Cosmic problems for condensed matter experiment

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Cosmic problems

Theory with some experiment:

- Outcomes of phase transitions (QCD, electroweak, GUTs, ...)
  - Defect formation
  - Anomalous production of baryon number

Theory with no experiment:

- Quantum processes in the presence of horizons
  - Generation of inflationary fluctuations (de Sitter horizon)
  - Hawking radiation (Schwarzschild horizon)
- Novel field theoretic phenomena *e.g.* duality

Underlying philosophy

Fundamental **processes** that are common to cosmology/particle physics and condensed matter can be tested in the lab.

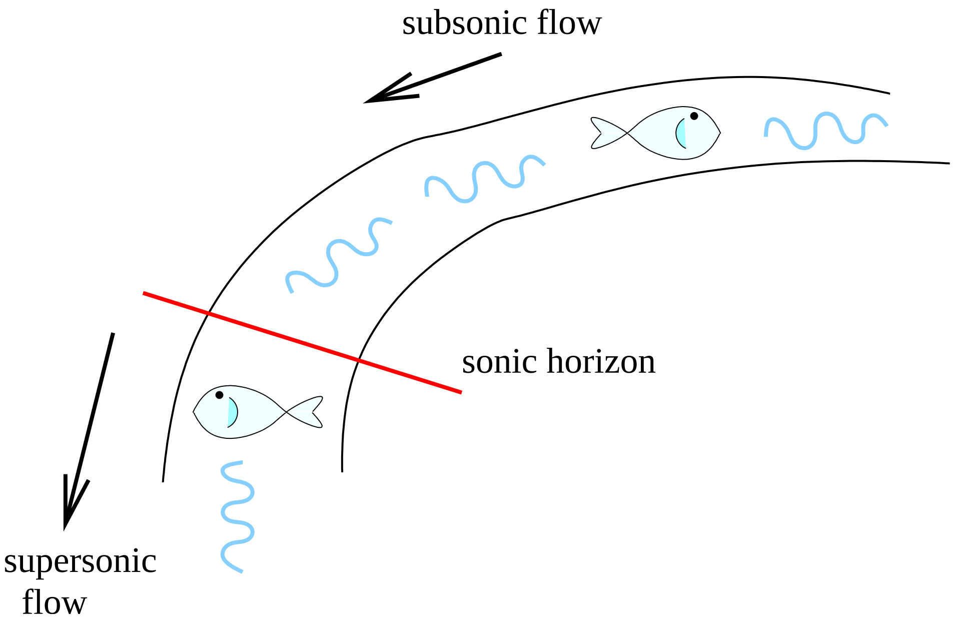
The **state** of the system (*e.g.* specrum of particles, cosmological expansion rate) cannot be justified based on laboratory experiment.

Ambiguities when fundamental processes cannot be distinguished from the state of the system (*e.g.* cosmological constant).

Dumbholes

Basic idea: Waterfall

Unruh, 1981



Fluid flow

Navier-Stokes equations:

$$\nabla \times \mathbf{v} = 0 \, , \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p - \nabla \Phi$$

Fluctuations (“sound”): $\rho = \rho_0 + \delta\rho$, $\mathbf{v} = \mathbf{v_0} + \nabla \phi$

Result:

$$\nabla_\mu \nabla^\mu \phi = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = 0$$

The metric

Fluid flow $g_{\mu\nu}$ given by:

$$ds^2 = (c_s^2 - v_0^2)dt^2 + 2v_0 dt dr - dr^2 - r^2 d\Omega^2$$

Painlevé-Gullstrand-Lemaître form of black hole metric†.

Horizon at $v_0 = c_s$. Sound cannot propagate upstream from horizon.

"Dumbhole" — a (smart) hole that cannot "speak"

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† Assuming spherically symmetric, stationary, convergent, background flow.

Hawking radiation

Quantum Field Theory around dumbhole leads to “Hawking sound” just as QED around a black hole gives Hawking light.

