

Quantum effects in constrictions of superconductors and semiconductors

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Quantum effects in constrictions of superconductors and semiconductors

Outline of talk:

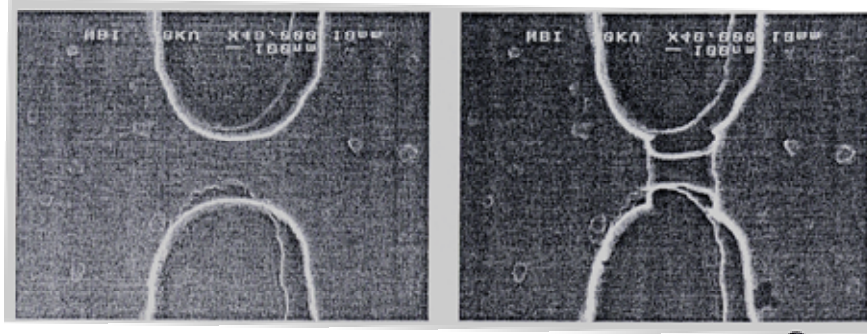
1. Diffusive constriction and heating effects
2. Superconducting microbridges
3. The ac Josephson effect ($f=2eV/h$)
4. Effects of nonequilibrium and Andreev scattering (shoulder and shgs)
5. Sharvin constriction (ballistic)
6. Quantum point contact (QPC) in a 2DEG and Conductance quantization ($G=2e^2/h$)
7. 0.7 structure and an activation energy revealed in bias spectroscopy
8. Resonances and non-linearity of QPCs
9. A combined Microbridge - point contact device
10. Detection of AC Josephson effect in QPC
11. Possible current quantization ($I=ef$) with Josephson radiation
12. Conclusion

Semiconductor/superconductor constriction

GaAs QPC



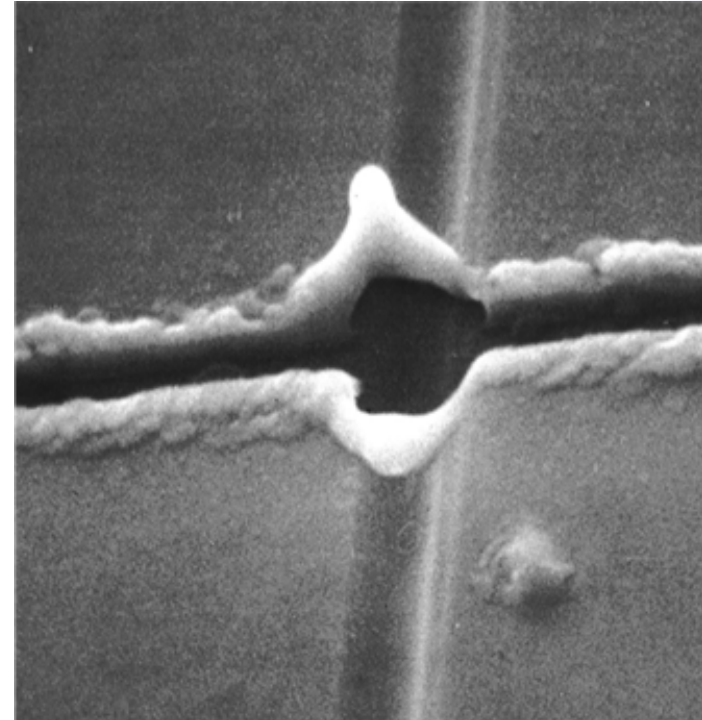
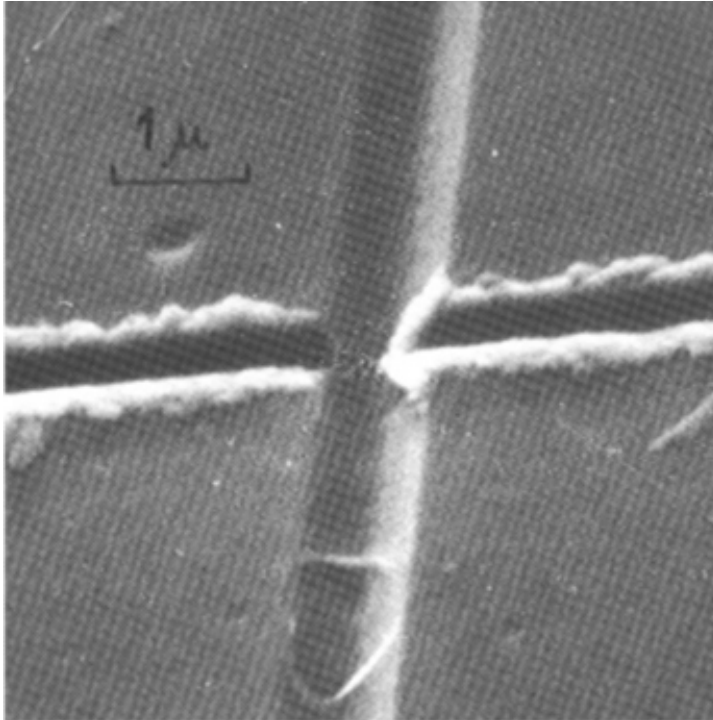
+ Al 2D microbridge + Al 3D microbridge



1 μm

S.E.Andresen, P.E.Lindelof, in Towards the Controllable Quantum States, (Eds. Takayanagi, Nitta, World Scientific 2003), pp.295

Double-scratched thin film tin microbridge



State of the art nanolithography in 1968. Razorblades with edge sharpness of about 100 nm was used. See P.E.Lindelof, Rep.Prog.Phys. 44, 949-1026 (1981) for a review. The right picture is the result of an accidental electrical pulse.

Diffusive Constrictions or microbridges, R and T

A neck of revolution with hyperbolic shape, resistivity ρ , a spec angle Θ and an opening diameter d much larger than the mean free path has a **resistance**

$$R = \rho/d \cotang(\Theta/2)$$

If $\rho = 10^{-7} \Omega\text{m}$ and $d=10^{-6} \text{m}$ and $\Theta=90^\circ$ then $R=0.1 \Omega$

-a typical value for a metallic microbridge. [J.C. Maxwell, "Electricity and Magnetism" (Oxford, 1873)]

A voltage V across the microbridge will generate a current, which due to the Joule heating will increase the **temperature in the middle of the constriction** to

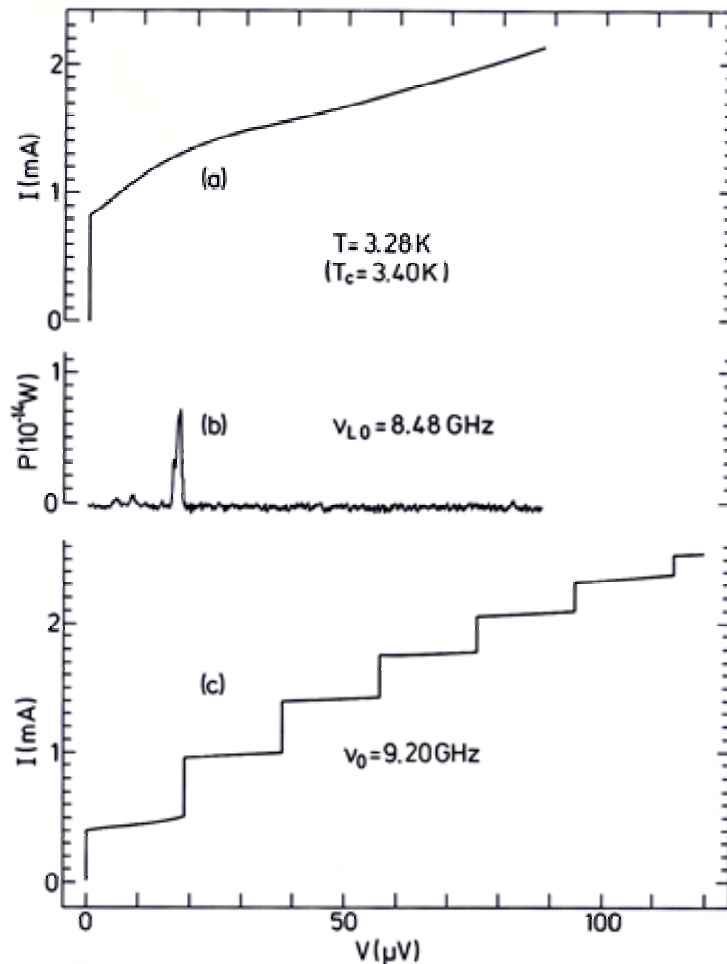
$$T = (T_o^2 + 3(eV/2\pi k_B)^2)^{1/2}$$

-Quite independent of materials constants, only dependent on the validity of Wiedemann-Franz law. [F. Kohlrusch, Ann.Phys. (Leipzig) 1, 132 (1900); R. Holm, "Electrical Contacts" (Springer, 1967)]

If ambient temperature is $T=1\text{K}$ then:

V (bias)	0.1 mV	1 mV	1 V
T (bridge temp.)	1.05 K	3.36 K	3205 K

Microbridge I-V characteristics, Josephson radiation microwave induced current steps



