} \frac{(\rho_s - \rho_1)}{\rho_1} \delta p$$

# Faceting of $^3$He crystals

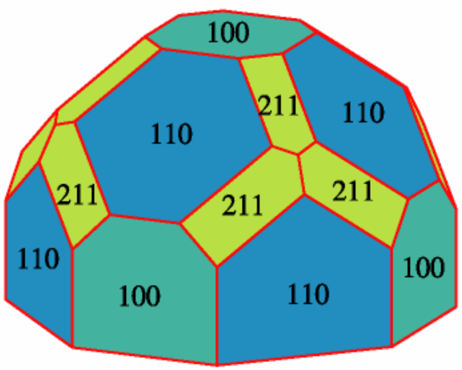
- Helium crystals are good model systems because the faceting can be studied along their melting curve over a wide range of temperature, down to $T = 0$, in principle

- Before the experiments with $^3$He crystals in LTL only three different types of facets had been observed in $^3$He (as in $^4$He) :

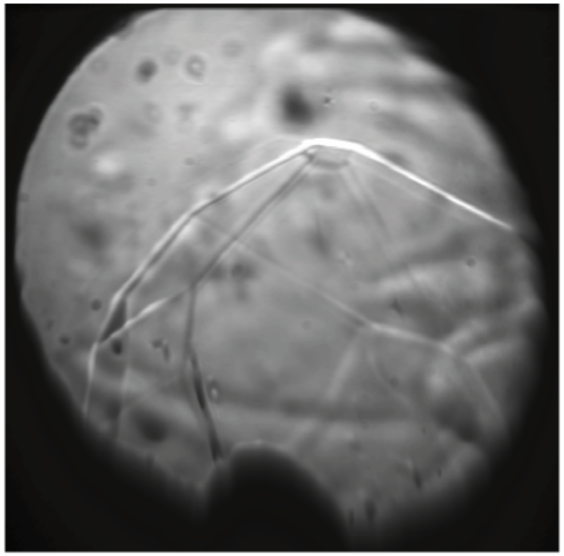
$d_{110} = 0.307$ nm

$d_{100} = 0.217$ nm

$d_{211} = 0.177$ nm



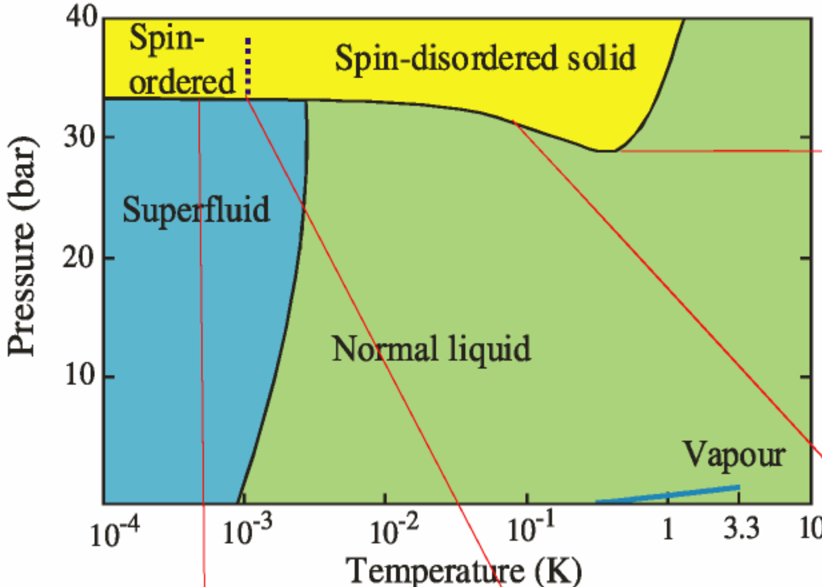

Computer-generated shape of a body-centered cubic (bcc) crystal with three types of facets

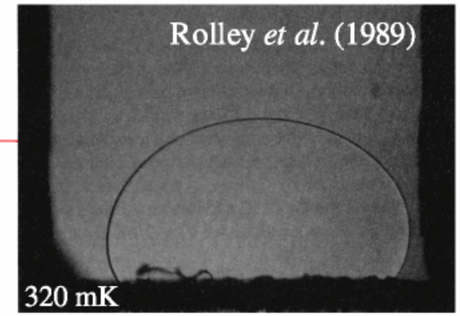


Growing bcc $^3$He crystal at 2.2 mK showing the (110), (100) and (211) types of facets. After Wagner *et al*. (1996).

- The growth rates of individual facets had not been measured and only an average growth rate of $^3$He crystals had been reported

# Our interferometric studies on the growth dynamics of faceted $^3$He crystals




$^3$He crystal in equilibrium, no facets; Rolley *et al.* measured also the surface tension of $^3$He crystals, $\gamma = 0.06$ erg/cm$^2$, with that value one obtains from the universal relation that $T_R$ for the (110) facets is 260 mK !

