Quantum phenomena at low temperatures, Lammi, January 2004

Superfluid turbulence

An overview

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Outline

- Overall aim: to give a brief overview of the subject, seeing how the various aspects fit together. Serves as introduction to later papers.

- Introductory comments about classical turbulence.

- Quasi-classical, full-developed, turbulence in a superfluid, especially ^{4}He.

- Dissipation in superfluid turbulence.

- Turbulence on thermal counterflow.

- The effect of a very viscous normal fluid: ^{3}He-B.

- The nucleation of superfluid turbulence.

Classical turbulence

- Classical flow is described by the Navier-Stokes equation:

$$\frac{\partial \mathbf{v}}{\partial t} + \underbrace{(\mathbf{v}\cdot\nabla)\mathbf{v}}_{\text{Non-linear inertial term}} = -\frac{1}{\rho}\nabla p + \underbrace{\nu\nabla^2\mathbf{v}}_{\text{Viscous term}}$$

ν = kinematic viscosity

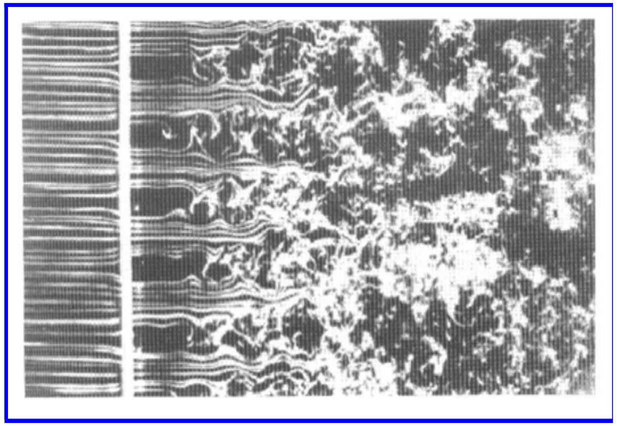
- Suppose that flow is characterized by a single characteristic velocity *U* and a single characteristic length *L*.

- $$\frac{\text{inertial term}}{\text{viscous term}} \sim \frac{LU}{\nu} = \mathrm{Re}$$

Re = Reynolds no.

- Re<<1: stable laminar flow

- Re>>1: laminar flow unstable → turbulence. For very large Re we get fully-developed turbulence.



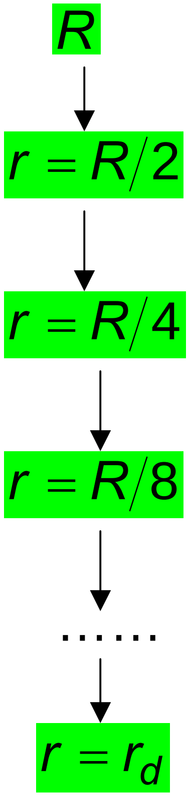
- Turbulence involves rotational motion. We often think of it loosely in terms of a superposition of eddies. Here we consider mostly fully-developed turbulence.

Inertial cascades in classical turbulence

- Suppose initially the flow produces large eddies, of size *R;* characteristic velocity *U,* with large Reynolds number $\mathrm{Re}_R = RU/\nu$.

- On scale *R* the viscous term can be neglected. The non-linear inertial term causes energy to flow into other, especially smaller but neighbouring, length scales, with a characteristic time $\tau_R = R/U$, the “turnover time”.

- This process continues, producing smaller and smaller eddies, characterized by sizes *r* and velocities u_r . Each size of eddy has associated with it a scale-dependent Reynolds number $\mathrm{Re}_r = ru_r/\nu$ and $\tau_r = r/u_r$. As long as there is no significant dissipation we have an ***inertial range cascade*** (Richardson cascade).