RSJ model:

$$I = I_c \sin\theta + \frac{h}{4\pi R} \frac{d\theta}{dt}$$

$$V_{dc} = R(I^2 - I_c^2)^{1/2}$$

$$V_1(t) = 2V_{dc} [I/I_c - ((I/I_c)^2 - 1)^{1/2}]$$

$$RI_c \text{ for } V_{dc} \gg RI_c$$

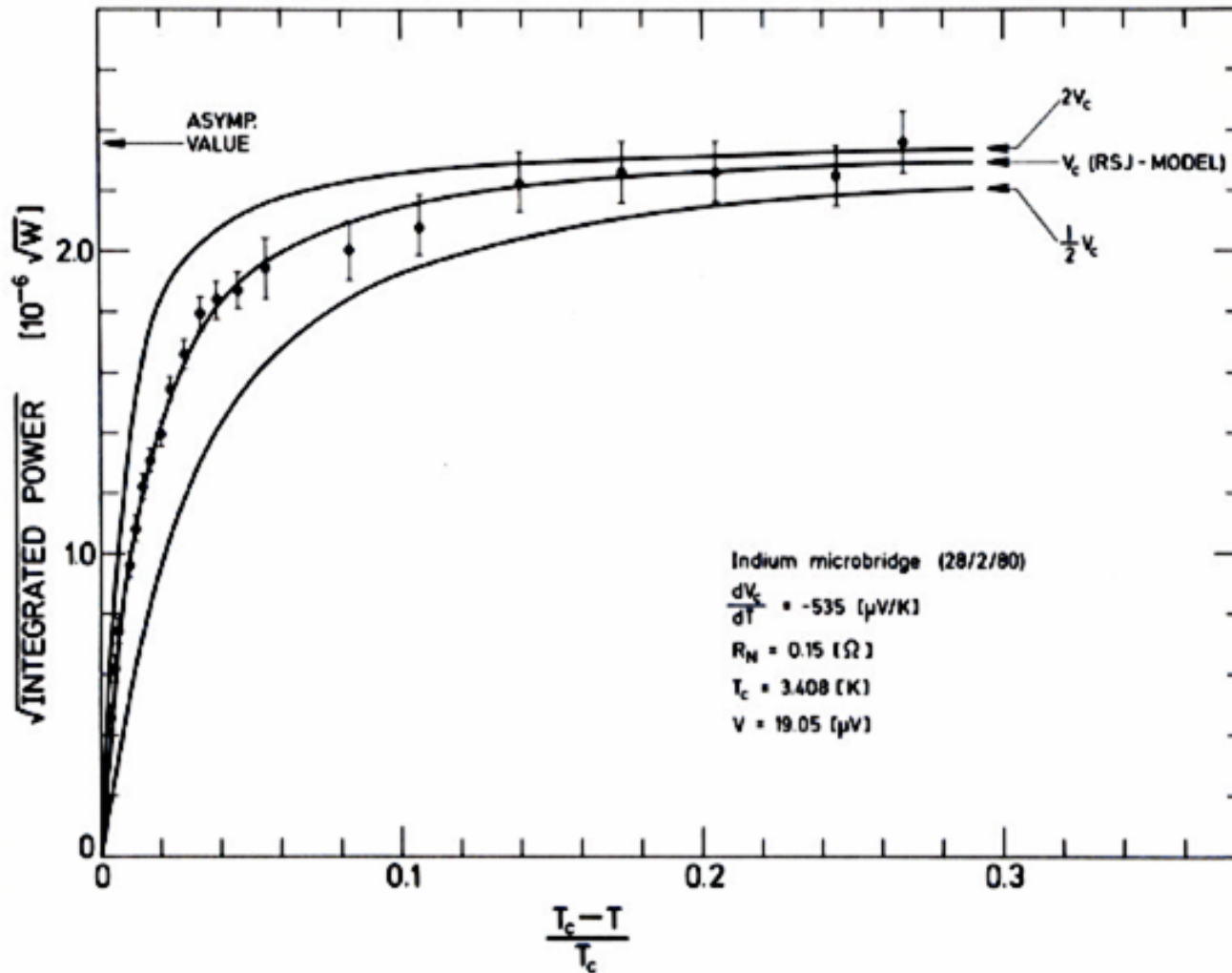
$$2V_{dc} \text{ for } V_{dc} \ll RI_c$$

$$I + I_\omega \cos(\omega t)$$

$$= I_c \sin\theta + \frac{h}{4\pi R} \frac{d\theta}{dt}$$

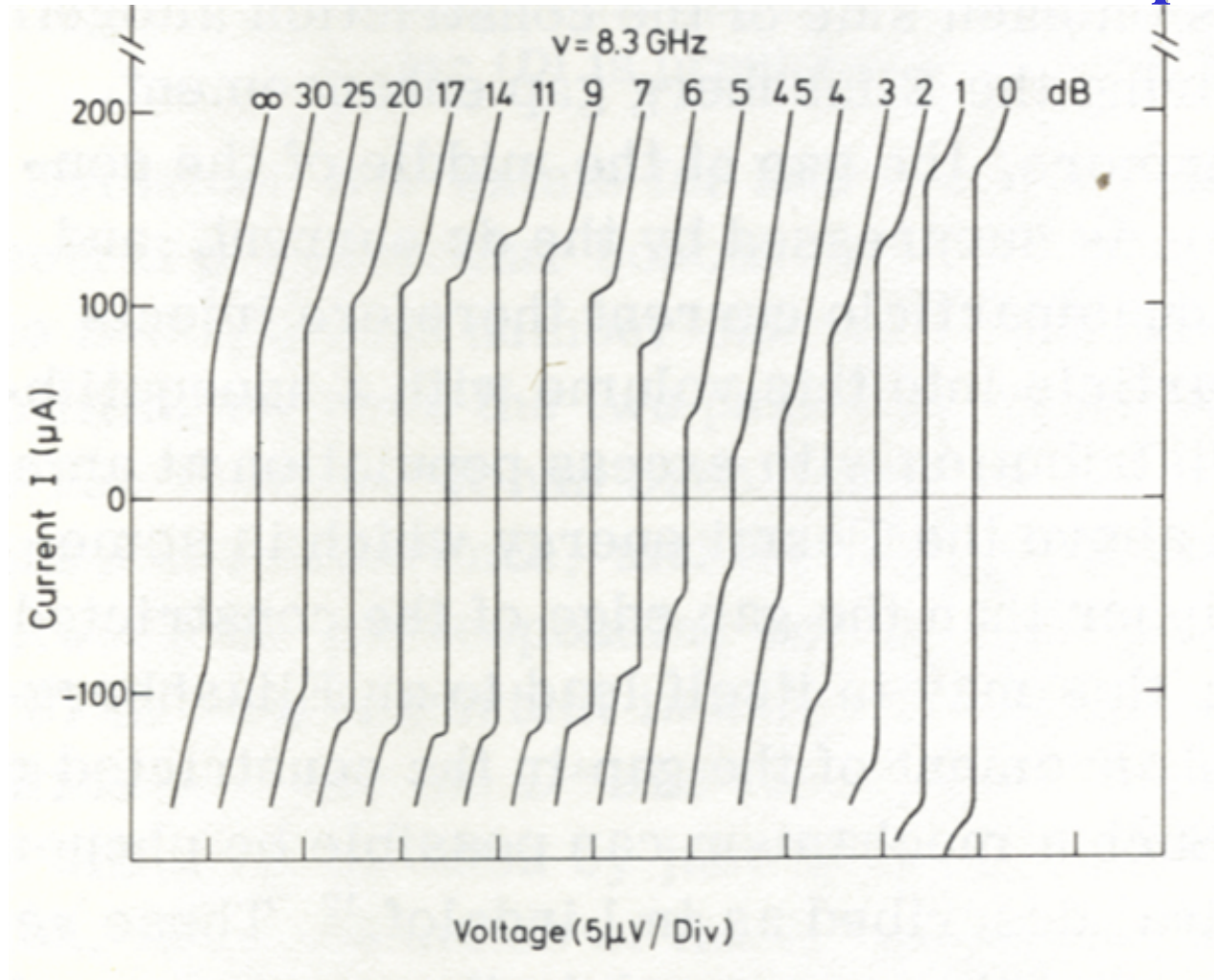
P.E.Lindelof, J.B.Hansen,
Fys.Tidskr. 82, 65 (1984)

A.C. Josephson amplitude vs. temperature, $\nu=9.2$ GHz



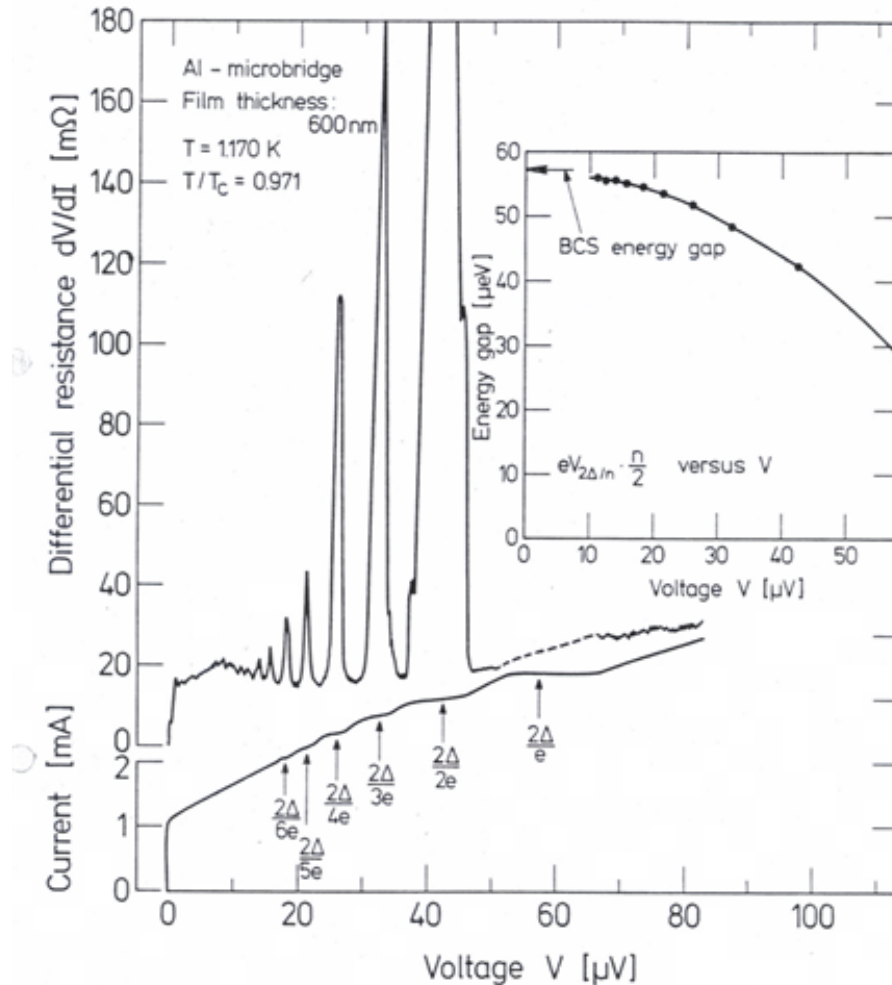
J.B.Hansen et al.,
in SQUID'80 (Eds.
Hahlbohm, Lübbig,
de Gruyter 1980),
pp. 29

