Racah Institute of Physics
Hebrew University of Jerusalem

Coulomb blockaded tunnel junction as a noise probe

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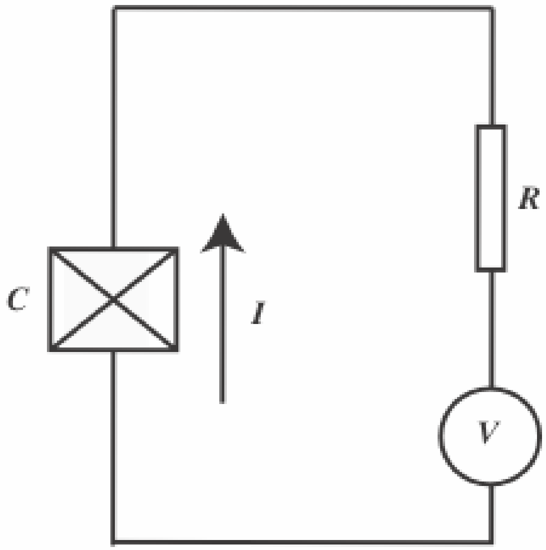
Content:

- Introduction: the Johnson-Nyquist noise and the Coulomb blockade of a Josephson junction
- Shot noise from the independent noise: the Gaussian approximation
- Taking into account that the shot noise is non-Gaussian: the odd moments and asymmetry of *IV* curves
- Conclusions

Lammi 2004

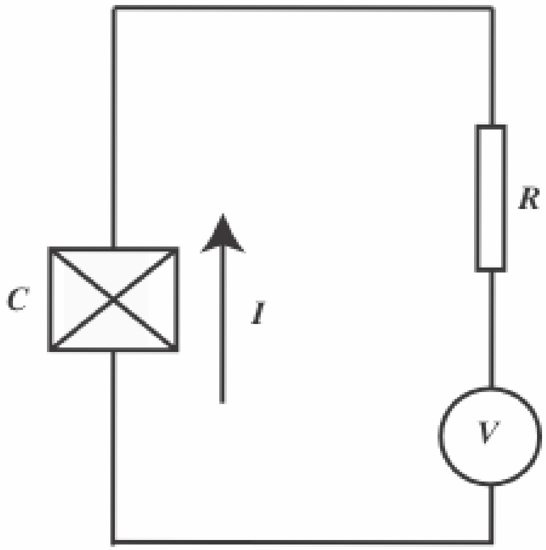


Hamiltonian for the Josephson junction

$$H = \frac{Q^2}{2C} - E_J \cos\varphi - \frac{\hbar}{2e} I\varphi$$

"kinetic" energy

"potential" energy



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phase φ → coordinate x

charge Q → momentum p

$[\hat{Q}, \hat{\varphi}] = 2ei$

$[\hat{p}, \hat{x}] = \hbar i$

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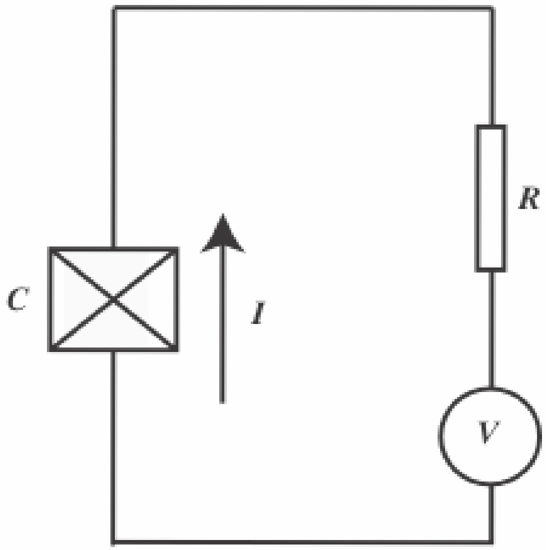

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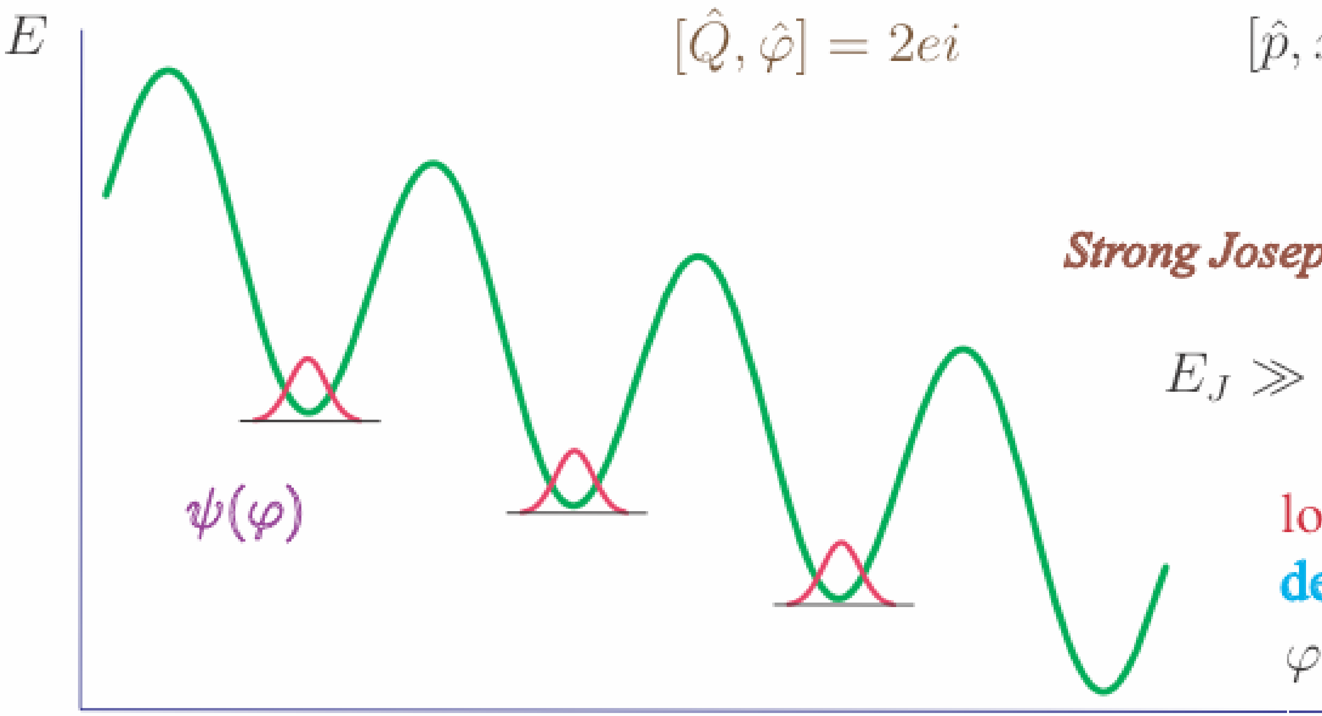