R

$r = R/2$

$r = R/4$

$r = R/8$

……

$r = r_d$

- What do we mean by significant dissipation? The time taken for viscosity to destroy an eddy of size *r* is given by $\tau_d = r^2/\nu$. Dissipation will have a significant effect only if $\tau_d \le \tau_r$; i.e. if $\mathrm{Re}_r \le 1$

- The inertial-range cascade will be terminated at $r = r_d$, where $\mathrm{Re}_{rd} = 1$

The Kolmogorov spectrum (homogeneous turbulence)

- There is a flow of energy at a rate ε per unit mass down the cascade.
- Eddies of size *r* have energy per unit mass $E_r = u_r^2$
- They lose this energy in a time $\tau_r = r / u_r$
- Therefore $\varepsilon = u_r^2 / \tau_r = u_r^3 / r$

- In a steady-state inertial range ε is independent of *r* as long as $r > r_d$
- Therefore in the inertial range $E_r = u_r^2 = \varepsilon^{2/3} r^{2/3}$

- This is a rough statement of the **Kolmogorov spectrum**, usually written more rigorously as $E(k) = C\varepsilon^{2/3} k^{-5/3}$

  where $E(k)dk$ is the turbulent energy per unit mass associated with Fourier components of the velocity field in the range *dk*, and $C \approx 1.6$

- The energy ε is ultimately dissipated by viscosity near the length scale r_d :

  $\varepsilon = \nu \langle \omega^2 \rangle$ where $\langle \omega^2 \rangle$ is the mean square vorticity.

Turbulence in a superfluid

- Must take account of: two-fluid behaviour, at least at high temperatures

rotation of superfluid component possible only through presence of quantized vortex lines,
circulation $\kappa = h/m_4$ or $h/2m_3$

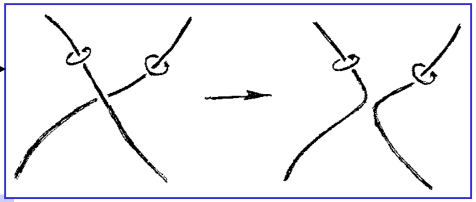
mutual friction $\mathbf{f}_D = -\gamma_0 \hat{\kappa} \times [\hat{\kappa} \times (\mathbf{v}_n - \mathbf{v}_L)] + \gamma_0' \hat{\kappa} \times (\mathbf{v}_n - \mathbf{v}_L)$

Magnus effect $\mathbf{f}_M = \rho_s \kappa \hat{\kappa} \times (\mathbf{v}_L - \mathbf{v}_s)$

balance of forces $\mathbf{f}_D + \mathbf{f}_M = 0$

viscosity of normal fluid very small for ^{4}He
very large for ^{3}He

vortex reconnections →



- Hence $\mathbf{f}_D = -\alpha \rho_s \kappa \hat{\kappa} \times [\hat{\kappa} \times (\mathbf{v}_n - \mathbf{v}_s)] - \alpha' \rho_s \kappa \hat{\kappa} \times (\mathbf{v}_n - \mathbf{v}_s)_s$

$\mathbf{v}_L = \mathbf{v}_s + \alpha \hat{\kappa} \times (\mathbf{v}_n - \mathbf{v}_s) - \alpha' \hat{\kappa} \times [\hat{\kappa} \times (\mathbf{v}_n - \mathbf{v}_s)]$

If $\alpha, \alpha' << 1$, $\mathbf{v}_L \approx \mathbf{v}_s$

Superfluid turbulent flow due to vortex lines

- Think about homogeneous turbulence, such as grid turbulence.

- The vortex lines in the superfluid will take the form of a more or less random tangle. Length of line per unit volume = L; mean vortex spacing = $\ell = L^{-1/2}$

- For a completely random tangle, characteristic velocity on length scale ℓ is

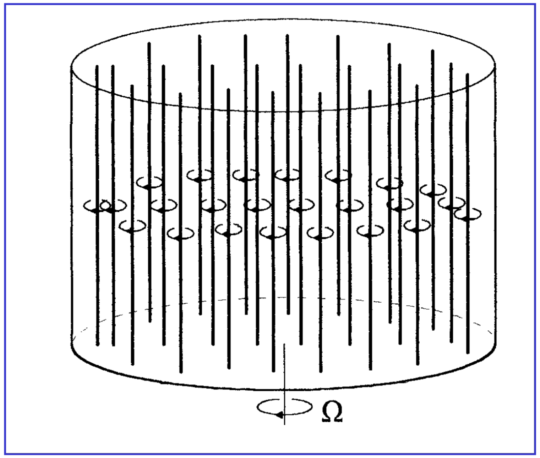
$$u_s = \kappa/\ell$$

and on larger length scales it is zero.