Large aluminium break junction. Microwave enhancement of I_c and induced steps



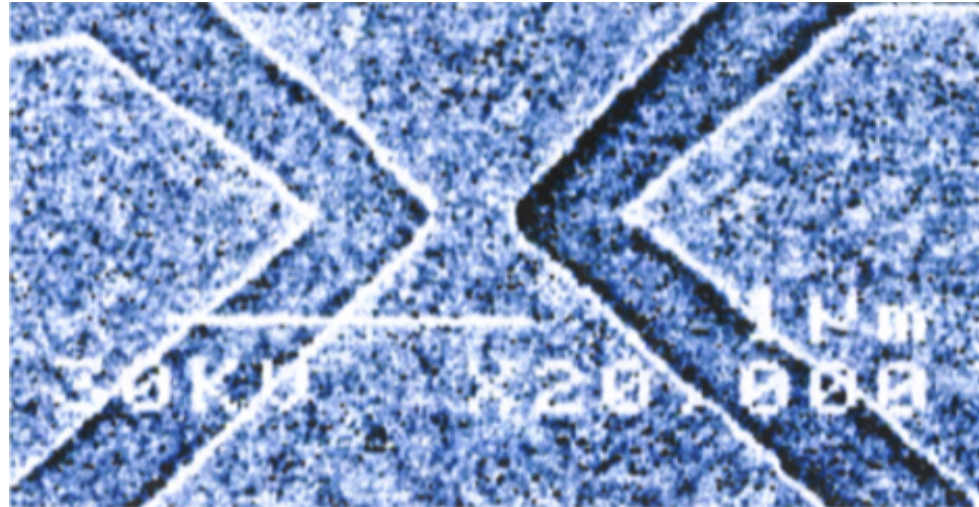
B.R.Fjordbøge et al., Phys.Rev.Lett. 37, 1302 (1976)

Subharmonic energy gap structure in superconducting microbridge



J.B. Hansen et al., Journal de Physique C6, 500 (1978)

Sidegated GaAs quantum point contact



1 μm

Sharvin constriction (ballistic) and quantization

Resistance of hole (Sharvin 1965):

$$\mathbf{R=h\lambda_F^2/(2e^2 \times \text{area})}$$

Resistance of 2-dimensional opening
(width d):

$$\mathbf{R=h\lambda_F/(4e^2 \times d)=h/2e^2 \times \lambda_F/2d}$$

If $n=\lambda_F/2d$ is quantized ($n=1,2,3,\dots$):

$$\mathbf{R=h/2ne^2}$$

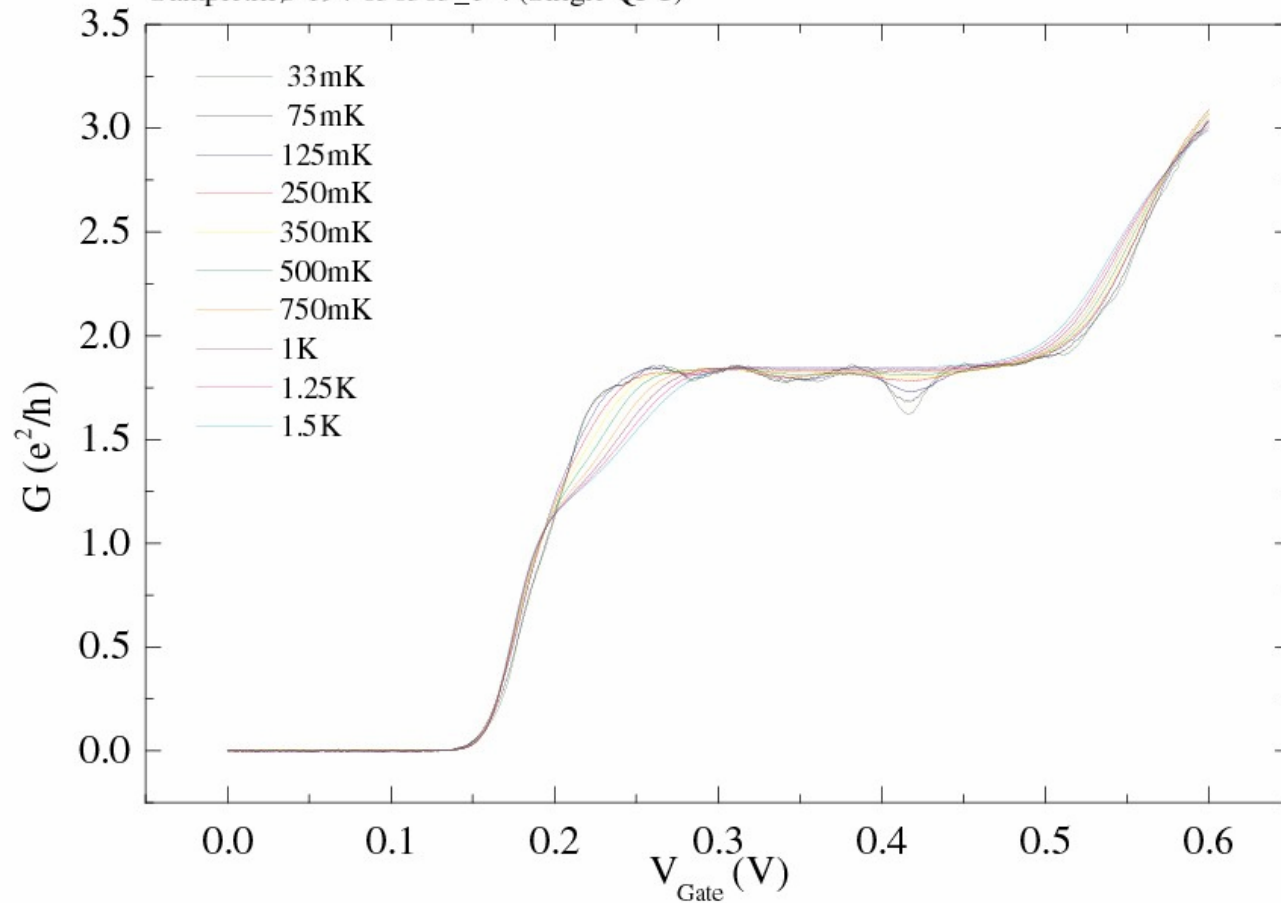
(conduction quantization)

Conductance vs. V_g and T

d.20.3.03 & 21.3.03

T:

Sample:hcø-194-030303_1-4 (Single QPC)



Saddle point potential model for QPC

$$V(x,y) = V_0 - \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_y^2 y^2.$$