# I

at the lowest temperature of about 0.5 mK we have reached; for a review, see Tsepelin *et al.*, J. of Low Temp. Phys. **129**, 489-530 (2002)

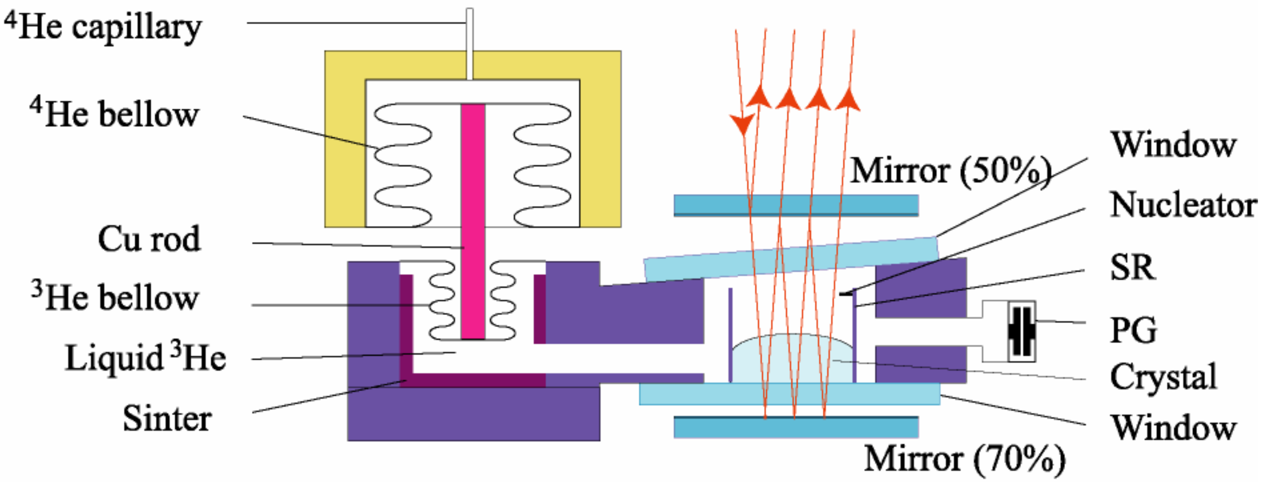
# II

near the magnetic ordering temperature $T_N = 0.93$ mK; talk by I.A. Todoshchenko in QFS 2003, Albuquerque (see also poster session here in Lammi)

# III

near 100 mK, the highest temperature at which the (110) facets have been seen on the bcc $^3$He crystals by Rolley *et al.* (1989); poster presentation by H.J. Junes here in Lammi

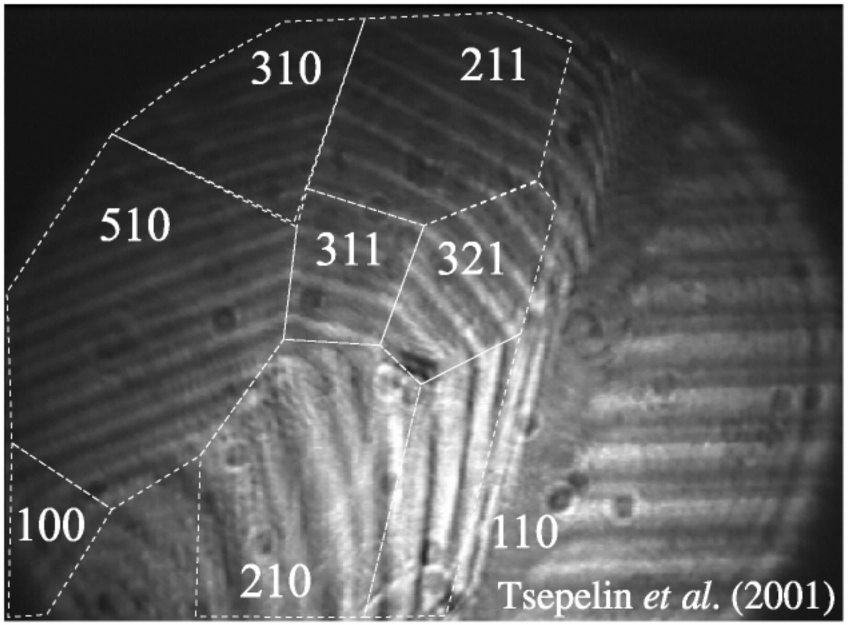
# Our experimental cell and a Fabry-Pérot interferometer