Strong Josephson coupling:

$$E_J \gg E_C = \frac{e^2}{C}$$

localized phase

delocalized charge

Weak coupling limit: $E_J \ll E_C$

localized charge, delocalized phase: $\psi_Q(\varphi) \propto \exp\left(\frac{iQ\varphi}{2e}\right)$

Time-dependent perturbation theory with respect to E_J:

$$H = \frac{Q^2}{2C} - \frac{E_J}{2}\left(e^{i\varphi} + e^{-i\varphi}\right)$$

$e^{i\varphi}$: creation operator

$e^{-i\varphi}$: annihilation operator

$$\langle Q + 2e|e^{i\hat{\varphi}}|Q\rangle \neq 0$$

$$\langle Q|e^{-i\hat{\varphi}}|Q + 2e\rangle \neq 0$$

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Matrix element (amplitude) for tunneling of a Cooper pair ($Q \to Q + 2e$):

$$\propto E_J \int_{t_i}^{t} \langle Q + 2e | e^{i\hat{\varphi}(t')} | Q \rangle dt'$$

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Probability of tunneling (Golden Rule): $W_{\rightarrow}(t) \propto E_J^2 \int_{t_i}^{t} dt_1 \int_{t_i}^{t} dt_2 \langle e^{i\hat{\varphi}(t_1)} e^{-i\hat{\varphi}(t_2)} \rangle$

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Tunneling rate (probability per second):

$$\Gamma_{\rightarrow} = \frac{dW_{\rightarrow}(t)}{dt} \propto \int_{-\infty}^{\infty} dt' \langle e^{i\hat{\varphi}(t_0)} e^{-i\hat{\varphi}(t_0 - t')}\rangle_{t_0}$$

Ingold & Nazarov (1992)

Josephson relation:

$$i\hbar\frac{d\varphi}{dt} = 2eV$$

Constant voltage bias:

$$\varphi(t) = \frac{2eVt}{\hbar} + \tilde{\varphi}(t)$$

$$I = 2e(\Gamma_{\rightarrow} - \Gamma_{\leftarrow}) = \frac{\pi e E_J^2}{\hbar}[P(2eV) - P(-2eV)]$$

$$P(E) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} dt\, e^{iEt}\left\langle e^{i\varphi(t_0)} e^{-i\varphi(t_0-t)}\right\rangle$$

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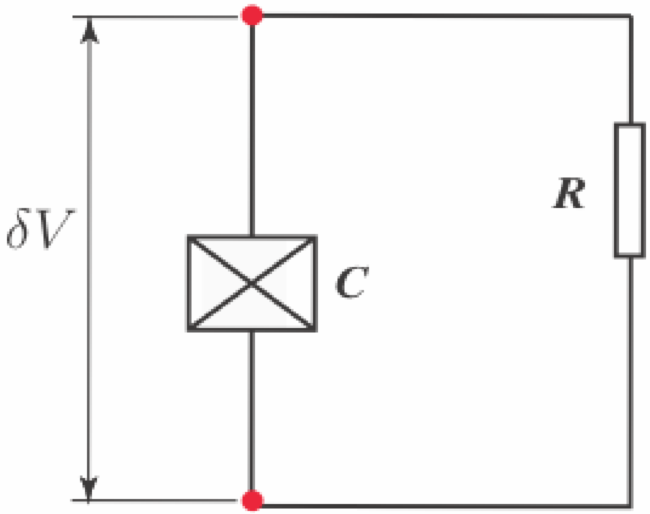
Assumption: Noise is Gaussian

$$\langle e^{i\hat{\varphi}(t_0)} e^{-i\hat{\varphi}(t_0-t)} \rangle_{t_0} = e^{J_0(t)}$$

$$J_0(t) = \langle [\varphi(t) - \varphi(0)]\varphi(0) \rangle$$

$$I = -\frac{2eE_J^2}{\hbar^2} \mathrm{Im} \int_0^{\infty} dt e^{J_0(t)} \sin\left(\frac{2eVt}{\hbar}\right)$$

Equilibrium (Johnson-Nyquist) noise in the environment (circuit)



$$\langle \delta V(t)\delta V(0)\rangle = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\mathrm{Re}Z(\omega)}{1-e^{-\beta\hbar\omega}} e^{-i\omega t}\hbar\omega\, d\omega$$

$$J_0(t) = \langle[\varphi(t)-\varphi(0)]\varphi(0)\rangle$$

$$= 2\int_{-\infty}^{\infty} \frac{d\omega}{\omega} \frac{\mathrm{Re}Z(\omega)}{R_Q} \frac{e^{-i\omega t}-1}{1-e^{-\beta\hbar\omega}}$$

$$\frac{1}{Z(\omega)} = \frac{1}{R} + i\omega C = \frac{1+i\omega\tau}{R}$$

$$R_Q = \frac{\pi\hbar}{2e^2} \qquad \tau = RC \qquad \beta = \frac{1}{T}$$

quantum resistance

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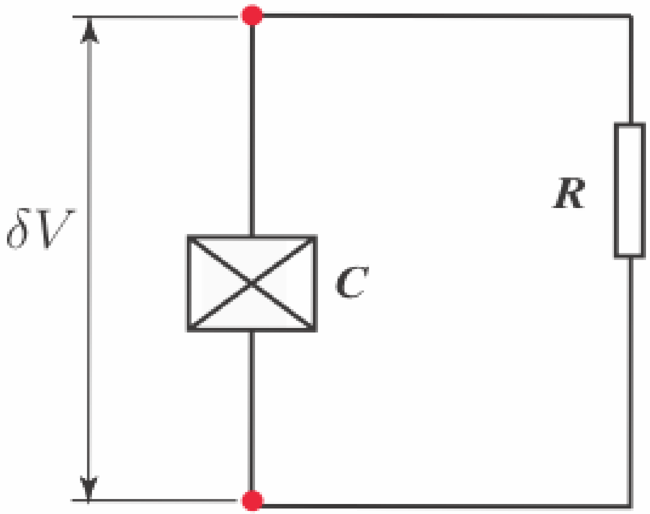