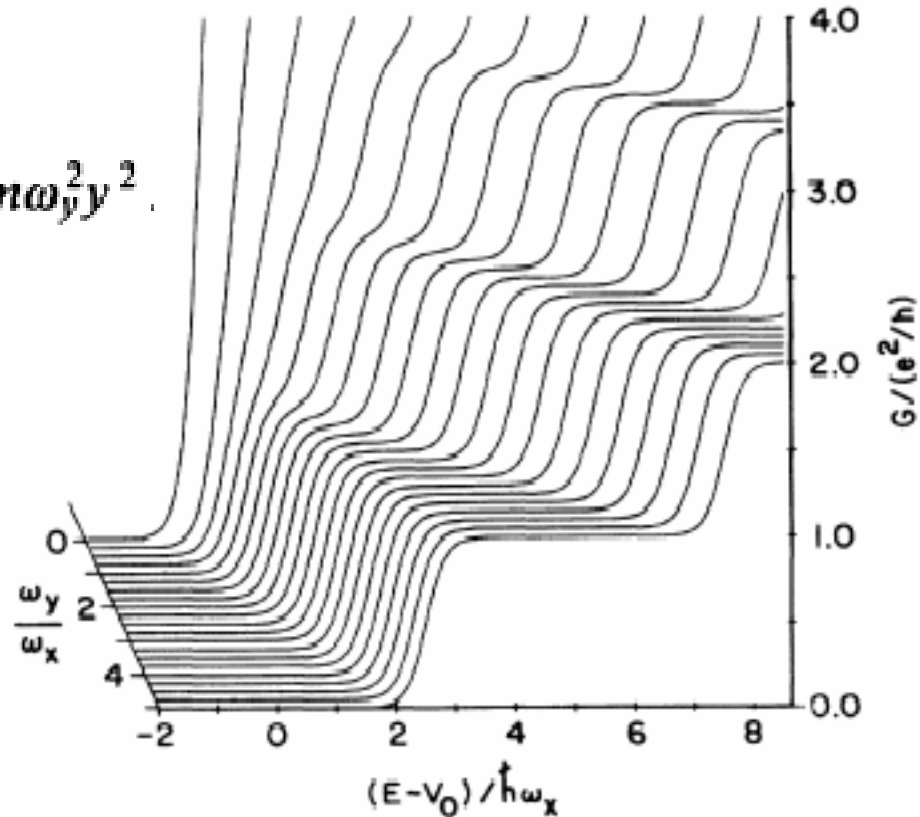
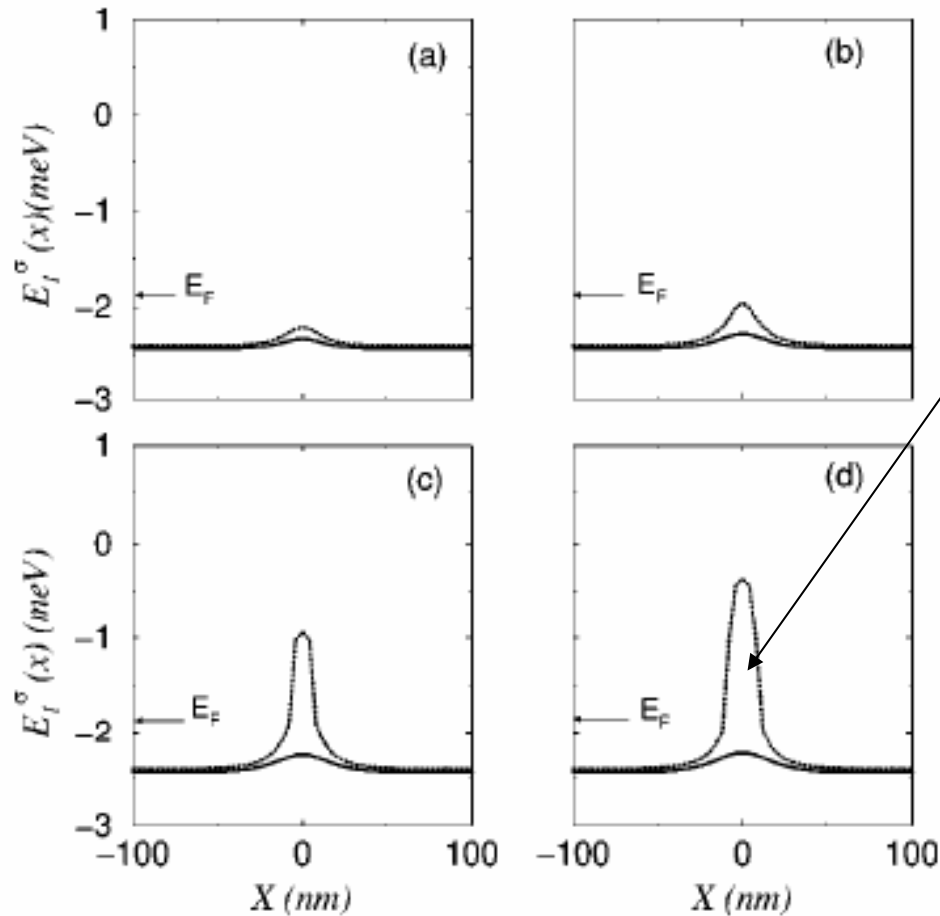


FIG. 2. Constriction conductance as a function of $(E - V_0) / \hbar \omega_x$ for differing saddle potentials characterized by a ratio ω_y / ω_x for ratios in an interval from 0 to 5 in increments of 0.25.

M. Büttiker,
Phys.Rev. B
41, 7909
(1990),

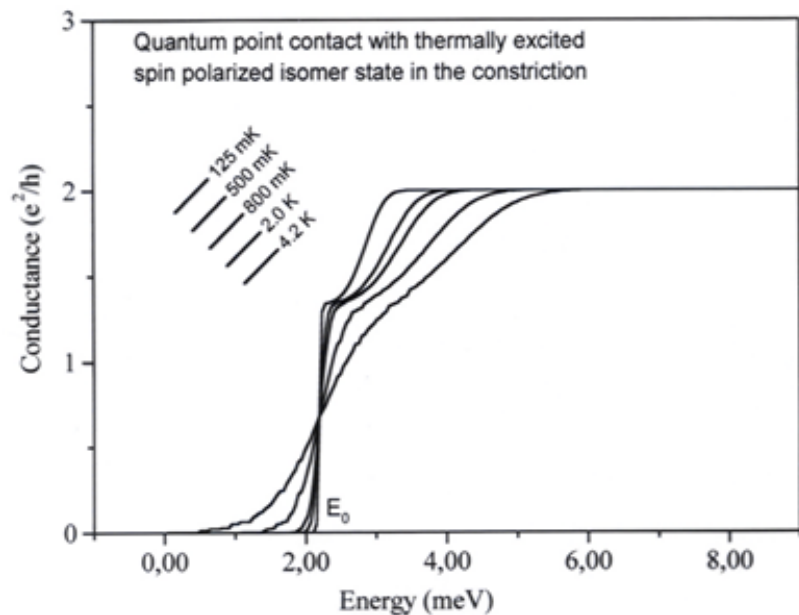
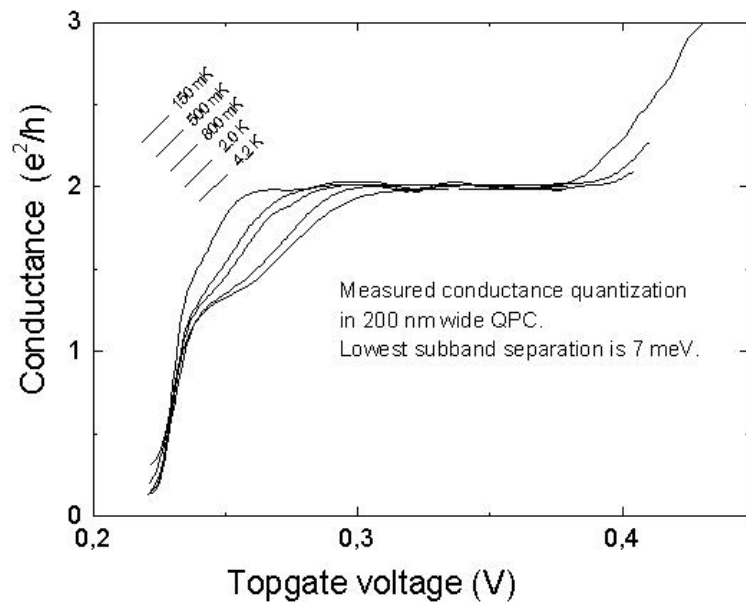
Spin density functional theory for QPC. Spin splitting in the center of the contact.



Roughly one
Bohr Magneton,
 $eh/2\pi m$

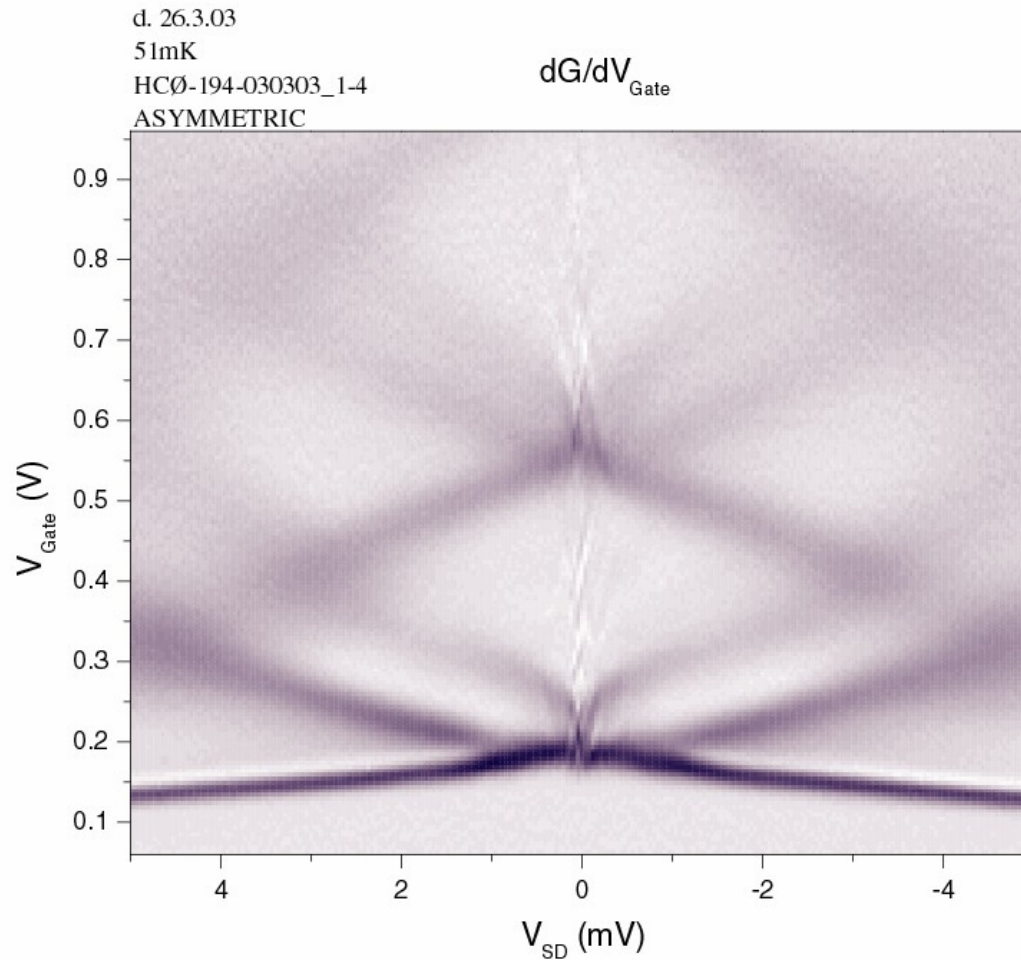
S.Wang &
K.F.Berggren,
Phys.Rev. B 57,
4552 (1998)