Hawking temperature:

$$T_H = \frac{1}{2\pi} \frac{\partial v_0}{\partial r}\bigg|_{hor} = (3 \times 10^{-7}\ K) \left[\frac{c_s}{300\mathrm{m/s}}\right] \left[\frac{1\mathrm{mm}}{R}\right]$$

where $R =$ horizon size

Practical matters

300 m/s in 1 mm $\sim 10^7 g$!

Lots of difficulties:

Container walls. Instabilities. Shocks. Extremely low Hawking temperature.

He^4 waterfall:

Roughness on walls $\Rightarrow$ flow instabilities such as rotons, vortices prior to horizon formation

Jacobson; Volovik

Generally, instabilities are very hard to avoid unless the sound speed is very low, in which case the Hawking temperature is also very low.

A generalization

Visser, 1998

$$T_{sH} = \left(\frac{\hbar}{2\pi k_B}\right) \frac{\partial}{\partial r}(c_s - v)\Big|_{hor}$$

where $c_s = c_s(t, \mathbf{x})$.

Instead of manipulating v, manipulate c_s.

Helium-3

He^3:

Jacobson and Volovik

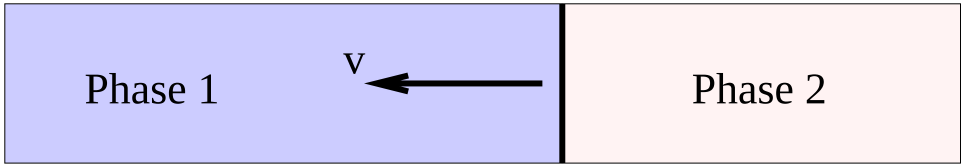
- Consider moving texture or domain wall. Fermionic quasiparticle velocity varies within the wall.

Estimate: $T_{sH} = 5\mu\mathrm{K}$.

At present reliable thermometry in He^3 only goes down to $100\mu\mathrm{K}$.

Propagating phase boundaries

gr-qc/0312069



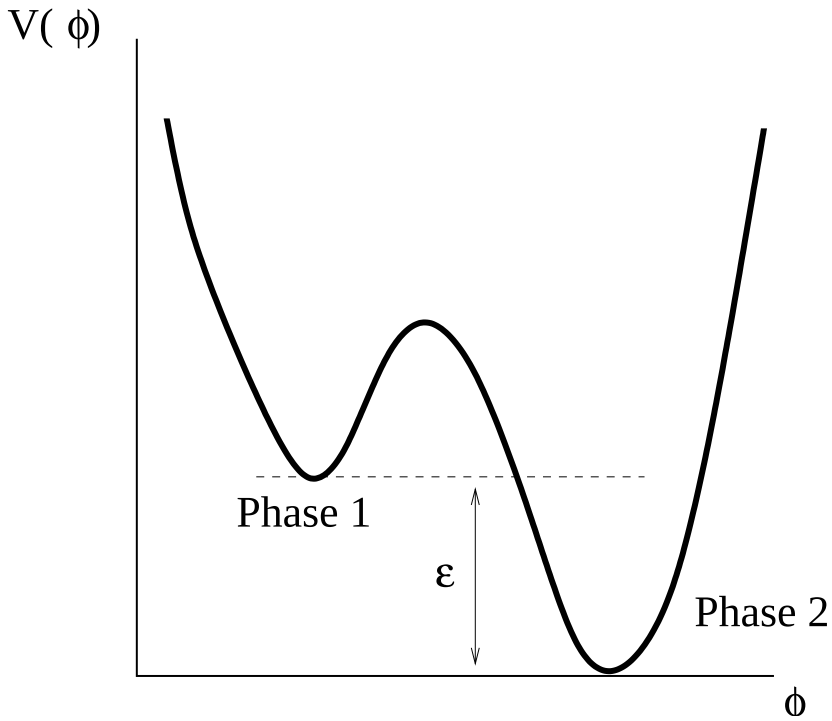
Material is at rest with respect to container.

$$c_1 > v > c_2$$

In rest frame of phase boundary, this is just like the Unruh setup.