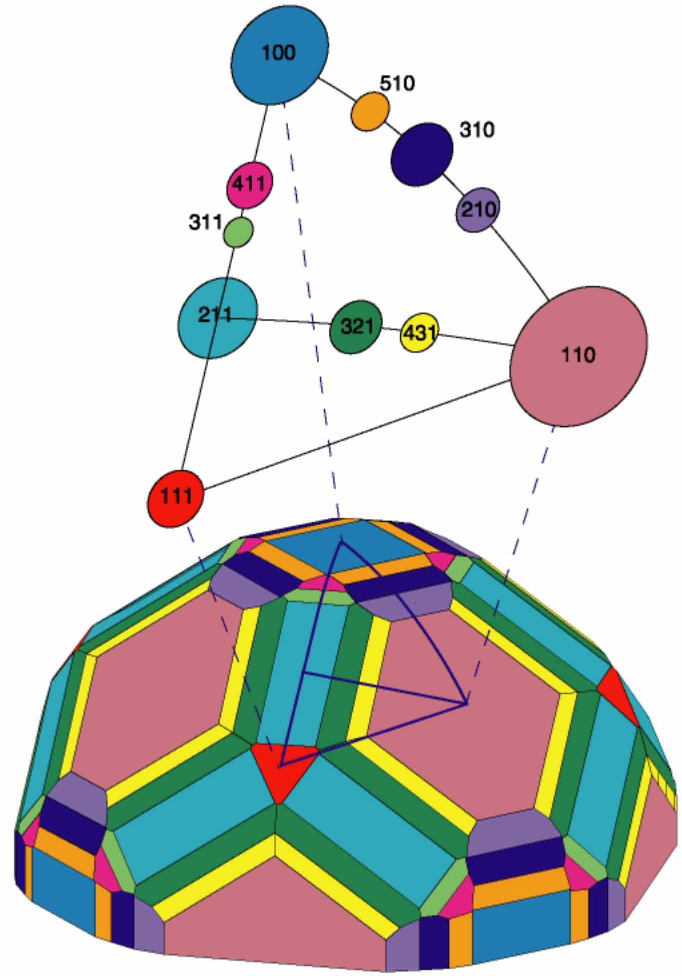

| | |
|---|---|
| Illumination: | from He-Ne laser ($\lambda$ = 632.8 nm) through an optical fiber |
| Imaging of interferograms: | by a slow scan CCD-sensor INSIDE the vacuum can of the nuclear demagnetization cryostat |
| Resolution: | about 5 $\mu$m in vertical and 15 $\mu$m in horizontal direction |

| | |
|---|---|
| Later: | the vibrating wire unit installed additionally for thermometry |

# $^3$He crystals show many different types of facets at 0.5 mK

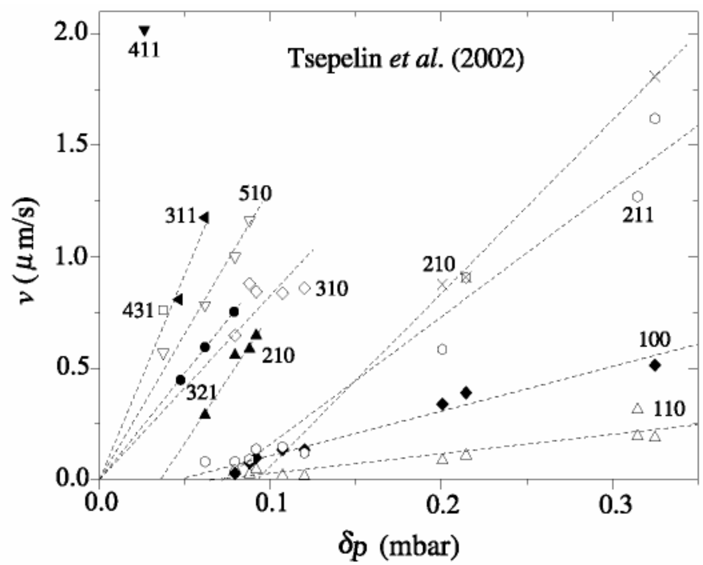


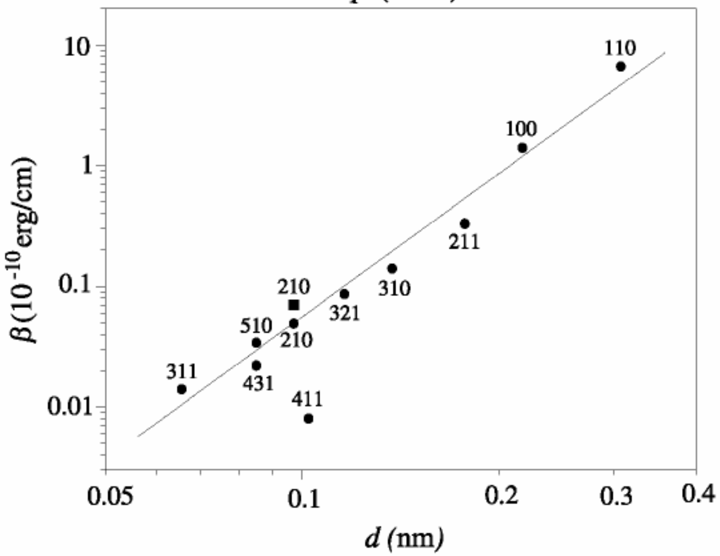

An interferogram of a growing $^3$He crystal at 0.55 mK. Adjacent fringes indicate a change in the crystal thickness by $\lambda/2(n_s - n_1) = 190\ \mu$m; the diameter of the illuminated area is about 8 mm