- Non-zero velocities on larger length scales can be achieved by partial or complete polarization of the vortex array.

- If there is no mutual friction vortices move with local superfluid velocity, and suitable arrays can mimic classical flow on scales larger than ℓ.

- The time-evolution of the superfluid velocity on scales greater than ℓ is also believed to mimic classical flow.



Quasi-classical superfluid turbulence

- This suggests that on scales larger than ℓ, in the absence of mutual friction, superfluid turbulence will be essentially classical.

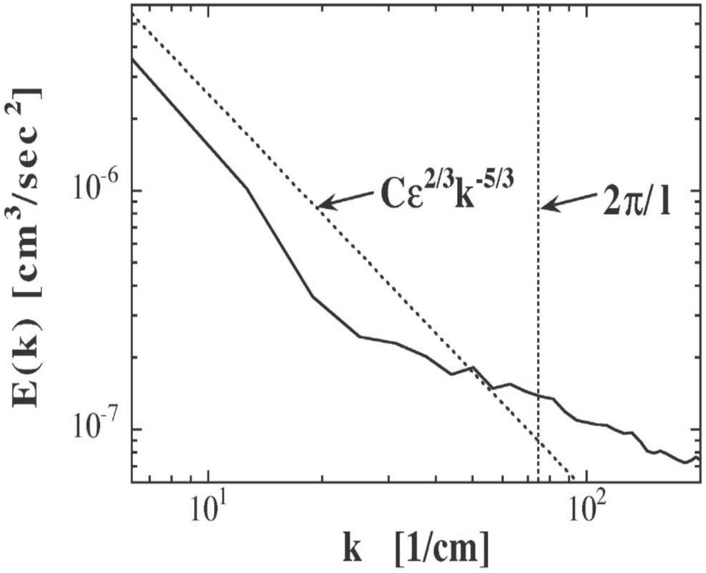
i.e. there will be an inertial-range cascade, with energy flowing at rate ε down the cascade; and there will be a Komogorov spectrum

$$E(k) = C\varepsilon^{2/3}k^{-5/3}$$

- There are two ways to avoid mutual friction
  - Work at very low temperature, where $\rho_n \approx 0$
  - Make $v_s = v_n$

- Experiments on fully-developed turbulence at very low temperatures, with ^{3}He-B or ^{4}He, are planned but not yet carried out.

Simulations (Tsubota et al) suggest that Kolmogorov spectrum is indeed set up.



- Evidence from quasi-classical turbulence in ^{4}He above 1K, when $v_s = v_n$

Quasi-classical turbulence in superfluid ^{4}He above 1K

- Maurer-Tabeling experiment: counter-rotating blades in helium; observe frequency spectrum of pressure fluctuations.

- Oregon towed-grid experiment: observe attenuation of second sound; measures time-dependence of *L*.

- Results consistent with following picture

  - Both fluids become turbulent, with same velocity field (Kolmogorov spectra) on scales greater than ℓ. I.e. this is a case where $v_s = v_n$

  - Possible only in ^{4}He, in which normal fluid has very low viscosity.

  - A numerical accident means that Kolmogorov dissipation length in normal fluid ~ ℓ: therefore matching of velocity fields on scales > ℓ is possible.