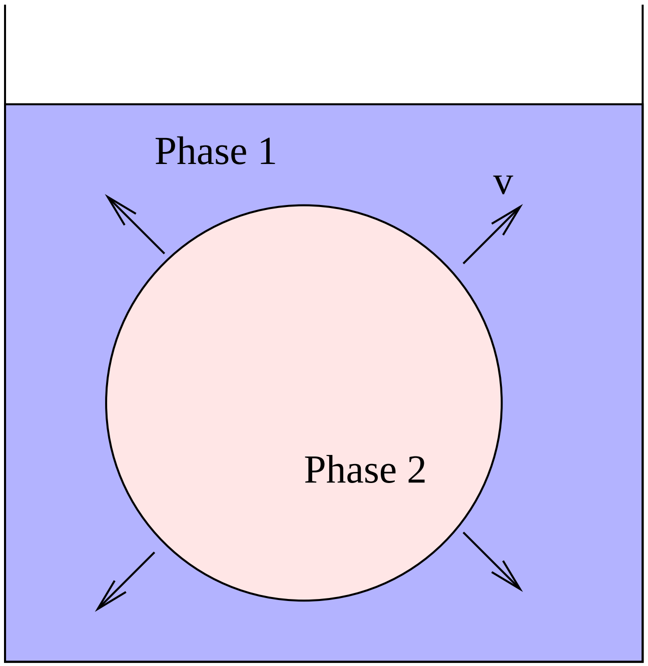
Phase boundary is a sonic horizon.

First order phase transitions



Supercooled system. Decay proceeds by bubble nucleation.

No container walls in 3d



Hawking temperature

$$T_{sH} = 0.04\ \mathrm{K} \left(\frac{\delta c_s}{300\ \mathrm{m/s}}\right) \left(\frac{100\text{Å}}{\xi}\right)$$

- ξ is the thickness of the phase boundary.
- v determines location at which gradient of c_s is evaluated.
- We will assume that T_{sH} is (roughly) independent of v as long as it satisfies $c_1 > v > c_2$.

Can this setup be realized experimentally?

B → A transition in ^{3}He

"Sound" quanta = fermionic quasiparticles

$c_{A\perp} = 3$ cm/s. $c_B = 55$ m/s or larger.

v_{BA} (*i.e.* growing bubble of B in A) observed to be as high as 67 cm/s. Buchanan et. al., 1986

However, since $c_B > c_{A\perp}$, we need $v_{AB} > 3$ cm/s.

$v_{AB} \sim$ several cm/s when driven by magnetic fields.

Bartkowiak et. al., 2000

Issues with the B-phase

Quasiparticles do not propagate on a Lorentzian metric in the B-phase.

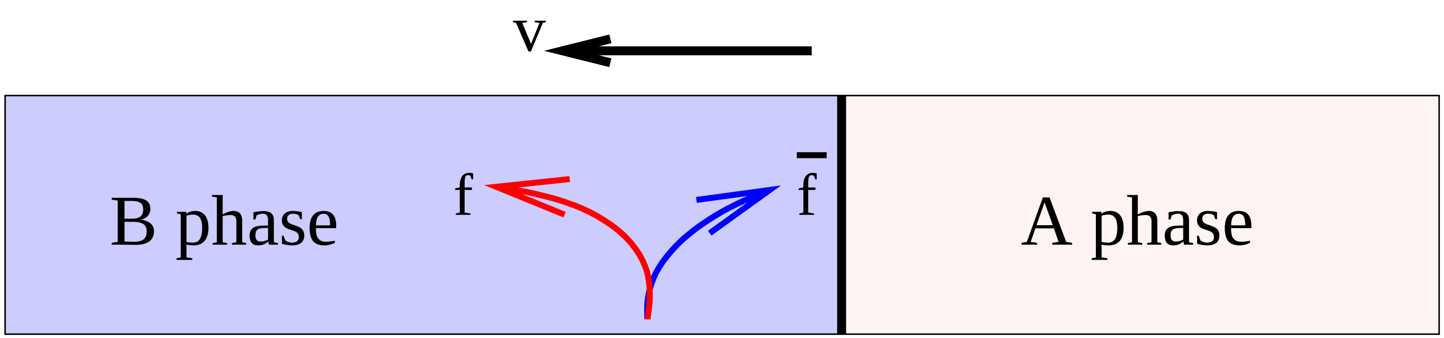
Therefore the metric analogy breaks down.