Computer-generated shape of a bcc crystal with eleven types of facets observed on $^3$He crystals

## Growth sequence of a single $^3$He crystal at 0.55 mK




- the slowest facets are of the (110) type; these facets have the largest step height (or the interplanar distance) in the bcc lattice

- the measured growth velocities of facets $v$ show a linear dependence of the applied overpressure $\delta p$

- spiral growth in the regime of the so called suppressed step mobility

$$v = \frac{v_{cr} d^2}{2\pi \beta} \frac{(\rho_s - \rho_l)}{\rho_l} \delta p$$

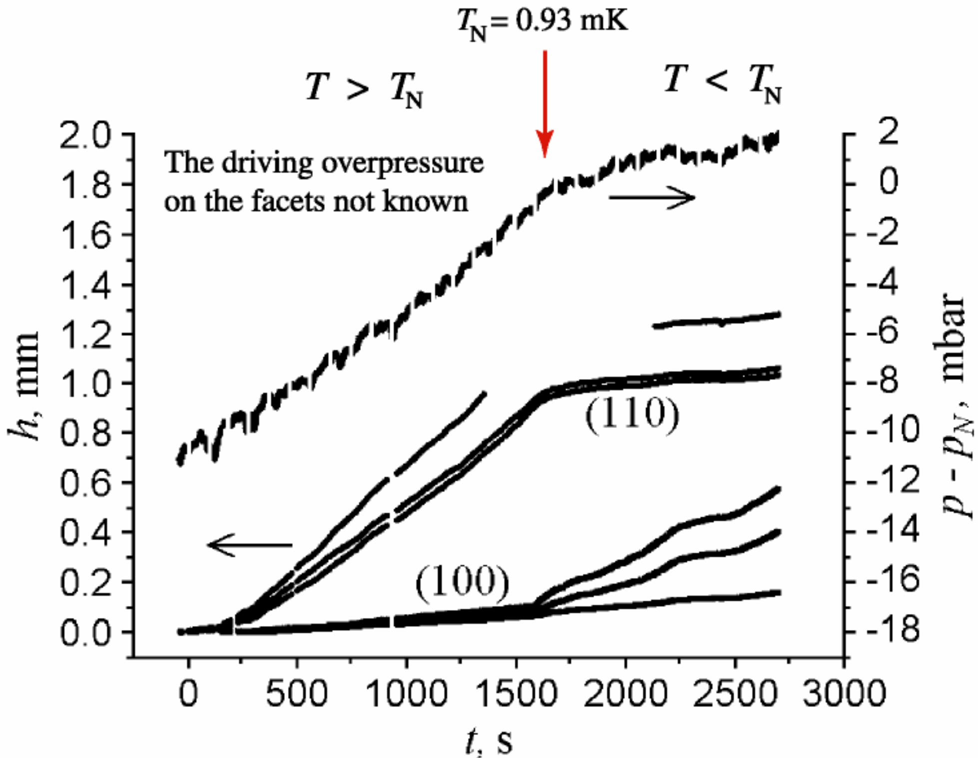
, $v_{cr} \approx 7$ cm/s (both the magnon velocity and the pair-breaking velocity)

- a linear fit to the calculated step free energies β gives a fourth order power dependence $\beta \propto d^4$, as predicted by the strong-coupling model, and

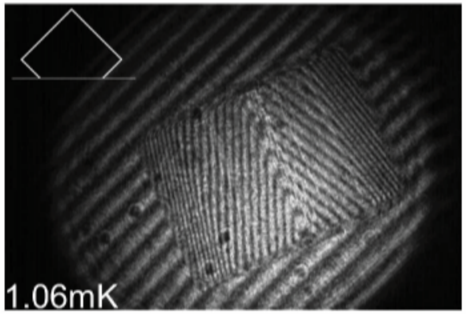
$$\frac{\beta_{110}}{\gamma d_{110}} = 0.36$$

Result: an evidence for a strong coupling of the superfluid/solid interface to the crystal lattice in $^3$He at 0.55 mK

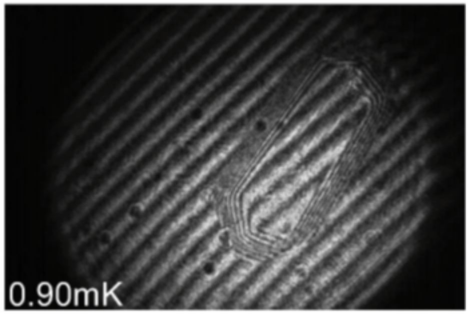
## Cooling the growing $^3$He crystal through $T_N$