Calculated $0.7 e^2/h$ Conductance plateau

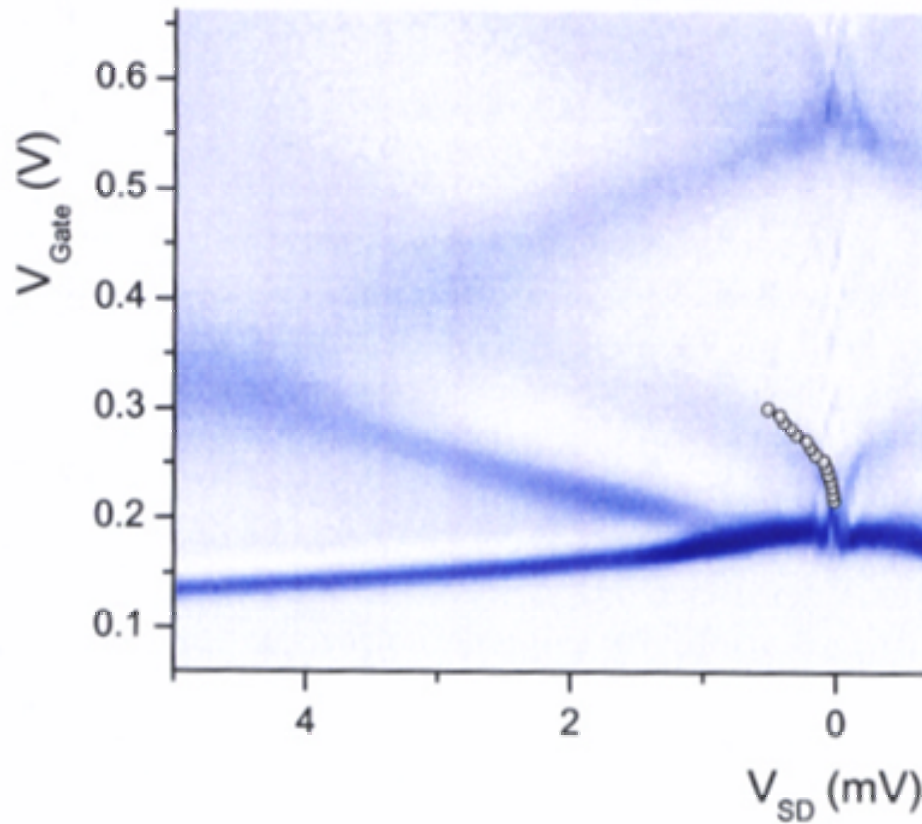


P.E.Lindelof, Proceedings of SPIE 4415, 77 (2001)

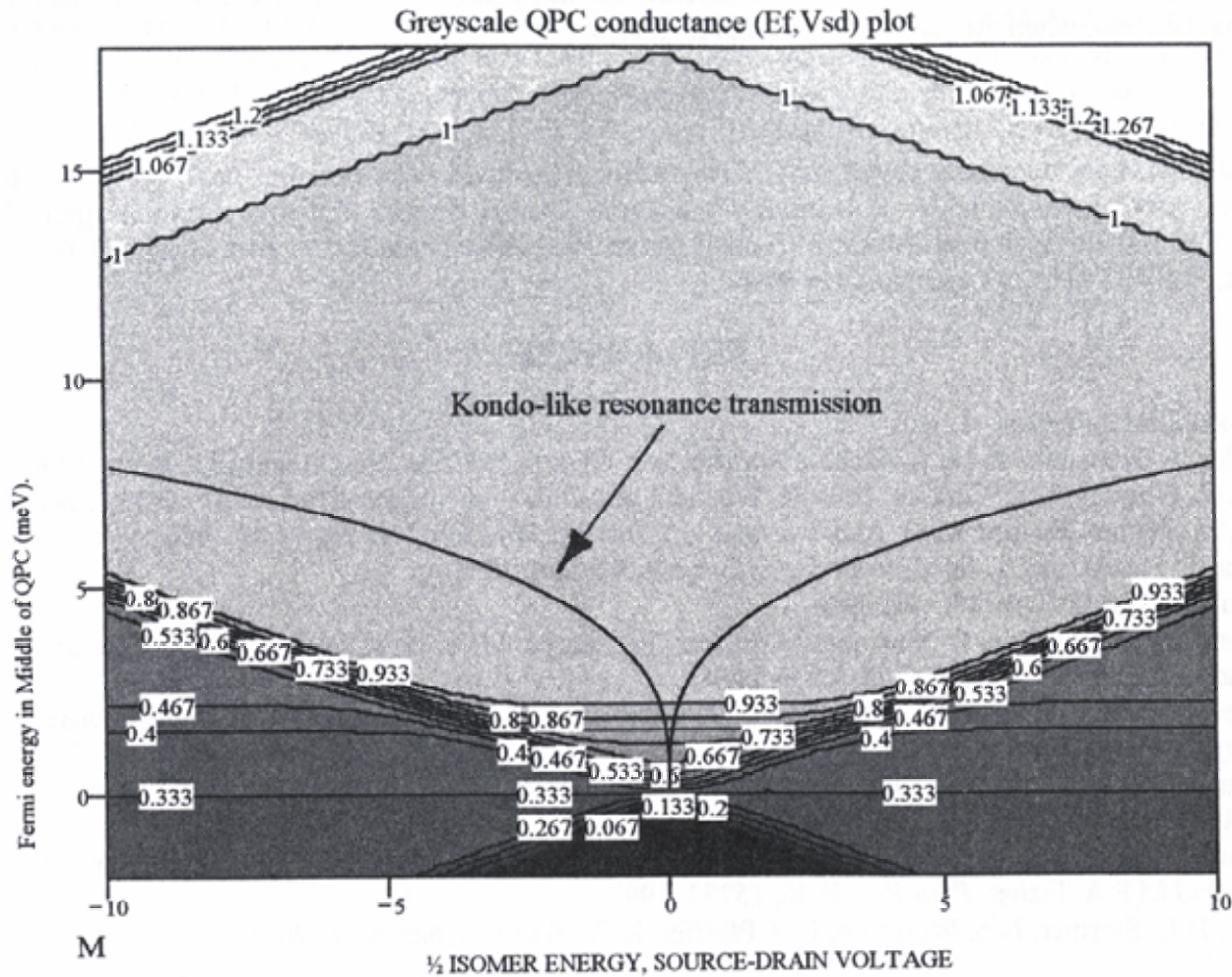
Transconductance vs. V_{SD} and V_G (Bias spectroscopy)



Activation energy of 0.7 feature inserted into bias-spectroscopy plot



Calculated Biasspectroscopy



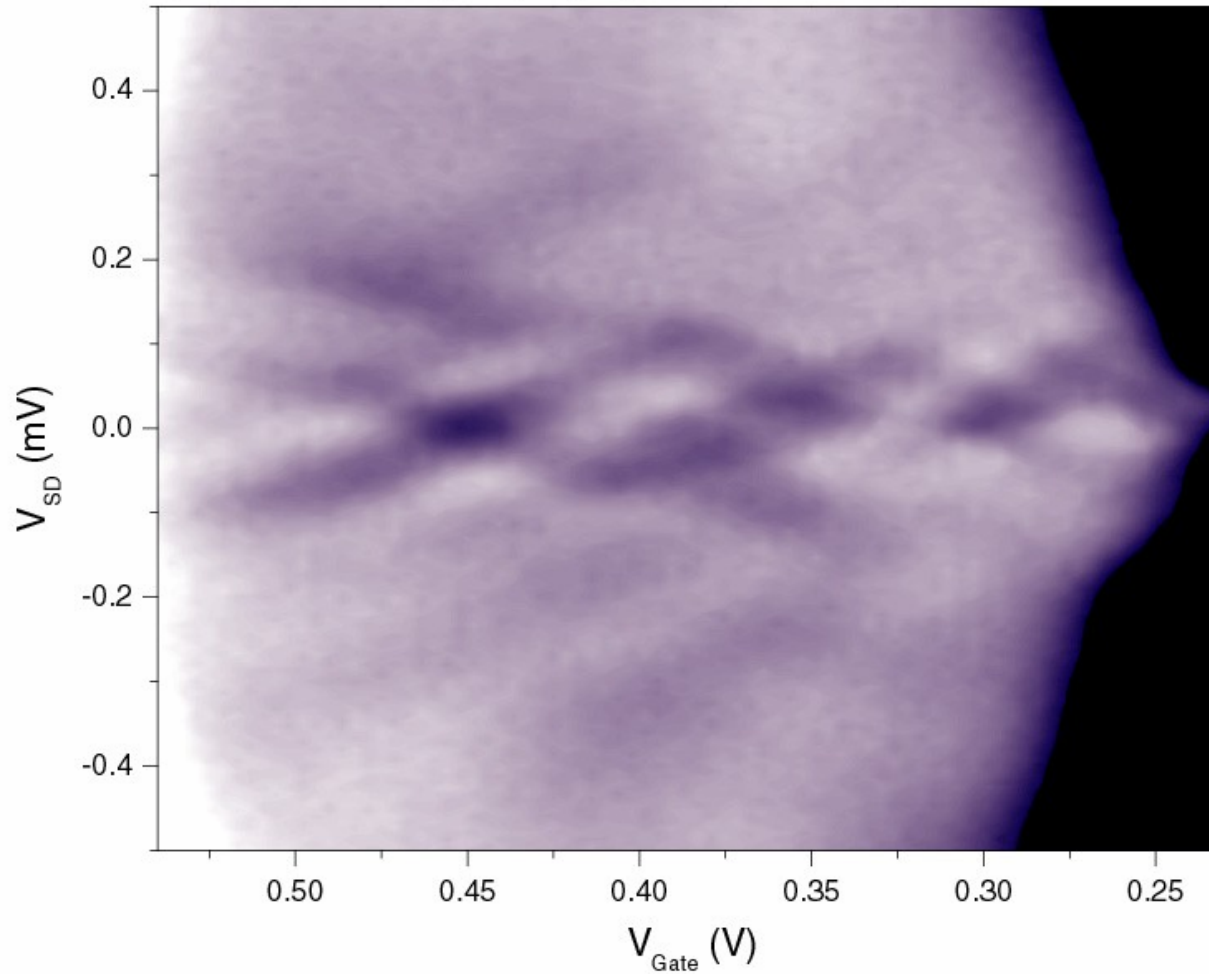
P.E.Lindelof,
Proceedings
of SPIE 4415,
77 (2001)