- We believe that this matching of the velocity fields occurs naturally, in the sense that a Kolmogorov spectrum is set up in both fluids independently. Mutual friction serves only to ensure exact matching of the two velocity fields. Evidence: classical Kolmogorov spectrum is found even when $\rho_n / \rho << 1$

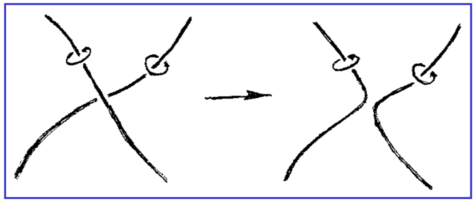
Dissipation on small length scales

- The existence of an inertial-range Kolmogorov spectrum

$$E(k) = C\varepsilon^{2/3}k^{-5/3}$$

for $k\ell < 1$ implies the existence of dissipation at rate ε for $k\ell > 1$

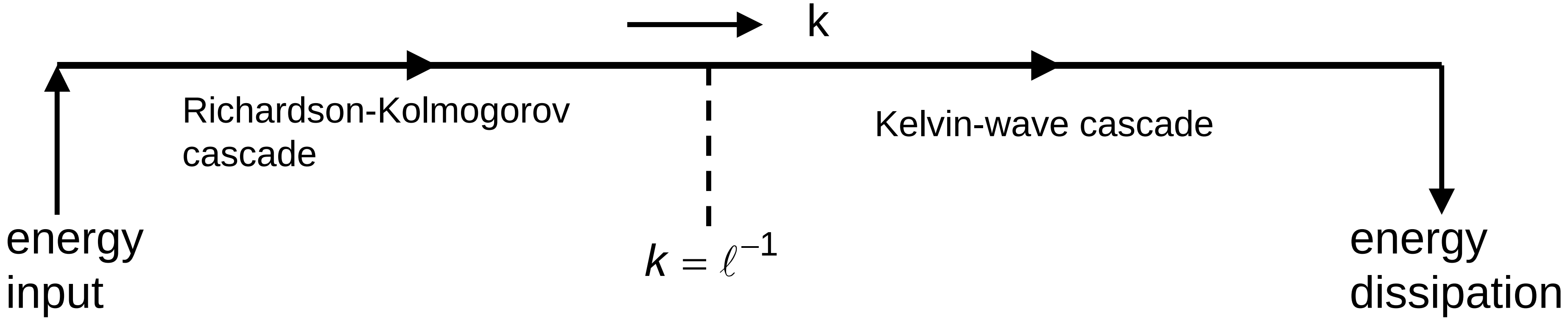
- What is the origin of this dissipation?
- The answer for ^{4}He depends on the temperature (discuss ^{3}He later).
  - For $T \geq 1\text{K}$ dissipation occurs on scale slightly less than ℓ by combination of mutual friction and normal-fluid viscosity (^{4}He only).
  - For $T < 1\text{K}$ dissipation is believed to occur at very small length scales and is associated with reconnections.



- Some energy is lost during reconnections.
- Reconnections can produce very small vortex rings that can escape from the turbulent region
- Reconnections leave kinks on the vortex lines

Dissipation on small length scales (cond)

- These kinks can be viewed as superpositions of harmonic Kelvin waves. The Kelvin waves build up in amplitude giving rise through non-linear coupling to a **Kelvin wave cascade** that takes energy to higher and higher wavenumbers. Eventually energy is transferred to Kelvin waves of very high wavenumber where it is dissipated by phonon radiation or by a very weak residual mutual friction. (Vinen, Tsubota and Mitani, Phys.Rev.Letters, 91, 135301 (2003))



k

Richardson-Kolmogorov cascade

Kelvin-wave cascade

energy input

$k = \ell^{-1}$

energy dissipation

- It can be shown, partly from experiment and partly from theory, that the energy dissipation can be written

$$\varepsilon = \nu'(\kappa L)^2$$

where $\nu' \sim \kappa$ is an effective kinematic viscosity. Compare $\varepsilon = \nu\langle\omega^2\rangle$

Turbulence associated with thermal counterflow in ^{4}He

- So far we have focussed on flows where mutual friction plays a minor role.