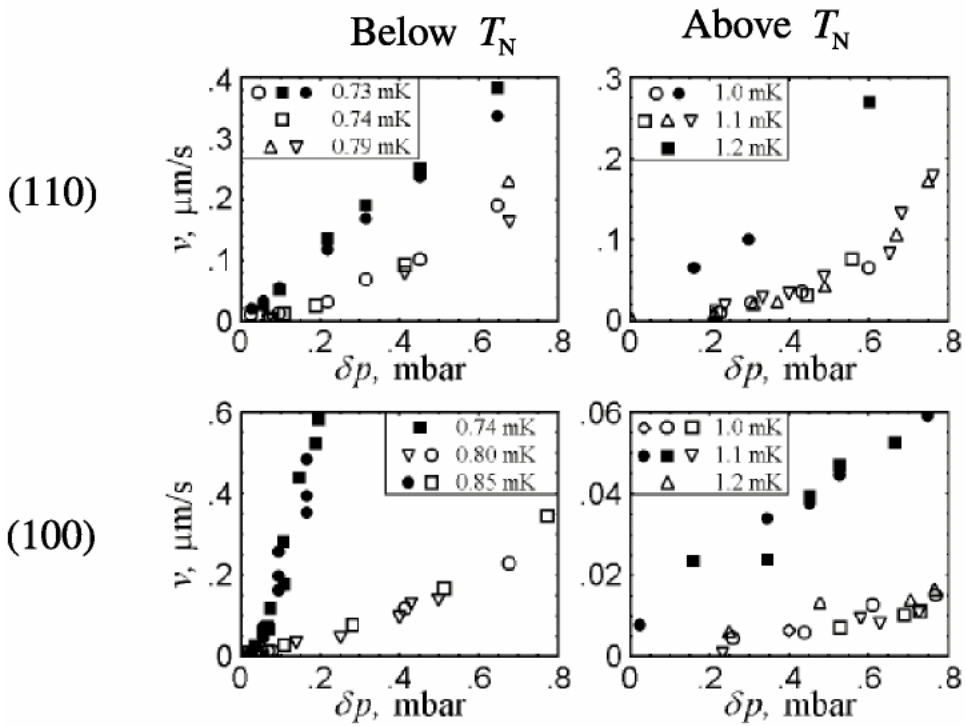



Above $T_N$, in the paramagnetic phase, the slowest facets are of the (100) type !

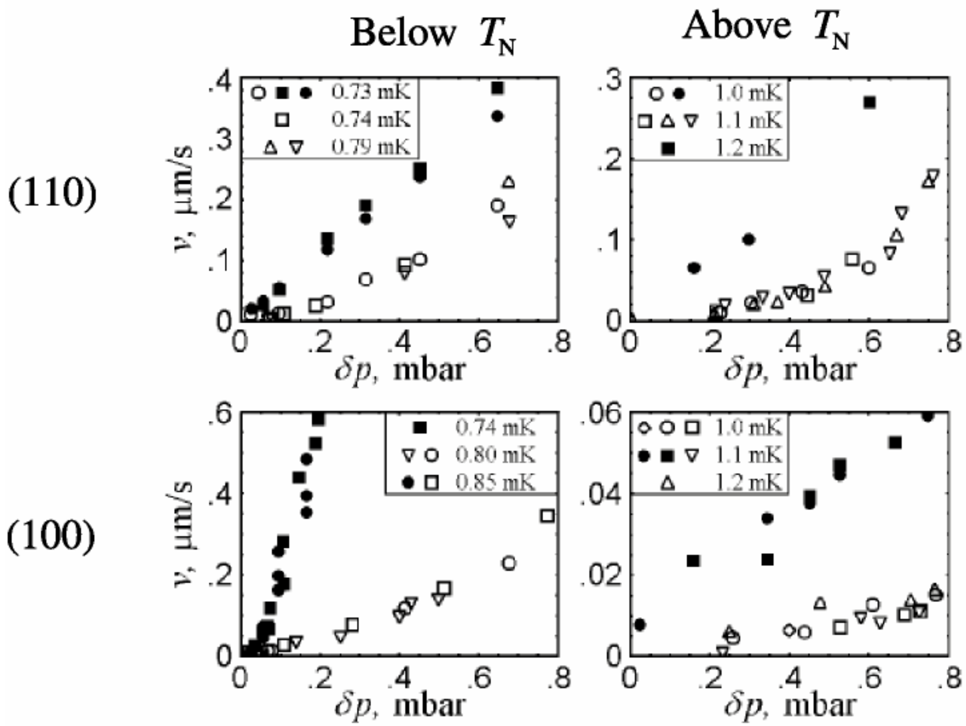



Below $T_N$, the slowest facets are of the (110) type as observed by Tsepelin *et al.*






The overpressure $\delta p$ on the facets obtained using the measured temperature of superfluid $^3$He (by the vibrating wire)