However the black hole analogy need not break down!

There is still a sonic horizon in which quasiparticles can fall in but from which they can't emerge†. Hawking radiation should still be emitted!

†Even a static AB interface is a reflecting mirror for quasiparticles impinging from the A phase side. But this is only an energy barrier preventing escape (not a "velocity barrier") and does not lead to Hawking radiation.

Hawking radiation without a metric



Vacuum fluctuations in the B phase produce an $f\bar{f}$ pair, say with f having positive energy and $\bar{f}$ having negative energy. $\bar{f}$ falls into the dumbhole, never to be seen again. f escapes, and forms the Hawking radiation.

Can this radiation be seen?

Experimental setup

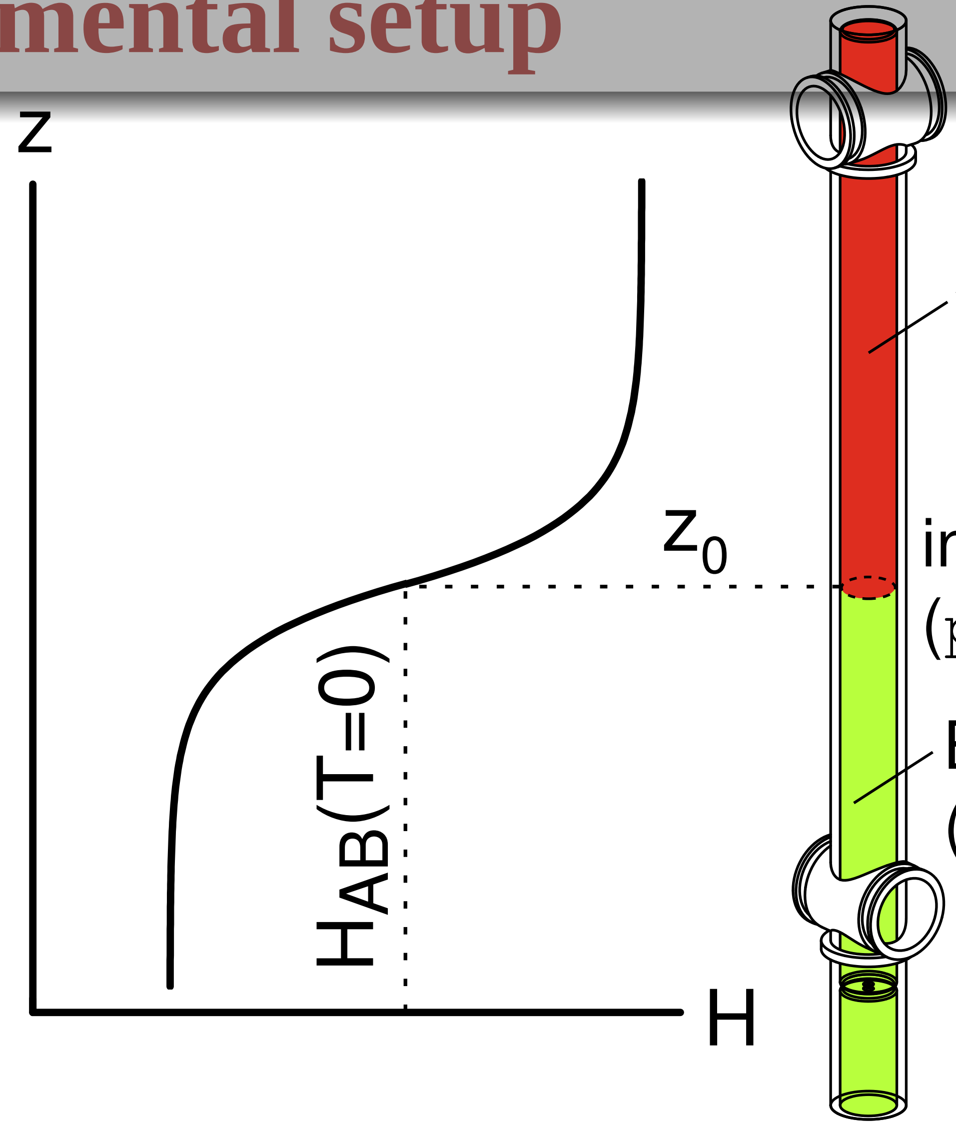


FIG. 29.1. Interface between two vacua stabilized by the vacuum pressure induced by external magnetic field. Interface is at $z = z_0$, where $H(z_0) = H_{AB}$. It separates true vacuum of ^{3}He-A, from the true vacuum of ^{3}He-B.

Oscillations of the He-3 AB interface

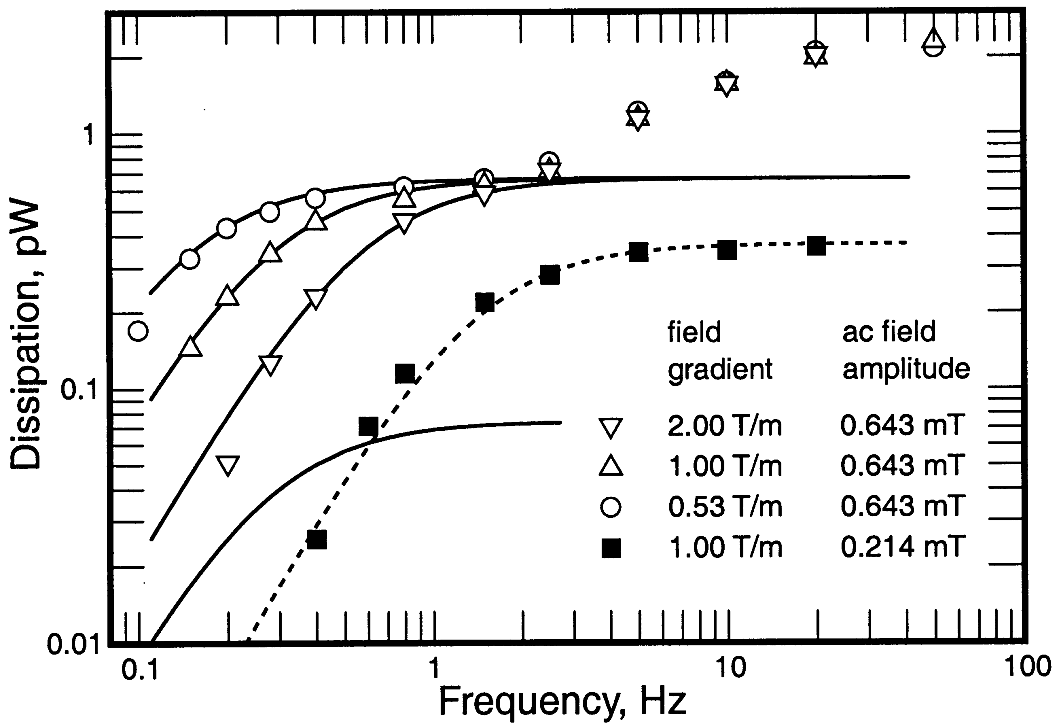


Fig. 2. The dissipation of an oscillating A–B boundary as a function of its frequency, see text.

$\nu = 1$ Hz $\Rightarrow v \sim 1$ cm/s

Bartkowiak et. al., 2000

Dissipation grows anomalously at $\nu \approx 3$ Hz (upper curves).