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$$R_Q = \frac{\pi\hbar}{2e^2} \qquad \tau = RC \quad \beta = \frac{1}{T}$$

quantum resistance

Long-time asymptotic: $J_0(t) \sim -\frac{2\pi}{\hbar}\rho T t$ $\qquad \rho = \frac{R}{R_Q}$

phase diffusion

Zero-temperature limit:

$$J_0(t) \sim -2\rho \ln \frac{t}{\tau} \qquad e^{J_0(t)} \sim \frac{1}{t^{2\rho}} \qquad \rho = \frac{R}{R_Q}$$

Conductance integral $G_0 = \frac{dI}{dV} \sim \int_0^\infty e^{J_0(t)} t\, dt$ is divergent at $\rho < 1$

Superconductor-Insulator transition at $\rho = 1$ $\qquad G_0 \propto T^{2\rho-2}$

In the insulator state ($\rho > 1$) at $T = 0$: $\quad I \propto V^{2\rho-1}$

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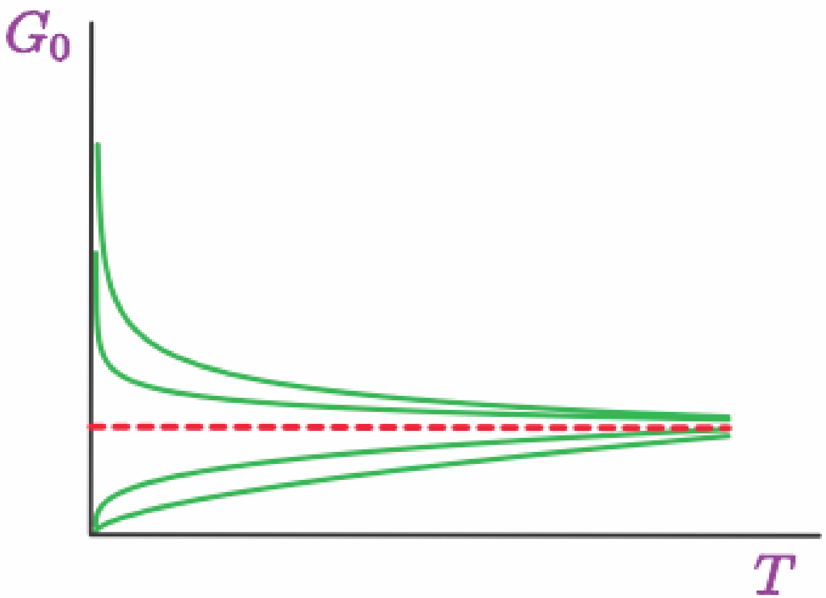
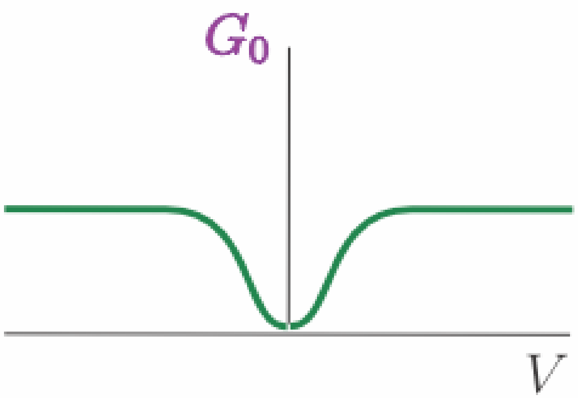
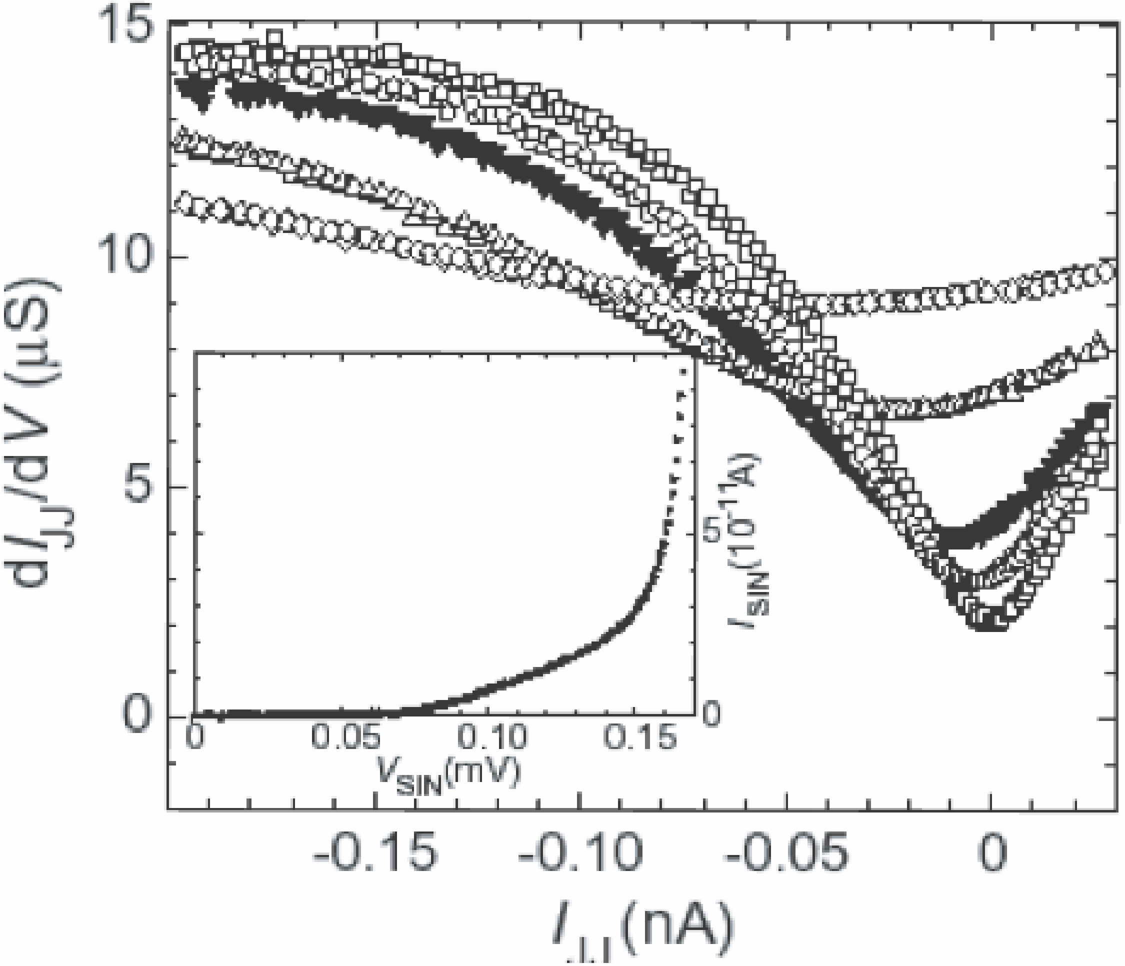