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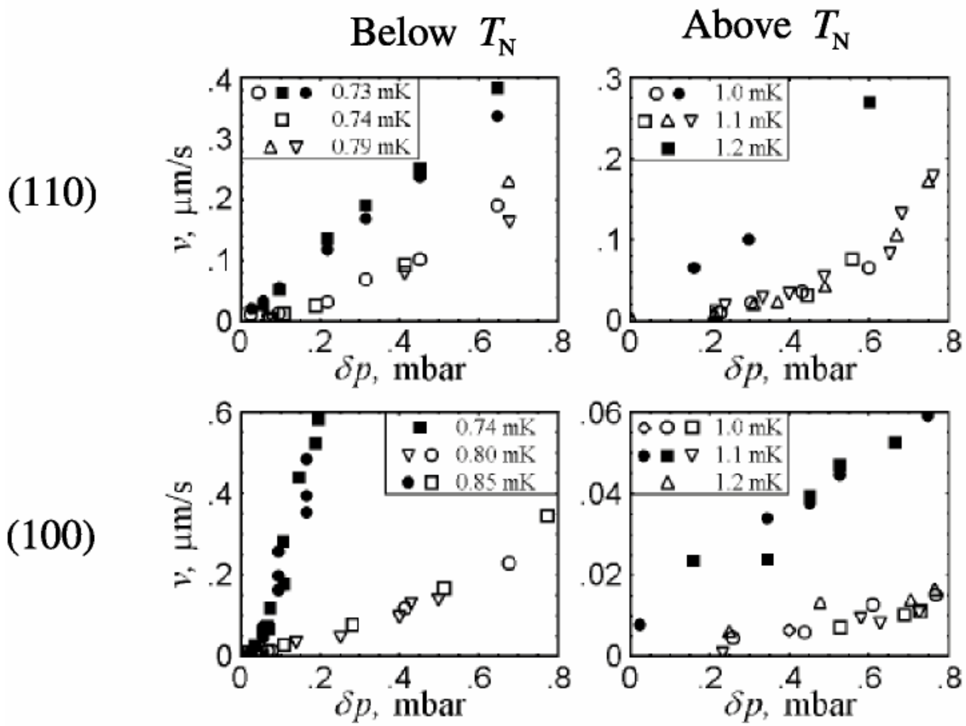
(1) there are two different growth mechanisms of facets, additionally to the spiral growth in the linear regime (observed by Tsepelin *et al.*) a slower growth mode with very small anisotropy of growth in the ordered state was observed

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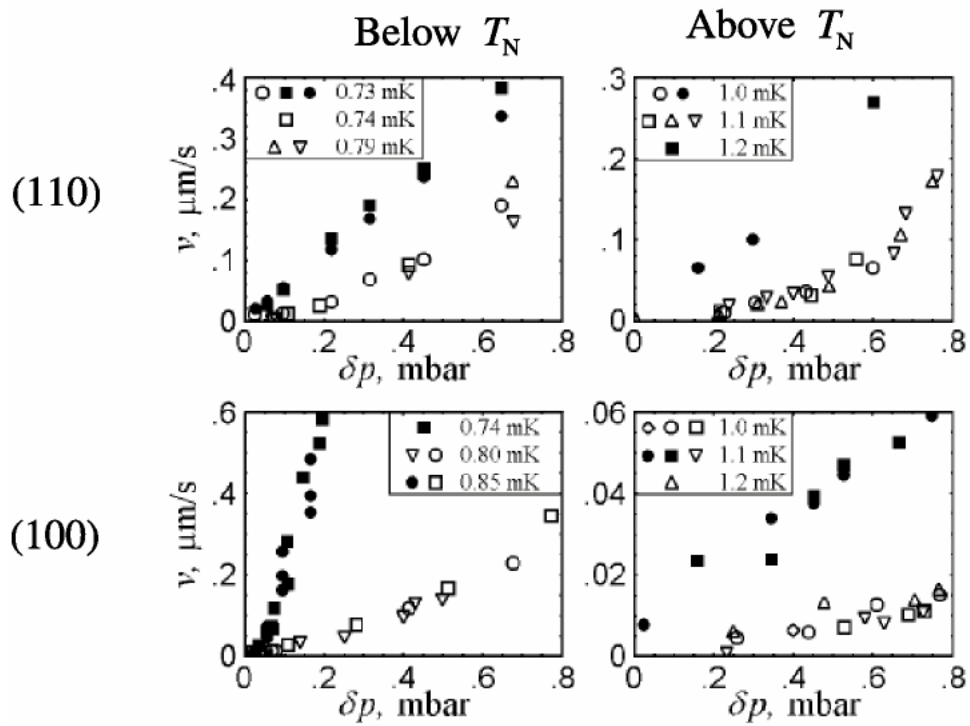
(2) the (110) facets do not "feel" the transition

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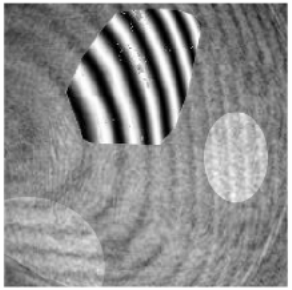
(2) the (110) facets do not "feel" the transition

(3) significant suppression (more than an order of magnitude) of the mobility of the (100) facets ABOVE the transition

# Growth dynamics of $^3$He crstals near 100 mK; the surface of a crystal as a pressure gauge

- To investigate quantitatively the growth dynamics of crystals, accurate measurements of the driving overpressure $\delta p = p_l - p_{eq}(T)$ are needed