Conductance resonances on $2e^2/h$ plateau

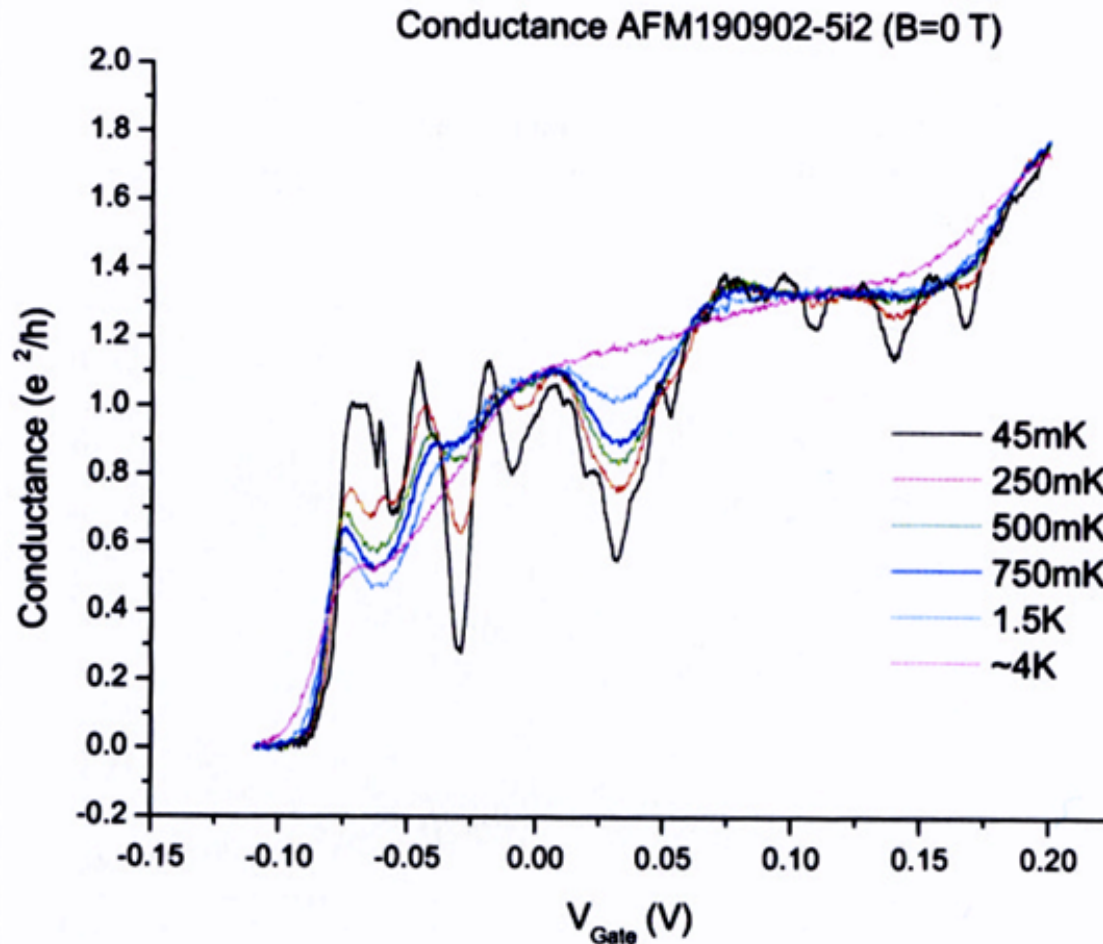
d.26.3.03

T: 63mK

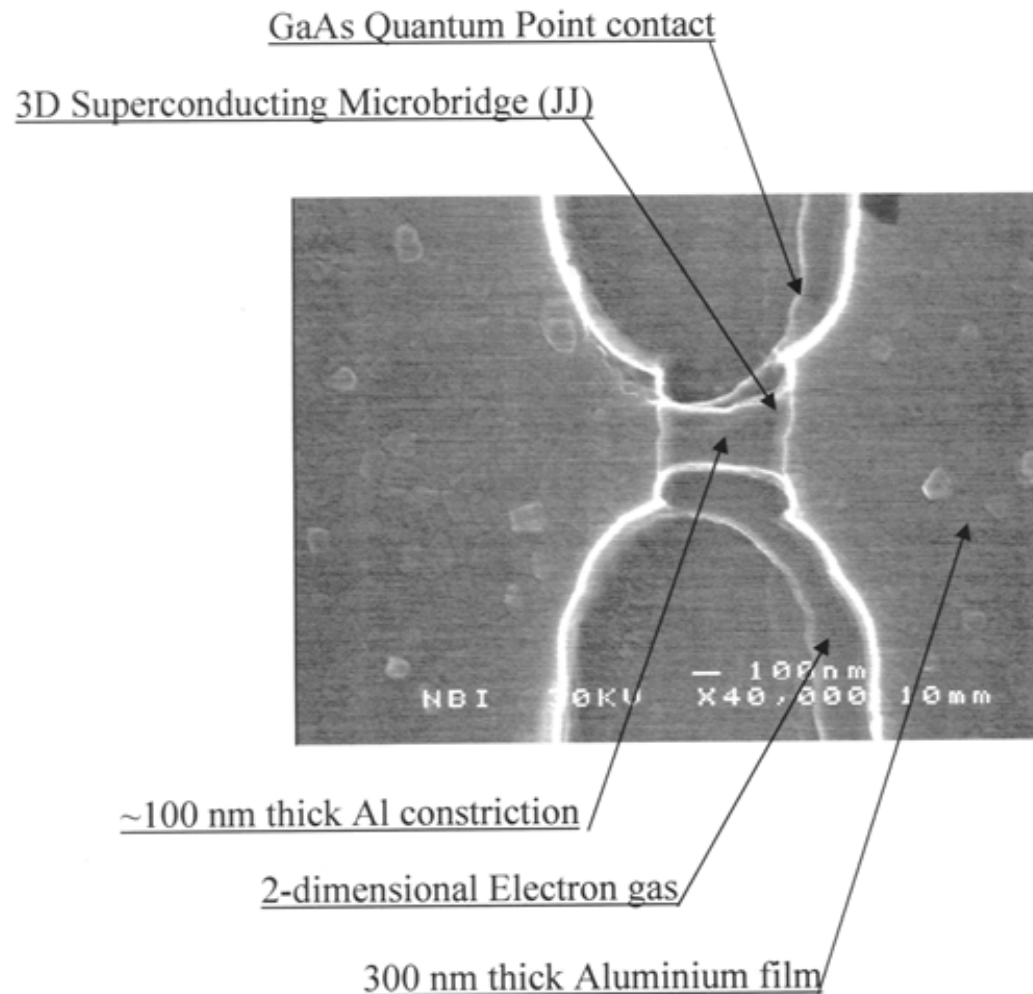
Sample:hcø-194-030303_1-4 (Single QPC)



GaAs Point contact Conductance resonances (AFM QPC with artificial impurity and large series resistance)