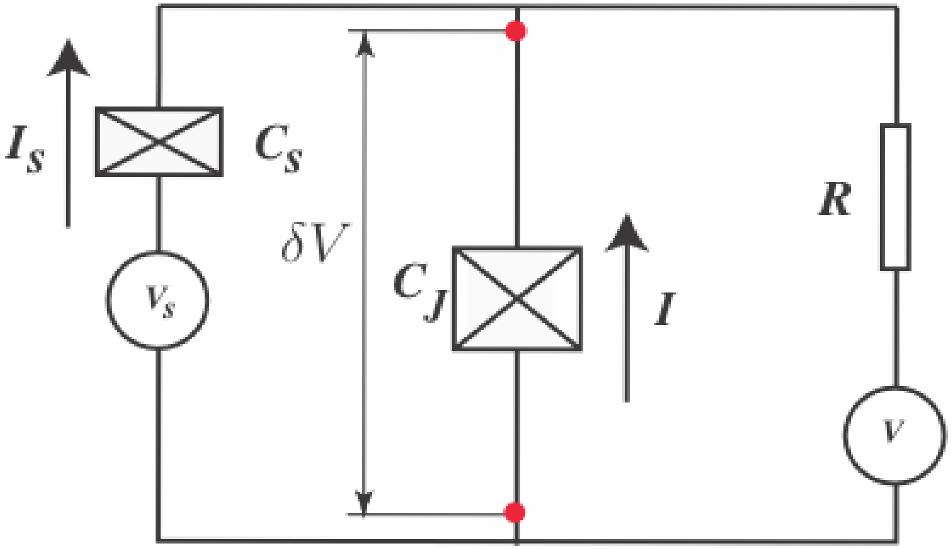
Independent shot-noise source

Delahaye, Lindell, Sillanpaa, Paalanen, Sonin, and Hakonen, cond-mat/0209076



Summary of observations:

- Zero-bias conductance grows with the noise current
- Shift of the conductance minimum from zero-bias

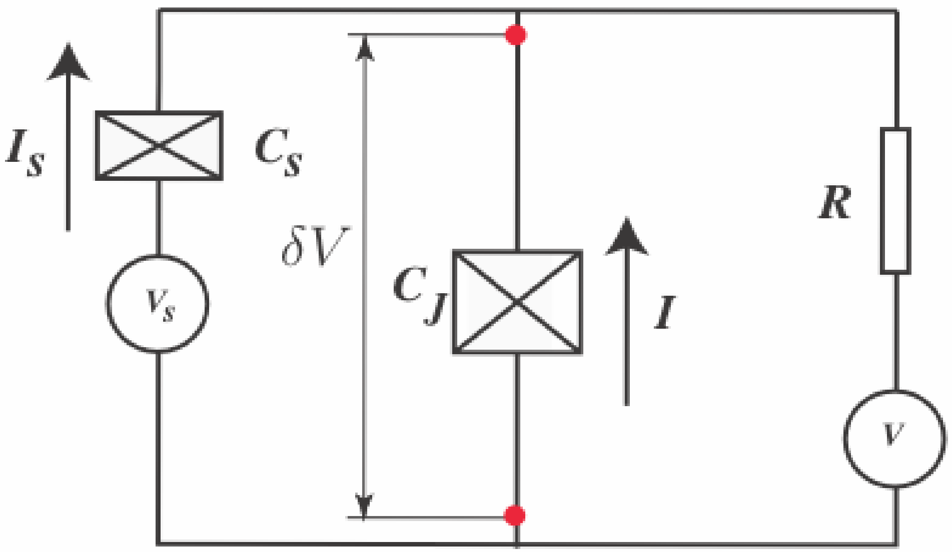


What voltage fluctuation δV
is produced by the current fluctuation δI_s?

$$\delta V = Z \delta I_s$$

$$\frac{1}{Z(\omega)} = \frac{1}{R} + i\omega C = \frac{1 + i\omega\tau}{R}$$

$$C = C_J + C_s$$



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Contribution of the shot noise to the phase fluctuation: $\varphi = \varphi_0 + \varphi_s$

$$J(t) = J_0(t) + J_s(t) \qquad J_s(t) = \langle [\varphi_s(t) - \varphi_s(0)]\varphi_s(0)\rangle$$

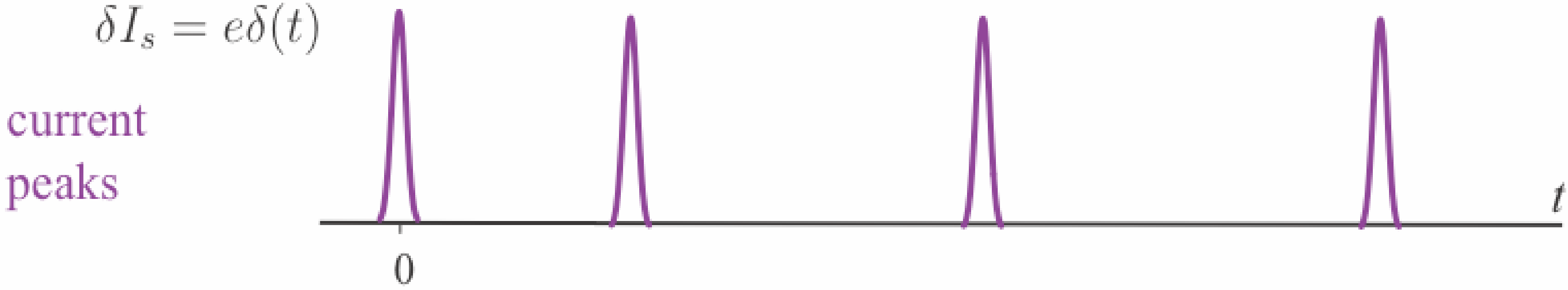
Long-time asymptotic: $J(t) = -\frac{2\pi}{\hbar}\rho B(T + T_N)t$ $\qquad T_N = \frac{1}{2}e|I_s|R$