- Near 100 mK in $^3$He large latent heat of crystallization and poor thermal conductivity of liquid; the measurements of the temperature of the liquid/solid interface practically impossible



Interferogram of a growing $^3$He crystal at 78.4 mK; the height difference between neighboring fringes is 200 μm; 3.5 x 3.5 mm²

- Fortunately, the crystal surface itself can serve as a very sensitive local (over)pressure gauge

$$\delta p = \text{const} - \rho_l g h - \frac{\rho \gamma K}{\rho_s - \rho_l},$$

where $K$ is the surface curvature and $h$ the crystal height

- The growth of the rough surface is linear $\delta p_R = \frac{v_R}{k}$, where $k$ is the growth coefficient of the rough surface

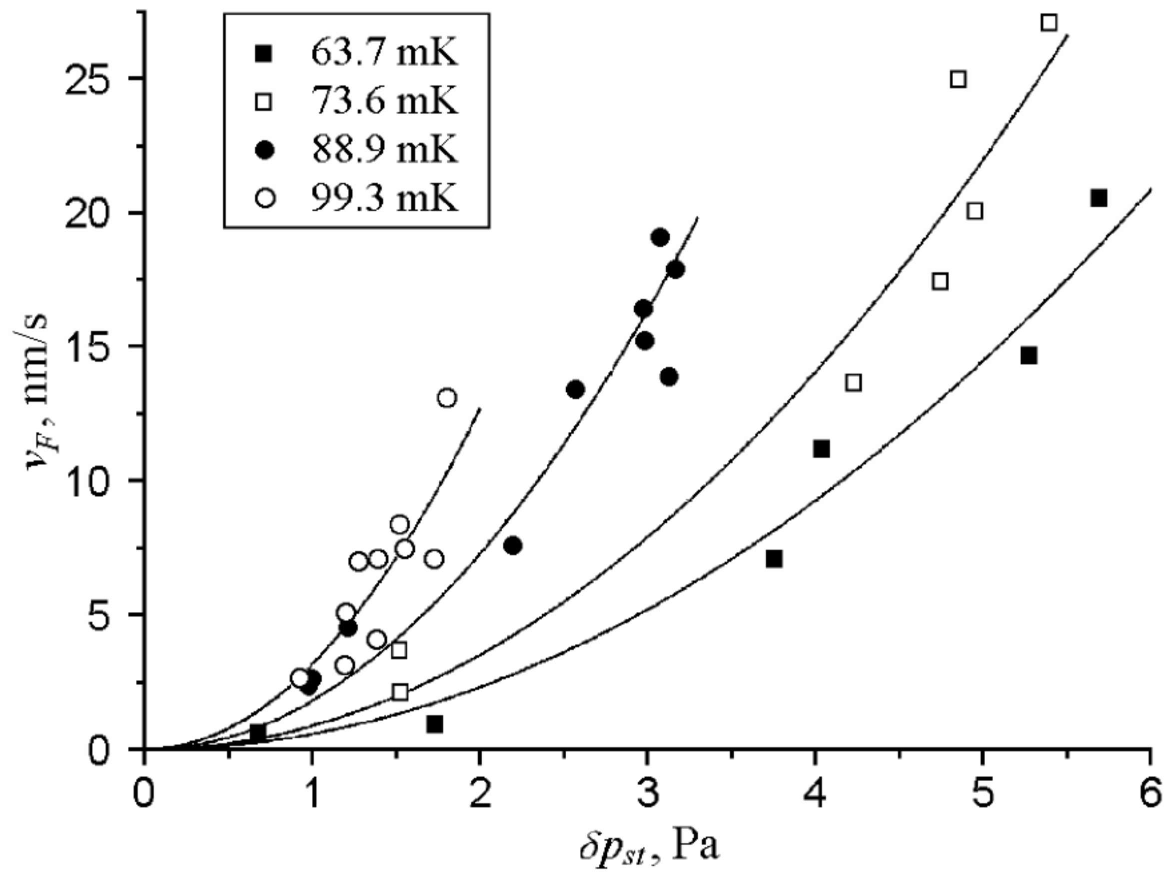
- Smooth facet has an additional growth resistance because the latent heat is released not uniformly but locally on the moving steps $\delta p_F = \frac{v_F}{k} + \frac{v_{st}}{\mu}$, where $\mu$ is the step mobility

- Finally, the effective overpressure $\delta p_{st}$, which drives the step motion is

