Entangled Hanbury Brown Twiss Effects with Edge States

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From Schottky to Bell

- Classical Shot Noise

W. Schottky, Ann. Phys. (Leipzig) **57**, 541 (1918).

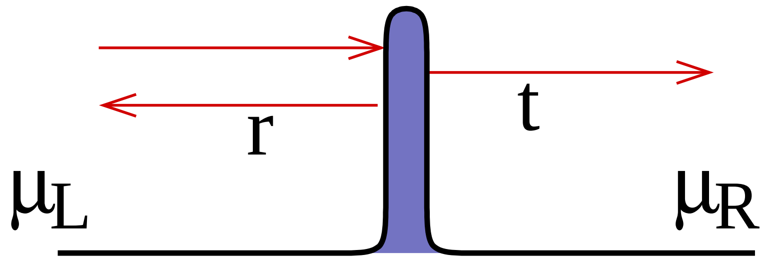
$$\langle(\Delta I)^2\rangle_\nu = 2e|\langle I\rangle|$$

- Quantum Shot Noise

$$|\Psi >_{inc} = \ e^{ikx}$$

$$|\Psi >_{ref} = re^{-ikx}$$

$$|\Psi >_{tra} = t\, e^{ikx}$$



- Entangled States

J. Bell, Physics **1**, 195 (1964).

Steps towards experimental realization

- Orbital entanglement.

P. Samuelsson, E.V. Sukhorukov and M. Büttiker, Phys. Rev. Lett. **91**, 157002 (2003).

- Zero-frequency measurement (Bell Inequality).

X. Maître, W. D. Oliver, Y. Yamamoto, Physica E **6**, 301 (2000).

N.M. Chtchelkatchev *etal.*, Phys. Rev. B **66**, 161320 (2002).

P. Samuelsson, E.V. Sukhorukov and M. Büttiker, Phys. Rev. Lett. **91**, 157002 (2003).

- Normal conductor, basic components.

C.W.J. Beenakker *et al*, Phys. Rev. Lett. **91**, 147901 (2003).

P. Samuelsson, E.V. Sukhorukov and M. Büttiker, cond-mat/0307473 (PRL in press).

C.W.J. Beenakker et al, cond-mat/0310199.

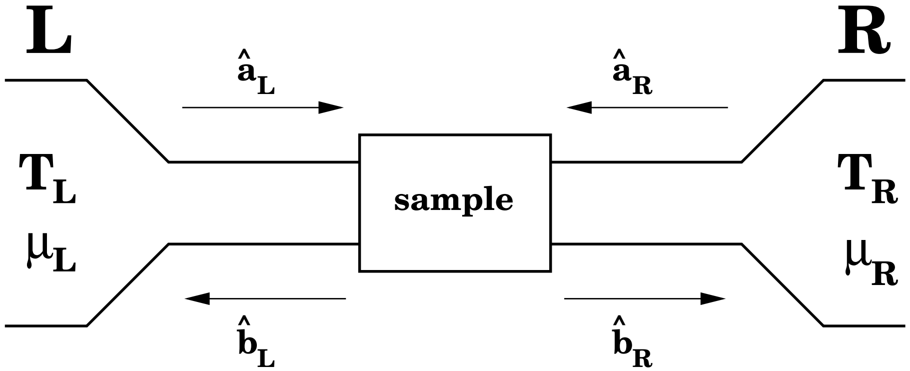
- Controllable geometry.

P. Samuelsson, E.V. Sukhorukov and M. Büttiker, cond-mat/0307473 (PRL in press).

Conductance and Shot Noise: Two terminal Conductors

- Scattering theory of electrical conduction.

$$s = \begin{pmatrix} r & t \\ t & r \end{pmatrix}$$



Eigenchannels: $t^\dagger t$ hermitian $\Rightarrow T_n$.

- Conductance is a function of transmission probabilities in every basis.