$G_0 \propto (T + T_N)^{2\rho - 2}$ $\qquad$ At $T = 0$: $\quad G \propto |I_s|^{2\rho-2}$

Nonanalytic dependence on the noise current! Valid only for small ρ

No explanation of the conductance minimum shift

Shot noise



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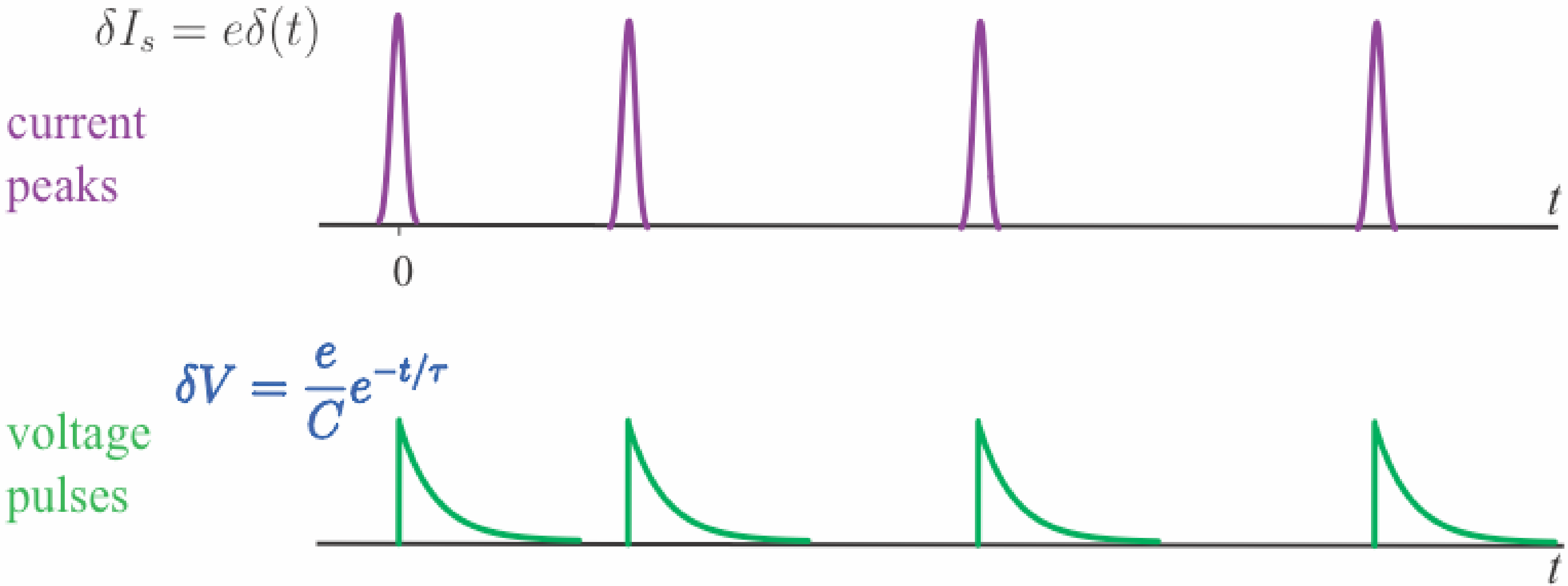
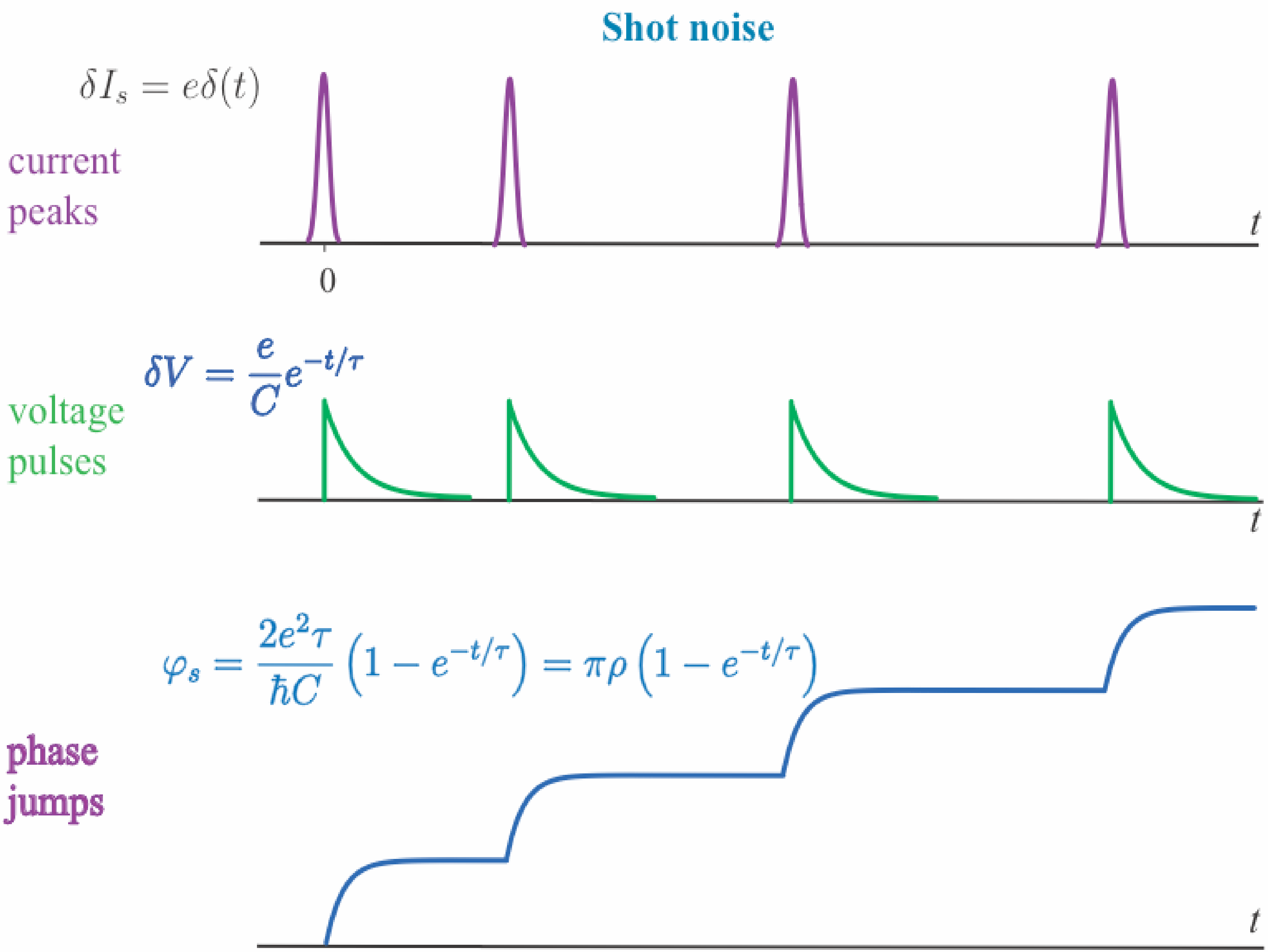