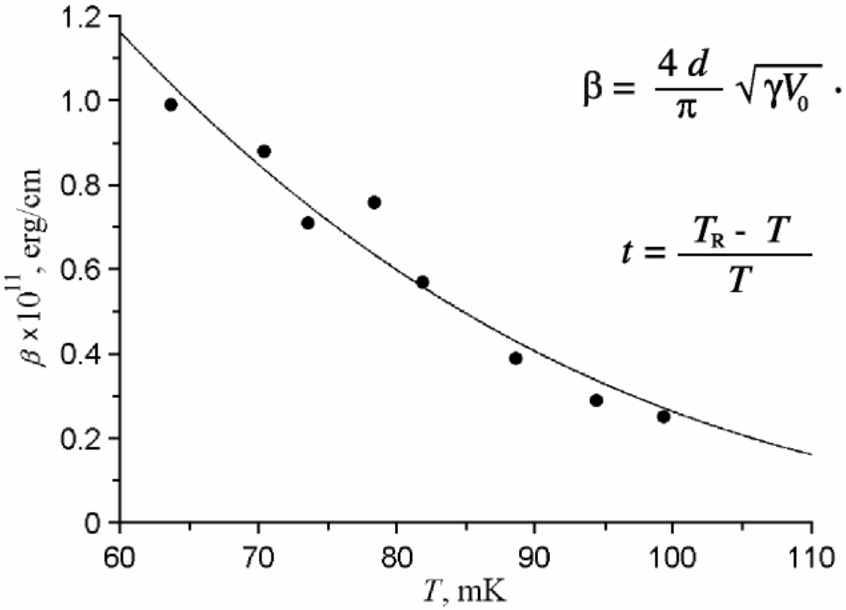
$$\delta p_{st} = \frac{v_{st}}{\mu} = \frac{v_R - v_F}{k} + (h_R - h_F)\rho_l g + \frac{\rho_l \gamma K_R}{\rho_s - \rho_l}$$

and can be found by measuring the heights, curvatures and velocities of both faceted and rough surfaces

Growth velocity $v_F$ of the (110) facets versus the effective overpressure $\delta p_{st}$



## Measured $\beta_{110}\,(T)$ on $^3$He crystals




$$\beta = \frac{4\,d}{\pi}\,\sqrt{\gamma V_0}\cdot\left\{\frac{8\pi\,V_0}{\Lambda_0^2\,T_R}\left(e^{\frac{T_R}{2T}}-1\right)\right\}^{\frac{1}{2\,t}}$$

$$t = \frac{T_R - T}{T}$$

$T_R$ = 260 mK (fixed)

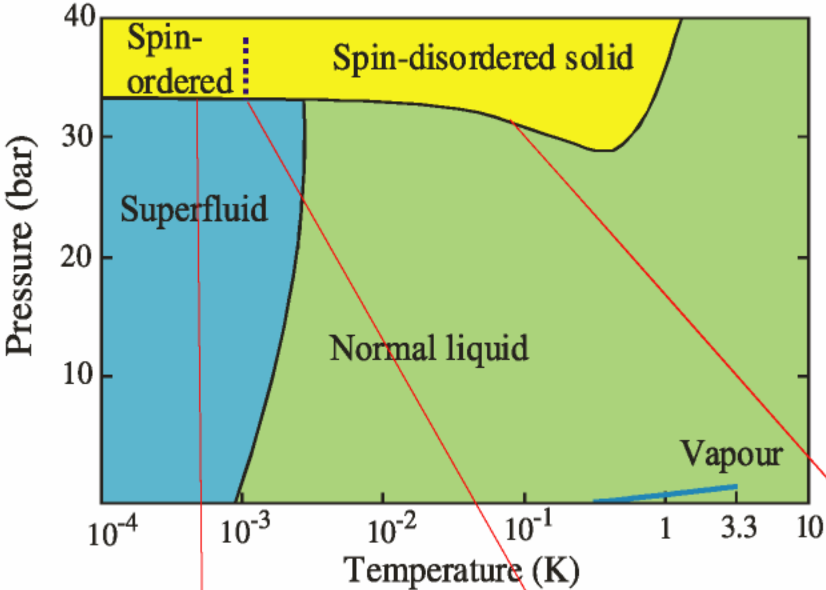
Result of a fit:

$\Lambda_0 = 11/d$

$V_0 = 1.3\cdot 10^{-5}\,\mathrm{erg/cm^2} = \dfrac{\gamma}{5000}$ !

Such a small pinning energy suggests for $\beta(T\rightarrow 0) = 4\cdot 10^{-11}$ erg/cm which is by an order of magnitude smaller than the value $\beta = 6.6\cdot 10^{-10}$ erg/cm obtained at 0.55 mK by Tsepelin *et al.*

# Summary




Weak versus strong coupling in $^3He$

(1) Does something happen at $T_N$ or/and at $T_C$ ?

or

(2) Are the step energy values overestimated at low temperatures ?

**I**

$^3He$ crystals show many different types of facets; an evidence for a STRONG coupling of the super-fluid/solid interface of $^3He$ to the crystal lattice

**II**

The nuclear ordering transition in a bulk solid $^3He$ does have an effect on the growth dynamics of facets; the supression of the mobility of the (100) facets above (!?) the transition, in the paramagnetic phase

**III**

The observed critical behavior of the step free energy of the (110) facets far from the roughening transition temperature, down to $T_R/4$, suggests that the coupling of the liquid/solid interface to the crystal lattice is uniquely WEAK in $^3He$