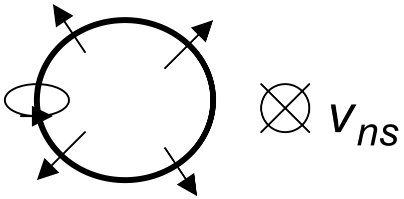
- Historically, the first type of superfluid turbulence to be discovered was one in which mutual friction plays an important role. No classical analogue.

- In thermal counterflow there is a heat flux $W = \rho ST\langle v_n \rangle$ and no net mass flow $\rho_s\langle v_s \rangle + \rho_n\langle v_n \rangle = 0$

- Focus on simplest case: everything spatially homogeneous.

- The two fluids are forced to move with different average velocities. Any turbulence in the superfluid component must then be severely affected by mutual friction. Does mutual friction kill the turbulence? No, it can actually serve to maintain it.

- To understand this, consider the behaviour of a vortex ring placed normal to $v_{ns} = v_n - v_s$.



The mutual friction will cause the ring to expand, thus generating extra vortex line (the ring must be sufficiently large for a given counterflow velocity).

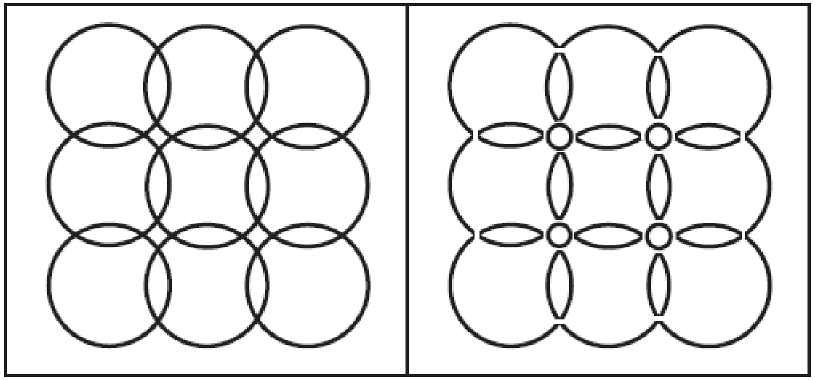
Neglect α' in the rest of this talk.

Thermal counterflow (cond.)

- This process, by itself, cannot lead to self-sustaining turbulence. We need an additional process: the vortex reconnection (Schwarz simulations).

Reconnections in an assembly of rings can lead to the generation of new rings, some of which have the right orientation for further expansion.

Really 3D: oversimplified



- Thus mutual friction plus vortex reconnections can lead to the maintenance of superfluid turbulence. (Strictly speaking there is a balance between generation and decay of turbulence, leading to a steady state, with constant time-averaged value of *L*.)

- Note that there need be no cascade. The turbulence need exist only on a scale of order ℓ.

- There is a single characteristic turbulent velocity of order κ/ℓ. Scaling then requires that

$$\frac{\kappa}{\ell} = \text{const. } v_{ns}; \quad \text{i.e. } L = \ell^{-2} = \text{const. } v_{ns}^2$$

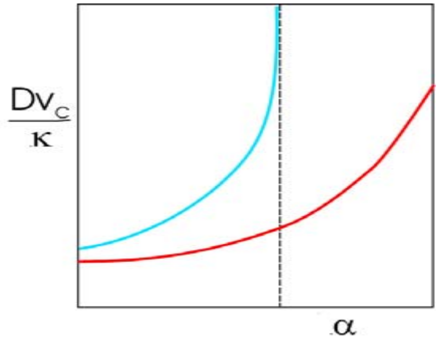
as observed

Counterflow turbulence (cond.)

- The Schwarz simulations relate to a flow of the normal fluid that is constrained to be spatially uniform. If this condition is relaxed we may find that the normal fluid flow becomes unstable (Melotte and Barenghi), leading perhaps to large-scale turbulence in both fluids. But this large-scale turbulence is not essential to the production of counterflow turbulence in the superfluid component.