$\delta I_s = e\delta(t)$
current
peaks
t
0
$\delta V = \frac{e}{C} e^{-t/\tau}$
voltage
pulses
t



Shot noise
$\delta I_s = e\delta(t)$
current peaks
0
t
$\delta V = \frac{e}{C} e^{-t/\tau}$
voltage pulses
t
$\varphi_s = \frac{2e^2\tau}{\hbar C}\left(1 - e^{-t/\tau}\right) = \pi\rho\left(1 - e^{-t/\tau}\right)$
phase jumps
t

Shot noise is not Gaussian: $\langle e^{i\varphi_s(t)} e^{-i\varphi_s(0)} \rangle \neq e^{\langle [\varphi_s(t) - \varphi_s(0)]\varphi_s(0) \rangle}$

Generalization of the expression for the current:

$$I = -\frac{2eE_J^2}{\hbar^2} \text{Im} \left\{ \int_0^\infty dt e^{J_0(t)} \left\langle \sin \left(\frac{2eVt}{\hbar} + \Delta\varphi_s \right) \right\rangle \right\}$$

$$\Delta\varphi_s = \varphi_s(t_0) - \varphi_s(t_0 - t)$$

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At $T = 0$:

$$I = -\frac{2eE_J^2}{\hbar^2} \mathrm{Im} \left\{ \int_0^\infty dt e^{J_0(t)} \left[\cos\frac{2eVt}{\hbar} \langle \sin\Delta\varphi_s \rangle + \sin\frac{2eVt}{\hbar} (\langle \cos\Delta\varphi_s \rangle - 1) \right] \right\}$$

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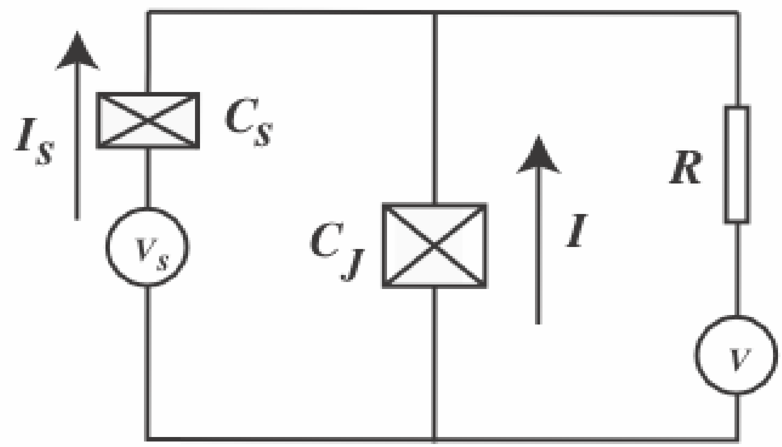
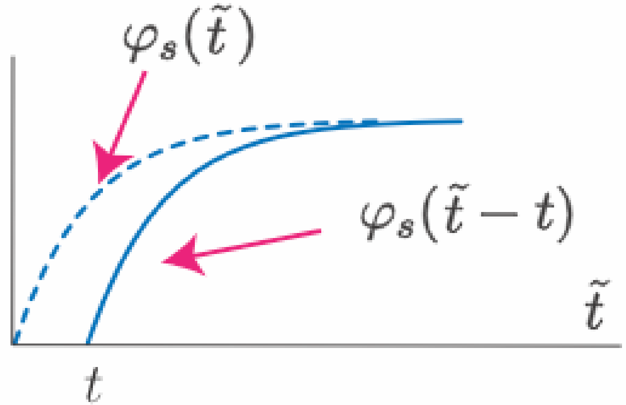
High noise current (strongly overlapping voltage pulses) $|I_s| \gg \frac{e}{\tau}$:

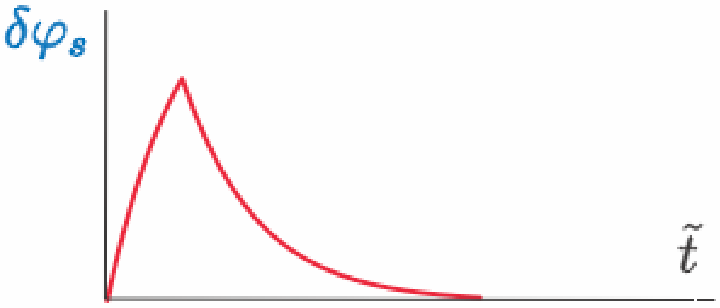
$$\Delta\varphi_s \approx \frac{2eV_{sn}t}{\hbar} \qquad V_{sn} = -I_s R$$

shift of conductance minimum



Low noise currents (sequence of well separated voltage pulses) $|I_s| \ll \frac{e}{\tau}$:





$$\Delta\varphi_s = \varphi_s(t_0) - \varphi_s(t_0 - t) = \sum_i \delta\varphi_s(t, t_0 - t_i)$$

$$\delta\varphi_s(t,\tilde{t}) = \mathrm{sign}(I_s)\pi\rho\left\{\Theta(\tilde{t})\left[1 - e^{-(\tilde{t})/\tau}\right] - \Theta(\tilde{t}-t)\left[1 - e^{-(\tilde{t}-t)/\tau}\right]\right\}$$

$$\langle\Delta\varphi_s^n\rangle = \sum_i \langle[\delta\varphi_s(t, t_0 - t_i)]^n\rangle = \frac{|I_s|}{e}\int_{-\infty}^{\infty} d\tilde{t}[\delta\varphi_s(t,\tilde{t})]^n$$

$$\langle\sin\Delta\varphi_s\rangle = \frac{|I_s|}{e}\int_{-\infty}^{\infty} d\tilde{t}\sin\Delta\varphi_s(\tilde{t},t)$$

$$\langle\cos\Delta\varphi_s\rangle - 1 = \frac{|I_s|}{e}\int_{-\infty}^{\infty} d\tilde{t}\left[\cos\Delta\varphi_s(\tilde{t},t) - 1\right]$$