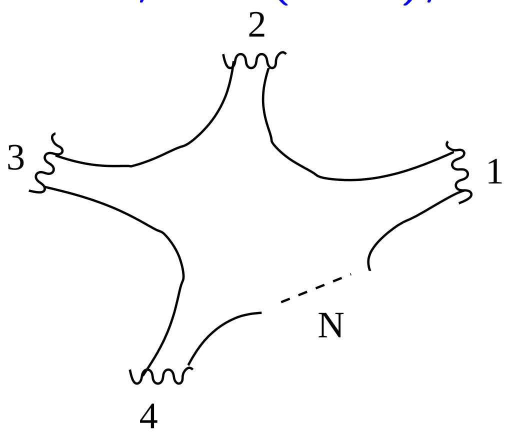
$$G = \frac{e^2}{h} Tr(t^\dagger t) = \frac{e^2}{h} \sum_n T_n$$

- Shot noise is a function of transmission probabilities ONLY in the eigen-channel basis.

$$S = 2e\frac{e^2}{h}|eV|Tr(r^\dagger r t^\dagger t) = 2e\frac{e^2}{h}|eV|\sum_n T_n(1 - T_n)$$

Correlations of shot noise: Multiterminal Conductors

M. Büttiker, Physica B **175**, 199 (1991); Phys. Rev. B **46**, 12485 (1992)



$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & \cdots & s_{1N} \\ s_{21} & s_{22} & s_{23} & \cdots & \\ s_{31} & s_{32} & s_{33} & & \\ \vdots & \vdots & & \ddots & \\ s_{N1} & & & & s_{NN} \end{pmatrix}$$

$$S_{\alpha\beta} = 2 \int dt \langle \Delta \hat{I}_\alpha(t) \Delta \hat{I}_\beta(0) \rangle$$

For $kT = 0$, $\alpha \neq \beta$

$$S_{\alpha\beta} = 2 \frac{e^2}{h} \int dE \, \mathrm{Tr} \left[B^\dagger_{\alpha\beta} B_{\beta\alpha} \right], \qquad B_{\alpha\beta} = \sum_{\gamma=1}^{M} s_{\alpha\gamma} s^\dagger_{\beta\gamma} (f_\gamma - f_0)$$

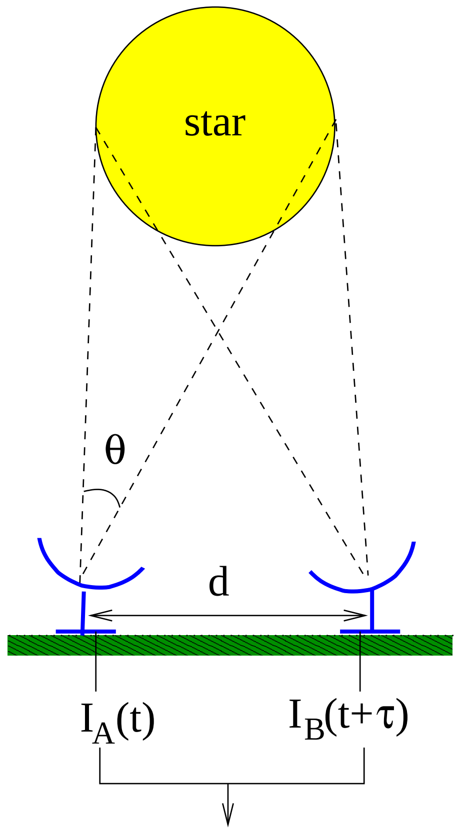
- Quantum partition noise, $M = 1$.
- Two particle Aharonov-Bohm effect, $M \geq 2$.
- Entanglement, $M \geq 2$.

Hanbury Brown Twiss Effect

Hanbury Brown and Twiss, Nature **177**, 27 (1956)

- Hanbury Brown Twiss: classical wave explanation.
- Purcell: quantum mechanical explanation Purcell, Nature **178**, 1449 (1956).

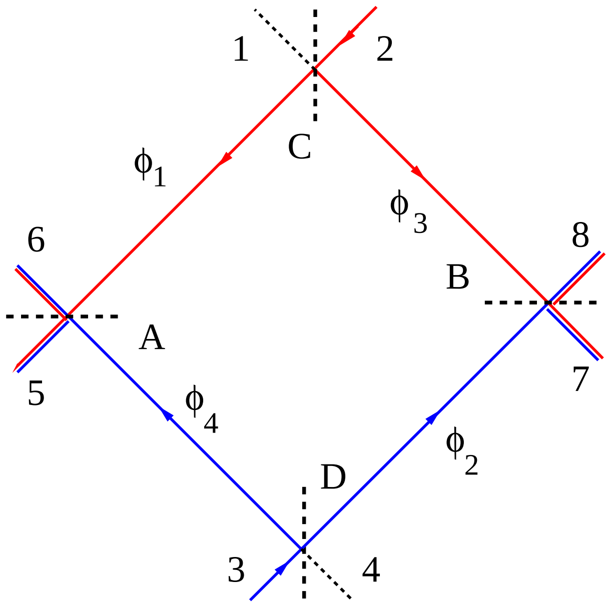
Indistinguishable particles $\Rightarrow$
i) Statistics,
ii) Exchange amplitudes.