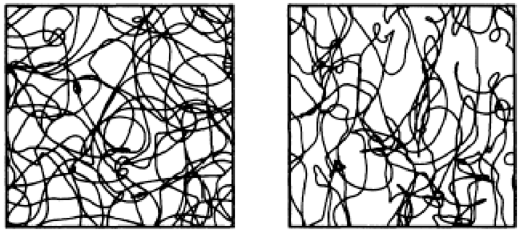
- In a channel of finite width (*D*) there is a critical velocity, v_C , below which the small-scale counterflow turbulence cannot be maintained owing to vortex annihilation at the boundaries (Schwarz).

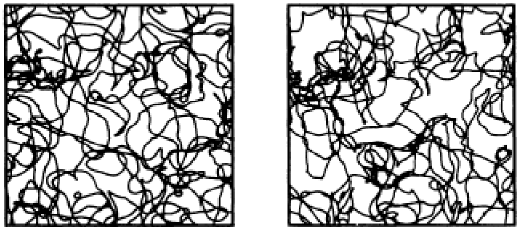
- This critical velocity increases if the mutual friction becomes very large $(\alpha \geq 1)$



- Increase in α changes vortex dynamics → increasing anisotropy→ decreasing reconnection rate? Need for further study (Tsubota).


$\alpha = 1.00$


$\alpha = 0.10$


$\alpha = 0.01$

Quasi-classical turbulence in ^{3}He-B

- The final case we consider is the possibility of quasi-classical (e.g. grid) turbulence in ^{3}He-B.

- Probably we can have such turbulence at very low temperatures, where the normal fluid is absent. Dissipation mechanisms may involve Caroli-Matricon states.

- But what happens at higher temperatures where there is a significant fraction of normal fluid? Remember that the normal fluid in ^{3}He-B is very viscous, so that turbulence in this component can hardly occur in practical sizes of channel.

- Then we must consider the effect of mutual friction on scales greater than ℓ.

- Volovik's recent work. Based on the idea that the mutual friction acts only on small length scales. Volovik's argument relates to a situation where the vortex lines are locally completely polarized, when a course-grained force of the mutual friction per unit mass can be written as

$$\mathbf{F}_D = \alpha\hat{\boldsymbol{\omega}} \times \left(\boldsymbol{\omega} \times \mathbf{v}_s\right) \qquad \left[v_n = 0\,;\ \boldsymbol{\omega} = \mathrm{curl}\mathbf{v}_s\right]$$

Volovik argues that because the mutual friction is proportional to ω, which is concentrated at small length scales for a Kolmogorov spectrum, the mutual friction itself acts only at small length scales.

Quasi-classical turbulence in ^{3}He-B (cond)

- Physically (and apart from geometrical factors)

$$F_D = [\alpha\kappa] L \langle v_L \rangle = [\alpha\kappa] L \langle v_s \rangle$$

Course-grained average velocities

- Therefore the force acts equally on all length scales greater than ℓ.

(This idea is used in many applications: eg the temperature gradient in a counterflow heat current; the use of second sound as a probe of vortex line density.)

- Given this result, how is a Kolmogorov spectrum affected by mutual friction?

- Mutual friction causes eddies of size *r* to be damped out with the time constant

$$\tau'_d = (\alpha\kappa L)^{-1} = \alpha^{-1}\kappa^{-1}\ell^2$$

independent of *r*

Therefore mutual friction is important if

$$\tau'_d < \tau_r$$

This condition can be expressed in terms of an effective scale-dependent Reynolds number