Odd momenta (asymmetry effects):

$$\langle \sin \Delta\varphi_s \rangle = \frac{I_s \tau}{e} \left\{ \frac{\pi}{2} + \mathrm{si}\left[\pi\rho\left(1 - e^{-t/\tau}\right)\right] \right.$$

$$\left. + \sin \pi\rho \left[\mathrm{ci}\pi\rho - \mathrm{ci}\left(\pi\rho e^{-t/\tau}\right)\right] - \cos \pi\rho \left[\mathrm{si}\pi\rho - \mathrm{si}\left(\pi\rho e^{-t/\tau}\right)\right] \right\}$$

Even momenta:

$$\langle \cos \Delta\varphi_s \rangle - 1 = \frac{|I_s| \tau}{e} \left\{ -\frac{t}{\tau} + \cos \pi\rho \left[\mathrm{ci}\pi\rho - \mathrm{ci}\left(\pi\rho e^{-t/\tau}\right)\right] \right.$$

$$\left. + \sin \pi\rho \left[\mathrm{si}\pi\rho - \mathrm{si}\left(\pi\rho e^{-t/\tau}\right)\right] + \mathrm{ci}\left[\pi\rho\left(1 - e^{-t/\tau}\right)\right] - \gamma - \ln\left[\pi\rho\left(1 - e^{-t/\tau}\right)\right] \right\}$$

$$\mathrm{si}(x) = -\int_x^\infty \frac{\sin t}{t}\, dt$$

$$\mathrm{ci}(x) = -\int_x^\infty \frac{\cos t}{t}\, dt$$

Expansion in $\Delta\varphi_s$ is invalid since $\rho \gg 1$.
No expansion in momenta!

The shape of the IV curve at small voltage

$$I = -\frac{2eE_J^2}{\hbar^2}\mathrm{Im}\left\{\int_0^\infty dte^{J_0(t)}\left[\cos\frac{2eVt}{\hbar}\langle\sin\Delta\varphi_s\rangle + \sin\frac{2eVt}{\hbar}(\langle\cos\Delta\varphi_s\rangle - 1)\right]\right\}$$

Expansion: $I = G_s(V_0 + V + aV^2 + bV^3)$

The shape of the IV curve at small voltage

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Expansion: $I = G_s(V_0 + V + aV^2 + bV^3)$

$$G_s = -\frac{4e^2E_J^2}{\hbar^3}\mathrm{Im}\int_0^\infty t\,dt e^{J_0(t)}[\langle\cos\Delta\varphi_s\rangle - 1]$$ conductance

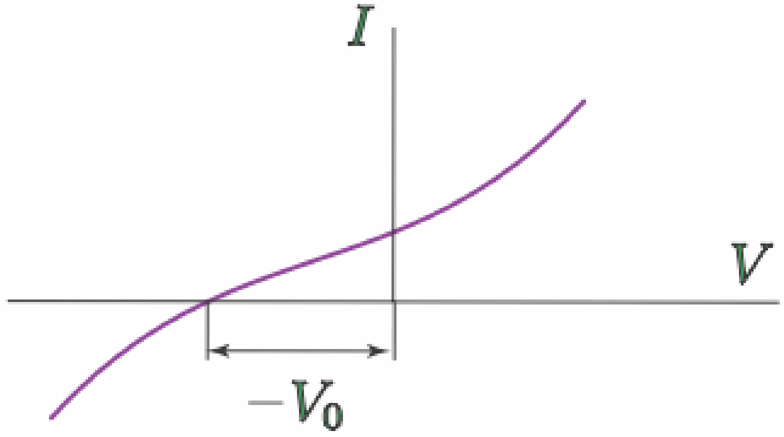
$$V_0 = -\frac{2eE_J^2}{\hbar^2 G_s}\mathrm{Im}\int_0^\infty dt e^{J_0(t)}\langle\sin\Delta\varphi_s\rangle$$ ratchet effect

$$a = \frac{4e^3E_J^2}{\hbar^4 G_s}\mathrm{Im}\int_0^\infty t^2\,dt e^{J_0(t)}\langle\sin\Delta\varphi_s\rangle$$ curvature of the curve conductance vs voltage

$$b = \frac{8e^4E_J^2}{3\hbar^5 G_s}\mathrm{Im}\int_0^\infty t^3\,dt e^{J_0(t)}[\langle\cos\Delta\varphi_s\rangle - 1]$$

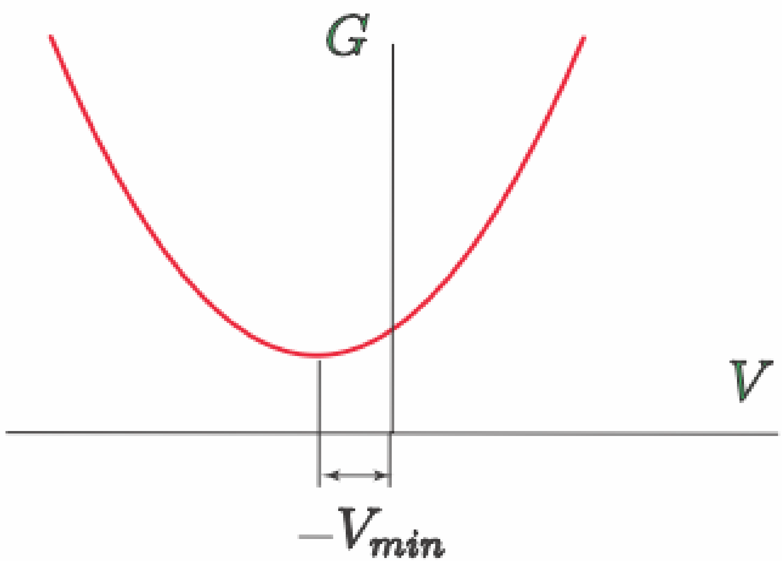
Shift of the conductance minimum: $V_{min} = -\mathrm{sign}(I_s)\frac{a}{3b}$