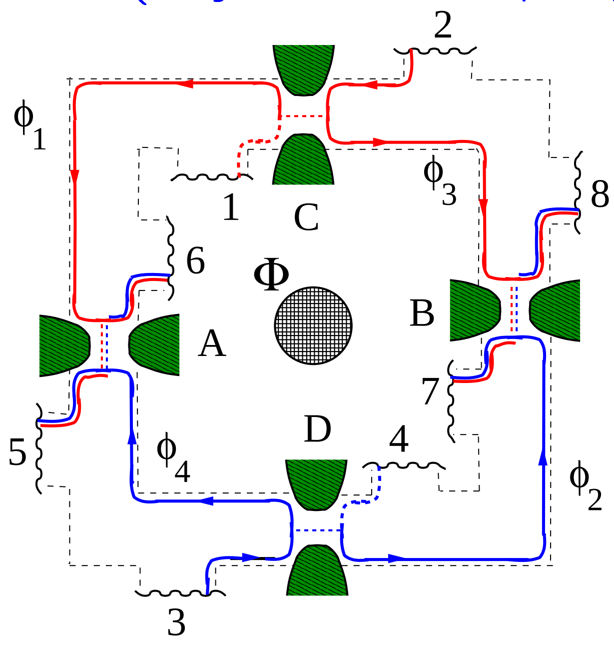


$$\int d\tau \langle \Delta I_A(t) \Delta I_B(t+\tau) \rangle = f\left(\frac{d\theta}{\lambda}\right)$$

Two-source HBT-interferometer.

Samuelsson, Sukhorukov, Büttiker, condmat/0307473 (Phys. Rev. Lett, in press)



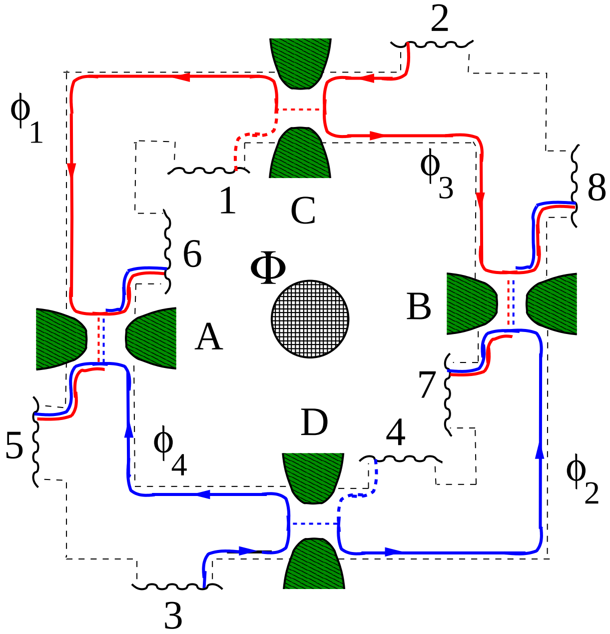


$$s_{52} = T_A^{1/2} e^{i\phi_1} T_C^{1/2} \Rightarrow G_{52} = -\frac{e^2}{h} T_A T_C$$

All elements of conductance matrix independent of Φ.

Two-particle AB-effect.

Samuelsson, Sukhorukov, Büttiker, condmat/0307473 (Phys. Rev. Lett, in press)



Fourth-order interference

$$S_{58} = -2\frac{e^2}{h}\int dE|s_{52}^* s_{82} + s_{53}^* s_{83}|^2 (f - f_0)^2$$

With $T_A = T_B = T_C = T_D = 1/2 \Rightarrow$

$$S_{58} = -\frac{e^2}{4h}|eV|\left[1 + \cos\left(\phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\frac{\Phi}{\Phi_0}\right)\right]$$

Correlations of properties of distant particles.