$$\mathrm{Re}'_r = \frac{u_r \ell^2}{\alpha\kappa r} < 1$$

But for a Kolmogorov spectrum $u_r = \varepsilon^{1/3} r^{1/3}$

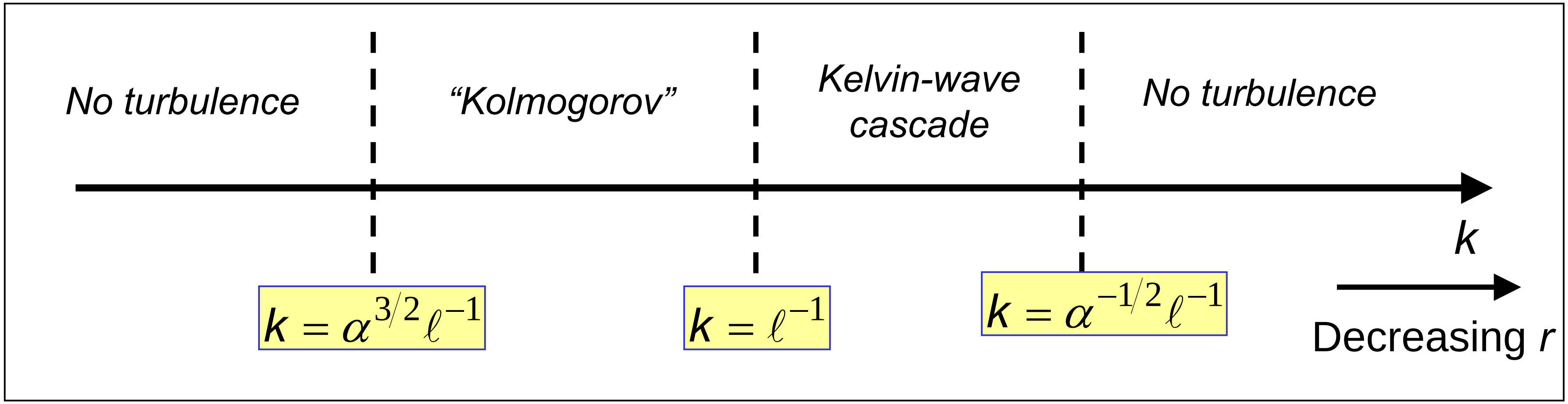
Therefore mutual friction is important if

$$\mathrm{Re}'_r = \frac{\varepsilon^{1/3}\ell^2}{\alpha\kappa r^{2/3}} < 1$$

Quasi-classical turbulence in ^{3}He-B (cond)

- Thus we find the strange result that Re'_r decreases with increasing *r* ; i.e. large eddies are in effect more strongly damped than small eddies.

- A more detailed analysis yields the following tentative picture.



No turbulence

"Kolmogorov"

Kelvin-wave cascade

No turbulence

$k = \alpha^{3/2}\ell^{-1}$

$k = \ell^{-1}$

$k = \alpha^{-1/2}\ell^{-1}$

k

Decreasing *r*

The nucleation of superfluid turbulence

- We have considered only the probable structure of fully-developed steady-state superfluid turbulence.

- There are interesting questions relating to the nucleation of turbulence in the superfluid component.

- How are the vortex lines formed initially? And do different particular forms of nucleation always lead to the same fully-developed turbulent state?

- In practical experimental situations we often do not know how nucleation occurs. We can try using computer simulations to guide us, but this can be difficult if we have no idea where to start!

- But interesting and relevant new experiments are starting to appear: e.g.

  Work in Helsinki on the nucleation of “turbulent spin-up” in ^{3}He-B [introduction of an effective Reynolds number $1/\alpha$ or $q=(1-\alpha')/\alpha$]

  Work in Lancaster on nucleation in a moving grid in ^{4}He.

Summary

- We have argued that quite often fully-developed superfluid turbulence has a basic structure similar to that of classical turbulence, with a inertial-range cascade feeding energy into small structures where it is dissipated by a variety of processes.

- Superfluid turbulence generated by thermal counterflow is an exception and has no classical analogue.

- In a superfluid like ^{3}He-B, in which the normal fluid is very viscous, the effect of mutual friction might be rather unexpected.

- There is still much speculation, with a need for new experiments and new and better theory.

Thank you!