Interface motion at low frequencies

Equation of motion:

$$-k(x - x_0(t)) - \gamma \frac{dx}{dt} = 0$$

where $x_0(t) = a \sin(\omega t)$ is the position if $\gamma = 0$. Then,

$$v(t) = v_0 \cos(\omega t - \phi) + O(e^{-\kappa t})$$

$$\kappa = \frac{k}{\gamma}\ , \quad v_0 = \frac{a\kappa\omega}{\sqrt{\kappa^2 + \omega^2}}\ , \quad \tan\phi = \frac{\omega}{\kappa}$$

Therefore $v_0 \propto \omega$ for $\omega << \kappa$ and v_0 is independent of ω for $\omega >> \kappa$. $P = \gamma v^2$ (force$\times$velocity) fits $\nu < 1$ Hz data for choice of γ (κ?).

Interface motion at high driving frequencies

At $\nu \gg 1$ Hz:

- Interface probably still oscillates at frequency $\sim \omega$.
- Data indicates that v departs from the low frequency behavior. Assume that the amplitude of v grows with ω.

For *illustrative* purposes take

$$v = \alpha\omega \cos(\omega t - \tilde{\phi})$$

where the amplitude α is an ω independent parameter in the frequency range of interest.

The exact form of v is not crucial for us. Even the power of ω in the amplitude could be different from 1.

Dissipation due to Hawking radiation

Hawking radiation occurs only when $v = \dot{z} > c_A$.

- *Assume* thermal Hawking radiation at T_{sH}.

For j “light” species of radiation, this gives†:

$$P_H = j\sigma_s T_{sH}^4 A \left(\frac{\delta t}{\tau}\right) = j\sigma_s T_{sH}^4 A \frac{1}{\pi} \cos^{-1}\left(\frac{c_A}{2\pi\alpha\nu}\right)$$

Write this as

$$P_H = \frac{P_0}{\pi} \cos^{-1}\left(\frac{\nu_*}{\nu}\right)$$

† $T_{sH} \approx$ 3mK $> \Delta_B \approx$ 1.7mK

Comparison with data

$$P_0 \approx 116 \left(\frac{j}{2}\right) \left(\frac{d}{4.3\text{mm}}\right)^2 \left(\frac{\delta c_s}{60\text{m/s}}\right)^2 \left(\frac{100\text{Å}}{\xi}\right)^4 \text{p}W$$

where d is the cell diameter (4.3 mm in the experiment). This is in the right range if $\xi \sim 300$ Å.

The ν dependence ($\cos^{-1}(\nu_*/\nu)$) qualitatively agrees with data if ν_* is treated as a free parameter.

How about the value of the critical frequency ν_*?

Comparison contd.

$$\nu_* = \frac{c_A}{2\pi\alpha}$$

If $\alpha = B_{AC}/\nabla B_0|_{\text{interface}}$ and the applied magnetic field is:

$$B(t,z) = B_0(z) + B_{AC}\ \sin(2\pi\nu t)$$

then,

∇B_0 T/m	δB mT	ν_* Hz
2.00	0.643	14.8
1.00	0.643	7.4
0.53	0.643	3.9
1.00	0.214	22.2

But no reason to adopt the low frequency $\alpha = B_{AC}/\nabla B_0$.
Explanation can only work if α is independent of ∇B_0.

Future investigations

Theory (with apologies):

- Interface motion at high frequencies.
- Derive Hawking radiation for AB interface (no metric).
- Determine other effects *e.g.* $1-$texture, accelerating boundary.

Experiment (with even greater apologies):

- Setup constant, not oscillating, v.
- Correlate power dissipated and v.
- Eliminate $1-$texture *e.g.* by inserting coaxial surface in container.
- Spectrum.

Conclusions

Can condensed matter experiment successfully study quantum field theory in curved spacetime?

Has condensed matter experiment already seen Hawking radiation? (If not, why not? After all: $P_0 \sim 1$ pW.)

Will condensed matter experiment shed light on other cosmic problems?