IV curve at low voltage bias



Effects of shot-noise asymmetry (odd moments):

1. Shift of the conductance minimum V_{min} (observed)
2. Ratchet effect V_0 (to be observed)



Odd moments:

Theory:

Levitov & Reznikov, cond-mat/011057

Beenaker ***et al.***, PRL, **90**, 176802 (2003)

Gutman & Gefen, PRB, **68**, 035302 (2003)

Experiment:

Reulet ***et al.***, PRL, **91**, 196001 2003)

Asymptotic expressions at $\rho \gg 1$:

$$G_s = \frac{\pi^2 E_J^2 C^3}{8e^5} \rho^{3/2} |I_s| \;\; V_0 = \text{sign}(I_s) \frac{e}{2C\sqrt{\rho}} \,,$$

$$a = \text{sign}(I_s) \frac{C\sqrt{\pi\rho}}{e} \,, \; b = \frac{\pi C^2 \rho}{24e^2} \qquad V_{min} = -\text{sign}(I_s) \frac{a}{3b} = -\text{sign}(I_s) \frac{8e}{C\sqrt{\pi\rho}}$$

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$$a = \mathrm{sign}(I_s) \frac{C\sqrt{\pi\rho}}{e}\,,\; b = \frac{\pi C^2 \rho}{24 e^2} \qquad V_{min} = -\mathrm{sign}(I_s) \frac{a}{3b} = -\mathrm{sign}(I_s) \frac{8e}{C\sqrt{\pi\rho}}$$

Convergence of integrals at long times $t \gg \tau$:

$$J_0(t) \propto -2\rho \ln t \qquad\qquad \langle \cos \Delta\varphi_s \rangle \propto t$$

convergent at $\rho > 1.5$

$$G_s \propto \int_0^\infty t\, dt e^{J_0(t)} \langle \cos \Delta \varphi_s \rangle \propto \int_0^\infty \frac{dt}{t^{2\rho-2}}$$

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Transition from low-impedance (non-analytic) to high-impedance (linear) dependence at $\rho = 1.5$:

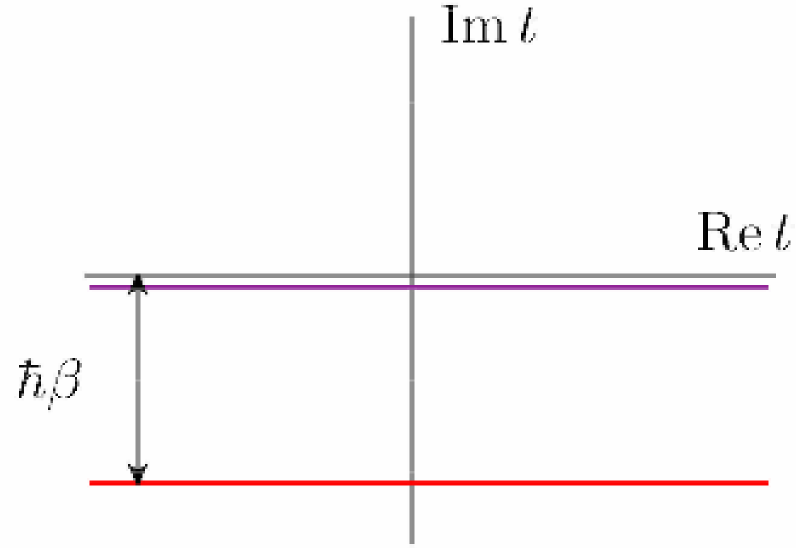
$G_s \propto |I_s|$ at $\rho > 1.5$

$G_s \propto |I_s|^{2\rho-2}$ at $\rho < 1.5$

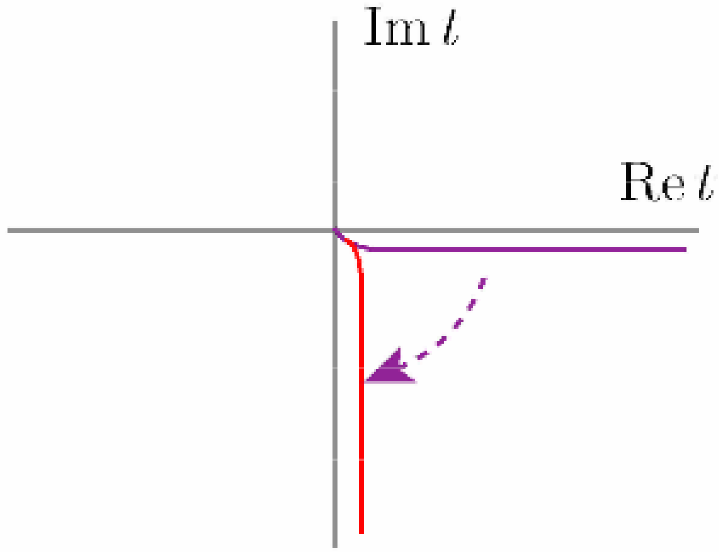
Numerical confirmation:
Pietila & Heikkila (unpublished)

Methods of integration in the plane of complex t

Johnson-Nyquist noise:



Shot noise,
high impedance $\rho \gg 1$:



Conclusions:

- The effect of shot noise from an independent source on the Coulomb blockaded Josephson junction in a high-impedance environment has been analyzed theoretically
- The theory took into account that the shot noise produces non-Gaussian phase fluctuations at the Josephson junction
- Non-Gaussian character of fluctuations results in asymmetry of IV curves:
  (i) the conductance minimum at nonzero voltage bias;
  (ii) the ratchet effect (nonzero current at zero voltage bias)
- The asymmetry effects are a clear manifestation of the odd moments of the shot noise, which are difficult for experimental detection by other methods
- The analysis demonstrates, that the Coulomb blockaded tunnel junction in a high impedance environment can be a sensitive noise detector

We expect that other types of noise (e.g., 1/f noise), produce similar effects, and the Coulomb blockaded ***normal*** junction also can be used as a noise detector