Einstein, Podolsky and Rosen, Phys. Rev. **47**, 777 (1935); Bell, Physics **1**, 195 (1964)

Spin entangled state

Spin $S = 0$ state (singlet), A,B detectors

$$\Psi = \frac{1}{\sqrt{2}} \left[\Psi_{\uparrow}(A)\Psi_{\downarrow}(B) - \Psi_{\downarrow}(A)\Psi_{\uparrow}(B) \right]$$

Spin $S = 1$ state (triplet),

$$\Psi = \frac{1}{\sqrt{2}} \left[\Psi_{\uparrow}(A)\Psi_{\uparrow}(B) + \Psi_{\downarrow}(A)\Psi_{\downarrow}(B) \right]$$

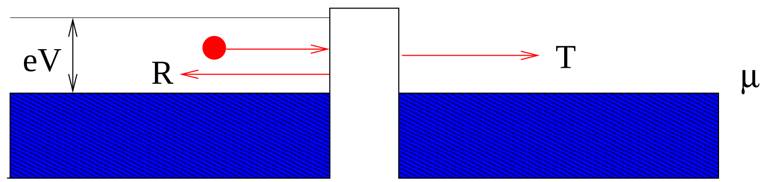
Orbitally entangled state

$$\Psi = \frac{1}{\sqrt{2}} [\Psi_1(A)\Psi_1(B) + \Psi_2(A)\Psi_2(B)]$$

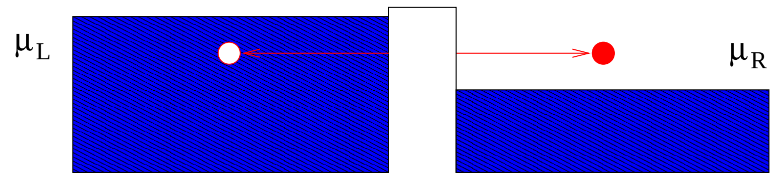
P. Samuelsson, E.V. Sukhorukov and M. Büttiker, Phys. Rev. Lett. **91**, 157002 (2003).

Reformulation of vacuum state

Electron picture of tunneling



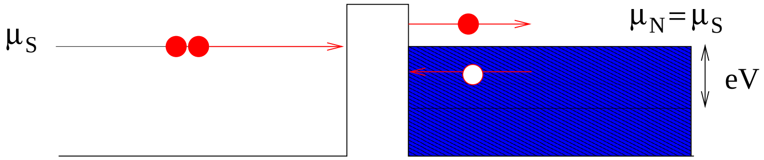
Electron-hole picture



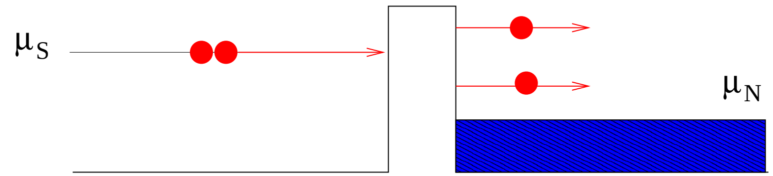
Beenakker *et al*, Phys. Rev. Lett. **91**, 147901 (2003)

Superconducting-normal hybrid structures

Bogoliubov-de Gennes picture



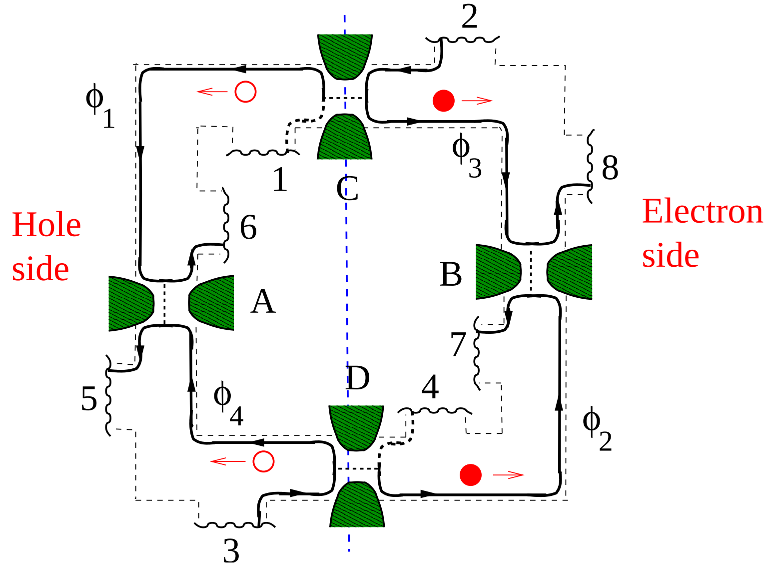
Pair-tunneling picture



Samuelsson, Sukhorukov and Büttiker, Phys. Rev. Lett. **91**, 157002 (2003)

Entanglement in the two-source HBT-setup.