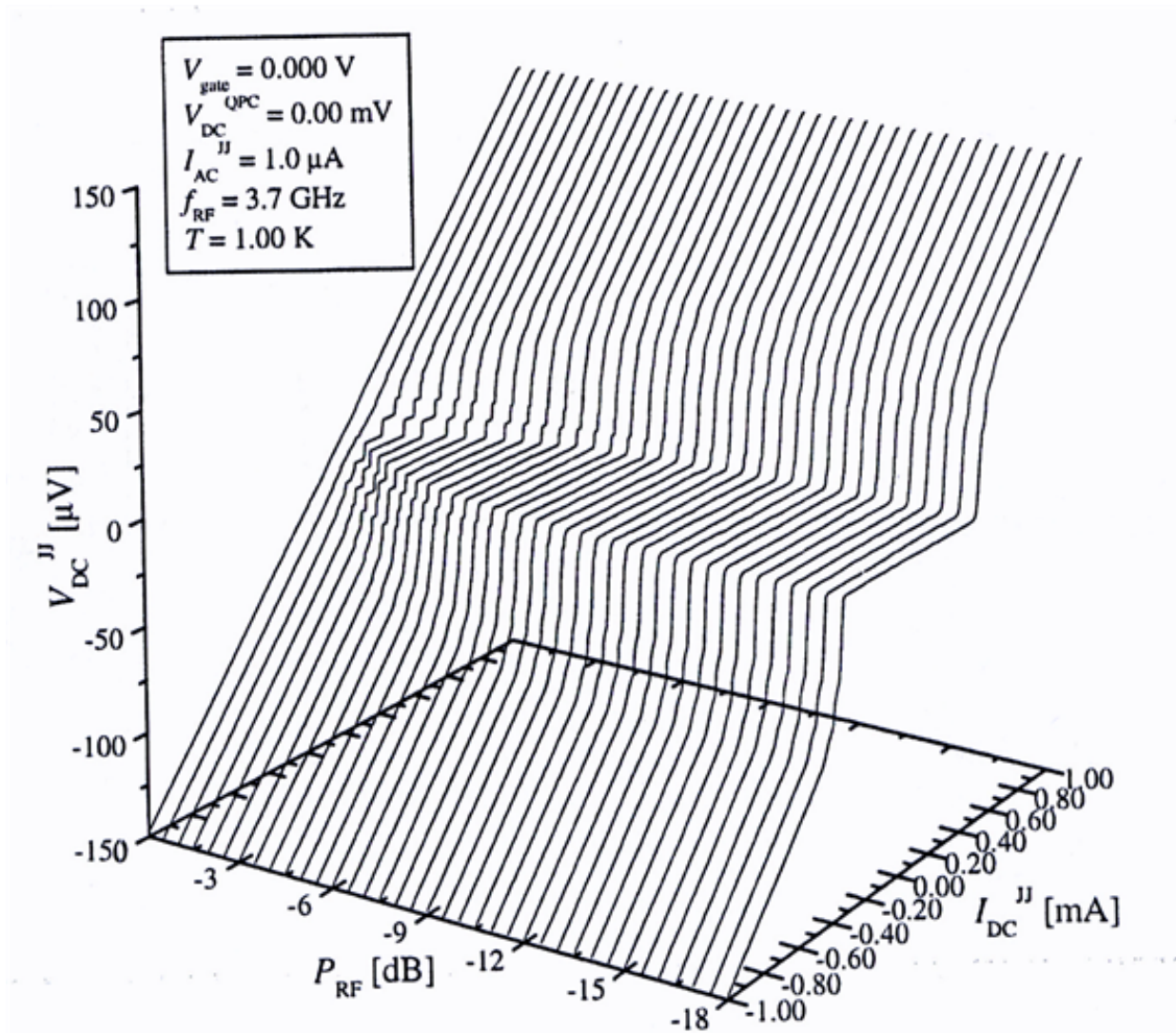


Superconducting Microbridge on QPC

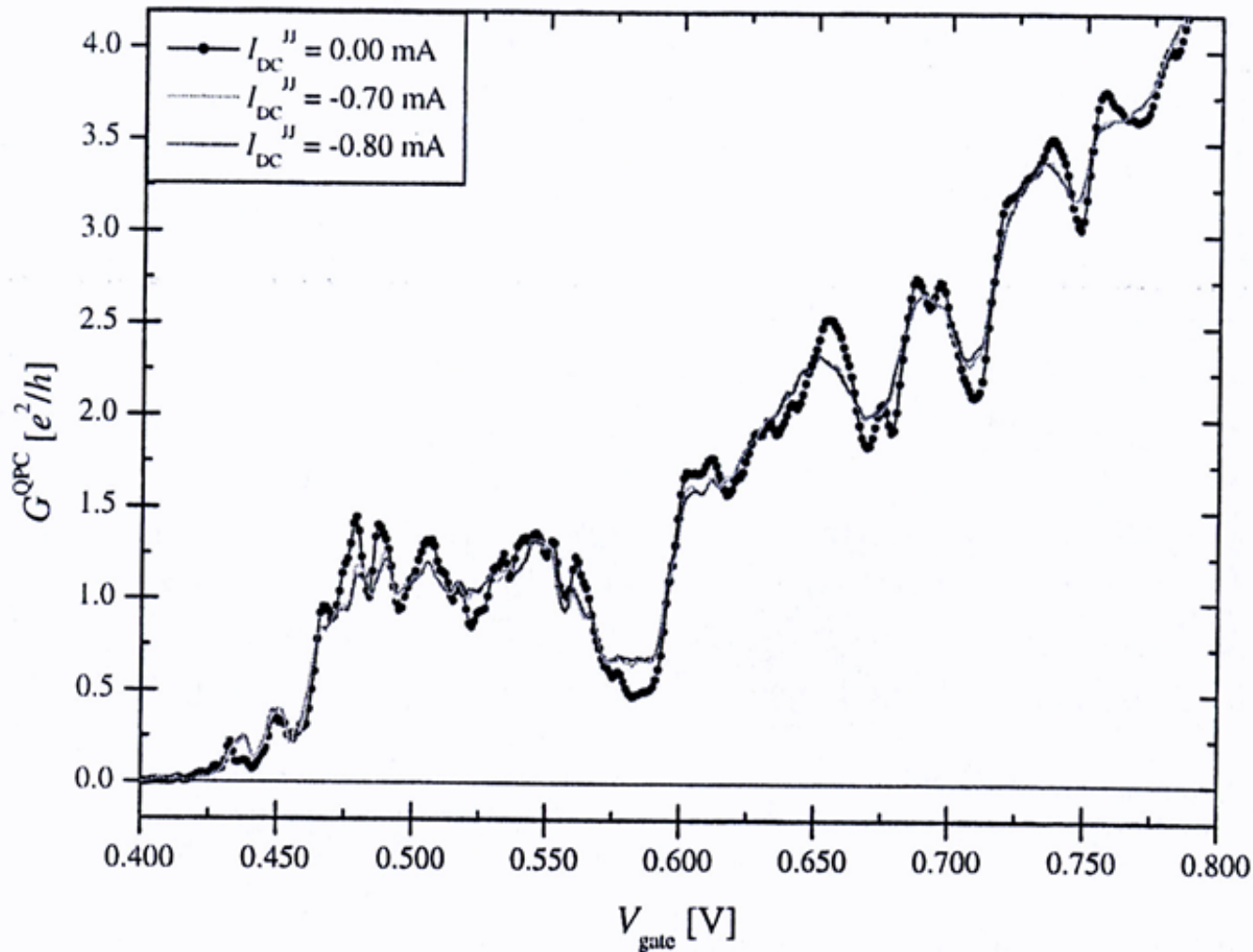


S.E.Andresen,
P.E.Lindelof, in Towards
the Controllable Quantum
States, (Eds. Takayanagi,
Nitta, World Scientific
2003) pp.295

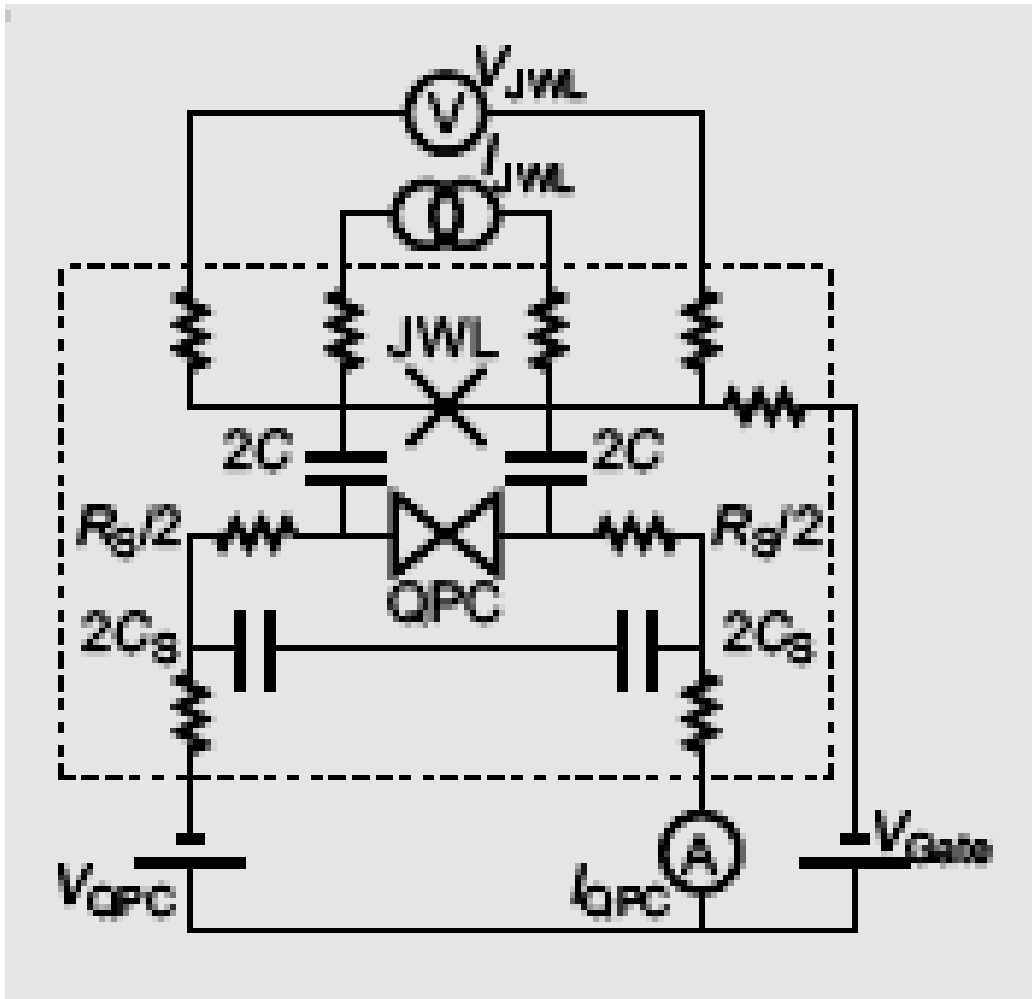
I-V characteristics for microwave irradiated superconducting microbridge



Conductance versus gatevoltage for GaAs point contact (3 different microbridge currents)