Samuelsson, Sukhorukov, Büttiker, condmat/0307473 (Phys. Rev. Lett, in press)



Tunnel limit $R_C = T_D = R \ll 1$, $\tau_C = \hbar/eV$, $\tau \sim \hbar/eVR$

$$|\Psi_{in}\rangle = \prod_{0<E<eV} c_2^\dagger(E) c_3^\dagger(E)|0\rangle \Rightarrow$$

$$|\Psi\rangle = |\bar{0}\rangle + \sqrt{R}\int_0^{eV} dE \left[c_{3B}^\dagger c_{3A} - c_{2B}^\dagger c_{2A}\right] |\bar{0}\rangle$$

Orbitally entangled electron-hole state

Entanglement test: Violation of Bell Inequality.

Bell Inequality:
Comparison of a classical, local theory with a quantum mechanical two-particle state. Here: Entanglement test.

With $\phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0 = 2\pi$ and

$$S_{A/B} = \begin{pmatrix} \cos\theta_{A/B} & -\sin\theta_{A/B} \\ \sin\theta_{A/B} & \cos\theta_{A/B} \end{pmatrix}$$

Noise correlators

$$S_{58} = S_{67} = -S_0 P_{++}, \quad S_{57} = S_{68} = -S_0 P_{+-}$$

$$S_0 = -(4e^2/h)|eV|R, \quad P_{\alpha\beta} = (1 + \alpha\beta\cos[2(\theta_A - \theta_B)])/4$$

Zero frequency noise correlator $\Leftrightarrow$ “Two particle joint detection probability”

$$E(\theta_A, \theta_B) = P_{++} + P_{--} - P_{+-} - P_{-+}$$

Clauser *et al*, Phys. Rev. Lett, **23**, 880 (1969)

Electron-electron entanglement through postselection.

Symmetric interferometer $T, R \approx 1/2$

- Electron-hole picture not appropriate.
- Incident electron state is a product state $\Rightarrow$ no intrinsic entanglement.

- Two-particle effects nevertheless persists.
- A Bell Inequality can be violated.

Explanation: Entanglement through "postselection" (measurement).

Glauber: Joint detection probability

$$P_{\alpha\beta} \propto \langle c_\beta^\dagger(t) c_\alpha^\dagger(t) c_\alpha(t) c_\beta(t) \rangle \propto S_{\alpha\beta} + 2\tau_C I_\alpha I_\beta$$

$$I_5 = I_6 = (e^2/h)TV, \quad I_7 = I_8 = (e^2/h)RV$$

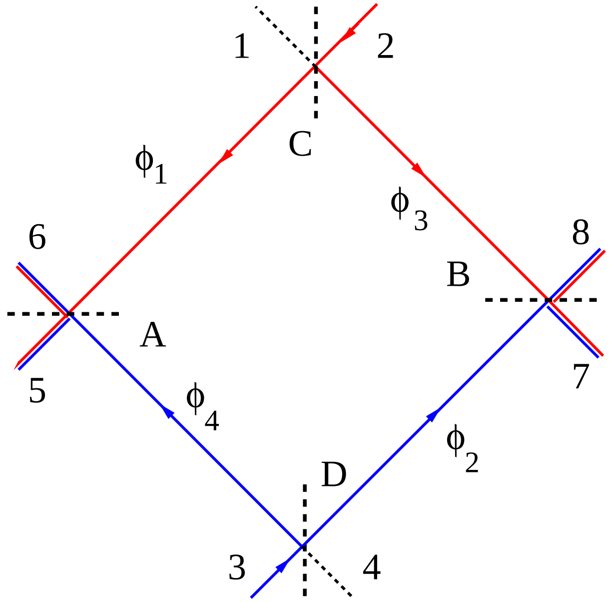
- Bell parameter (Bell Inequality: $|S_B| \leq 2$)

$$S_B^{max} = 2\sqrt{1 + \cos^2 \phi_0}, \;\; \phi_0 = \phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0$$

- Dephasing, $0 < \gamma < 1$ $\Rightarrow$ $\quad S_B^{max} = 2\sqrt{1 + \gamma^2 \cos^2 \phi_0}$

γ =visibility of two-particle AB-effect.

Epilogue: Thermal Photon versus thermal Electron sources



Joint detection probability $\Rightarrow$ (narrow band filters)

$$P_{\alpha\beta} \propto \langle c^\dagger_\beta(t) c^\dagger_\alpha(t) c_\alpha(t) c_\beta(t) \rangle \propto S_{\alpha\beta} + 2\tau_C I_\alpha I_\beta$$

$$S_B^{max} = (2/3)\sqrt{1 + \cos^2\phi_0}$$

No violation of Bell Inequality

Difference: In a time interval $\tau_C = \hbar/eV$ the electron source emits only one electron

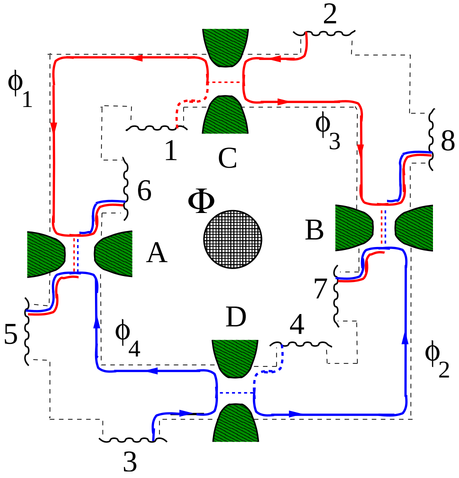
Summary

- Edge states and QPC's (or gates) permit to generate electrical analogs of optical geometries: controllable geometry

- Two-particle Aharonov-Bohm-(exchange)-effects.

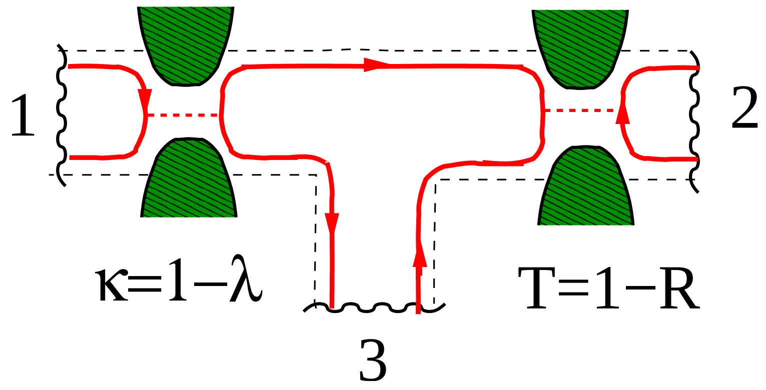
- Asymmetric interferometer: Violation of a Bell Inequality connected to orbitally entangled electron-hole states.

- Symmetric interferometer: Violation of a Bell Inequality due to postselected orbital electron-electron entanglement.



Current correlations, Oberholzer experiment.

Oberholzer *et al*, Physica E, **6**, 314 (2000)



Bias configuration

$$\mu_1 = \mu_0 + e|V|, \qquad \mu_2 = \mu_3 = \mu_0$$

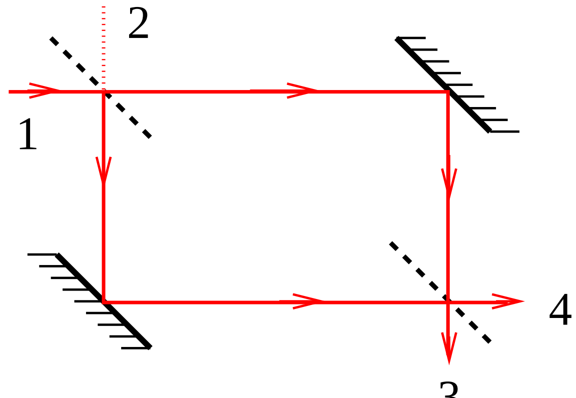
- Current correlators

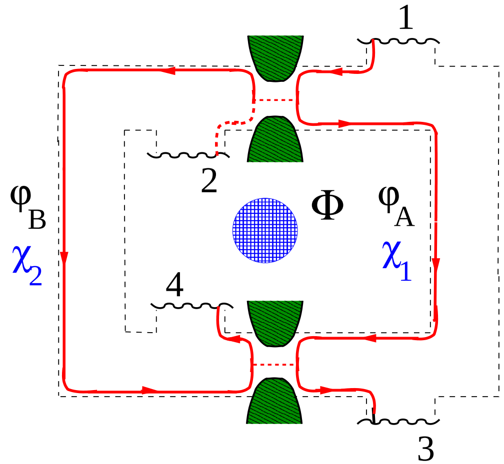
$$\langle \Delta I_2 \Delta I_3 \rangle = -2\frac{e^2}{h}|eV|\kappa^2 TR, \qquad \langle (\Delta I_2)^2 \rangle = 2\frac{e^2}{h}|eV|\kappa T(1-\kappa T)$$

$$\langle (\Delta I_3)^2 \rangle = 2\frac{e^2}{h}|eV|\kappa R(1-\kappa R)$$

Optical and electrical Mach-Zender interferometer.

Ji *et al*, Nature, **422**, 415 (2003)





Second order interference ⇔ One particle Aharonov-Bohm effect

$$s_{31} = \frac{1}{2}\left[e^{i(\phi_A - \chi_1)} + e^{i(\phi_A - \chi_2)}\right]$$

$$G_{31} = \frac{e^2}{2h}[1 + \cos(\phi_A - \phi_B - 2\pi\Phi/\Phi_0)]$$

where $\chi_1 - \chi_2 = 2\pi\Phi/\Phi_0$, $\Phi_0 = h/e$.

Joint detection probability, timescales and statistics.

Joint detection probability

$$P_{\alpha\beta} \propto \langle c_\beta^\dagger(t) c_\alpha^\dagger(t+\tau) c_\alpha(t+\tau) c_\beta(t) \rangle$$

HBT-geometry, thermal sources ($\omega_0 - \Delta\omega/2 < \omega < \omega_0 + \Delta\omega/2$)

$$\begin{aligned} P_{\alpha\beta} \quad &\propto \quad |s_{2\alpha}|^2|s_{2\beta}|^2\,[1 \pm g(\tau)] + |s_{3\alpha}|^2|s_{3\beta}|^2\,[1 \pm g(\tau)] \\ &+ \quad |s_{2\alpha}|^2|s_{3\beta}|^2 + |s_{3\alpha}|^2|s_{2\beta}|^2 \pm g(\tau)\left[s_{2\alpha}^* s_{3\beta}^* s_{2\beta} s_{3\alpha} + s_{2\alpha} s_{3\beta} s_{2\beta}^* s_{3\alpha}^*\right] \end{aligned}$$

with

$$g(\tau) = \sin^2(\Delta\omega\tau)/(\Delta\omega\tau)^2$$

and $-(+)$ corresponding to electrons (photons), $\Delta\omega = eV/2\hbar$ for electrons.

Short timescales, $\tau \ll 1/\omega$

- Photons bunch, electrons antibunch.
- Interference term depend on statistics.

Short time observables.

Time-dependence of current cross-correlators

$$\langle \Delta I_\alpha(t) \Delta I_\beta(t+\tau) \rangle = \frac{e^2}{h^2} \int\int dE dE' e^{i(E-E')\tau/\hbar}$$

$$\times \sum_{\gamma,\delta} s^*_{\alpha\gamma} s_{\alpha\delta} s^*_{\beta\delta} s_{\beta\gamma} \left[f_\gamma(E) - f_0(E) \right] \left[f_\delta(E') - f_0(E') \right]$$

for no scattering between α and β,

$$s_{\alpha\beta} = 0$$

This is the case for the HBT-geometry.

Integrals restricted to energies around E_F $\Rightarrow$ Only low-frequency observables for arbitrary τ.

Proportional to joint detection probability $\Rightarrow$

$$\langle c^\dagger_\beta(t) c^\dagger_\alpha(t+\tau) c_\alpha(t+\tau) c_\beta(t) \rangle \propto \langle I_\alpha(t) I_\beta(t+\tau) \rangle$$