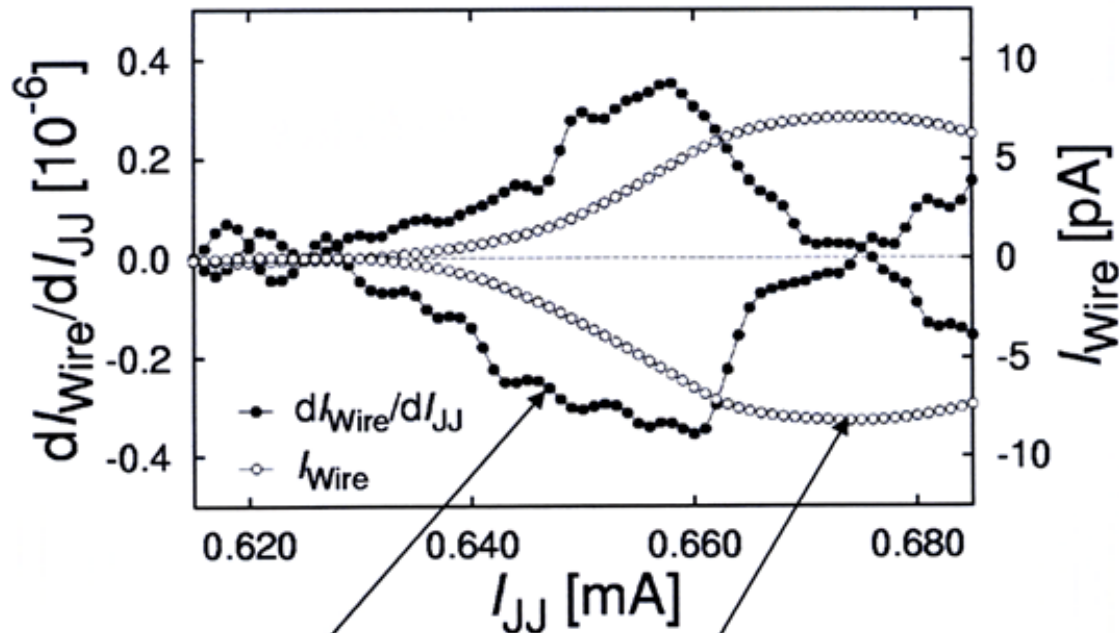
Microbridge-QPC equivalent diagram



S.E.Andresen, P.E.Lindelof,
in Towards the Controllable
Quantum States, (Eds.
Takayanagi, Nitta, World
Scientific 2003), pp. 295

Rectification of AC Josephson effect in QPC

$T=0.3$ K
 $R_{JJ}=0.2$ Ohm
 $I_c=0.63$ mA
 $R_{QPC}=7$ kOhm
 $V_{gate}=0.7$ V

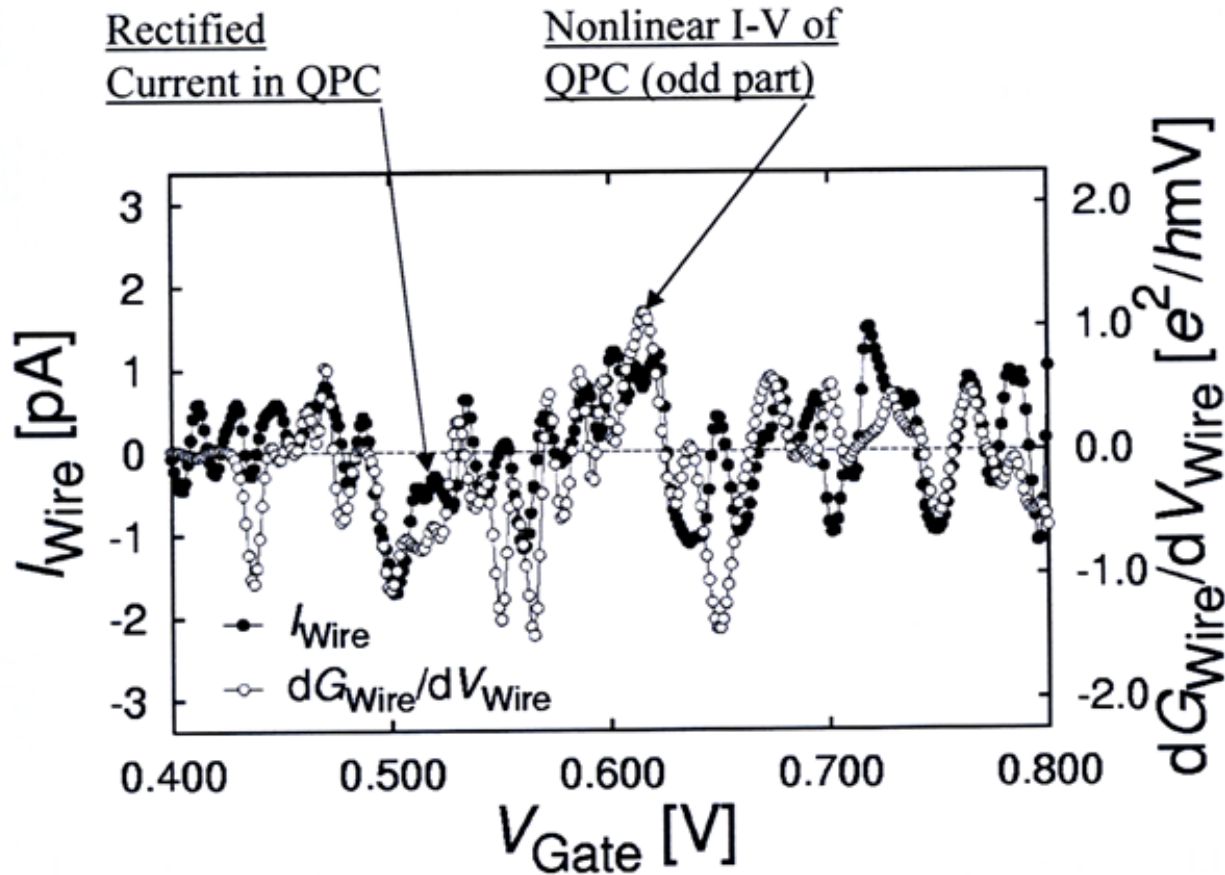


Differential
Correlation..

Rectified current in QPC..

S.E.Andresen,
P.E.Lindelof, in Towards
the Controllable Quantum
States, (Eds. Takayanagi,
Nitta, World Scientific
2003), pp. 295

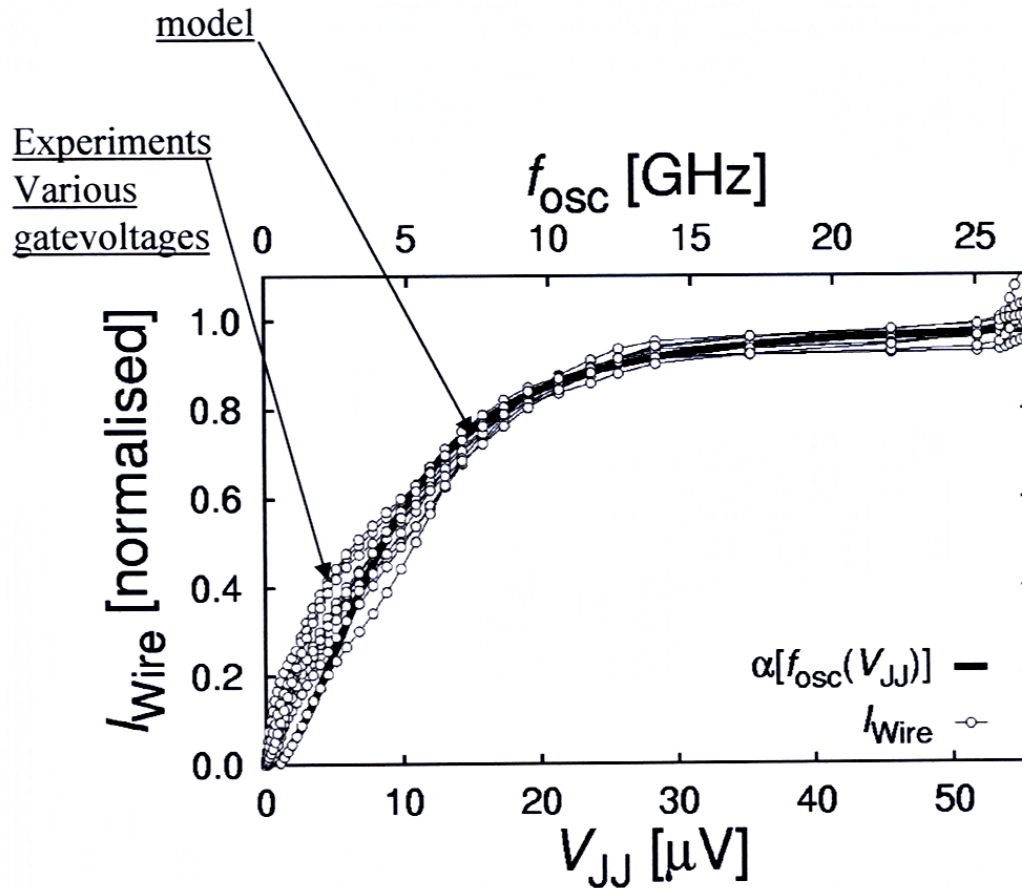
Rectified AC Josephson Effect & Measured QPC Nonlinearity



Result of comparison: AC Josephson amplitude: $\sim 20 \mu\text{V}$

S.E.Andresen,
P.E.Lindelof, in Towards
the Controllable Quantum
States, (Eds. Takayanagi,
Nitta, World Scientific
2003), pp. 295

Rectified current for increasing AC Josephson Frequency

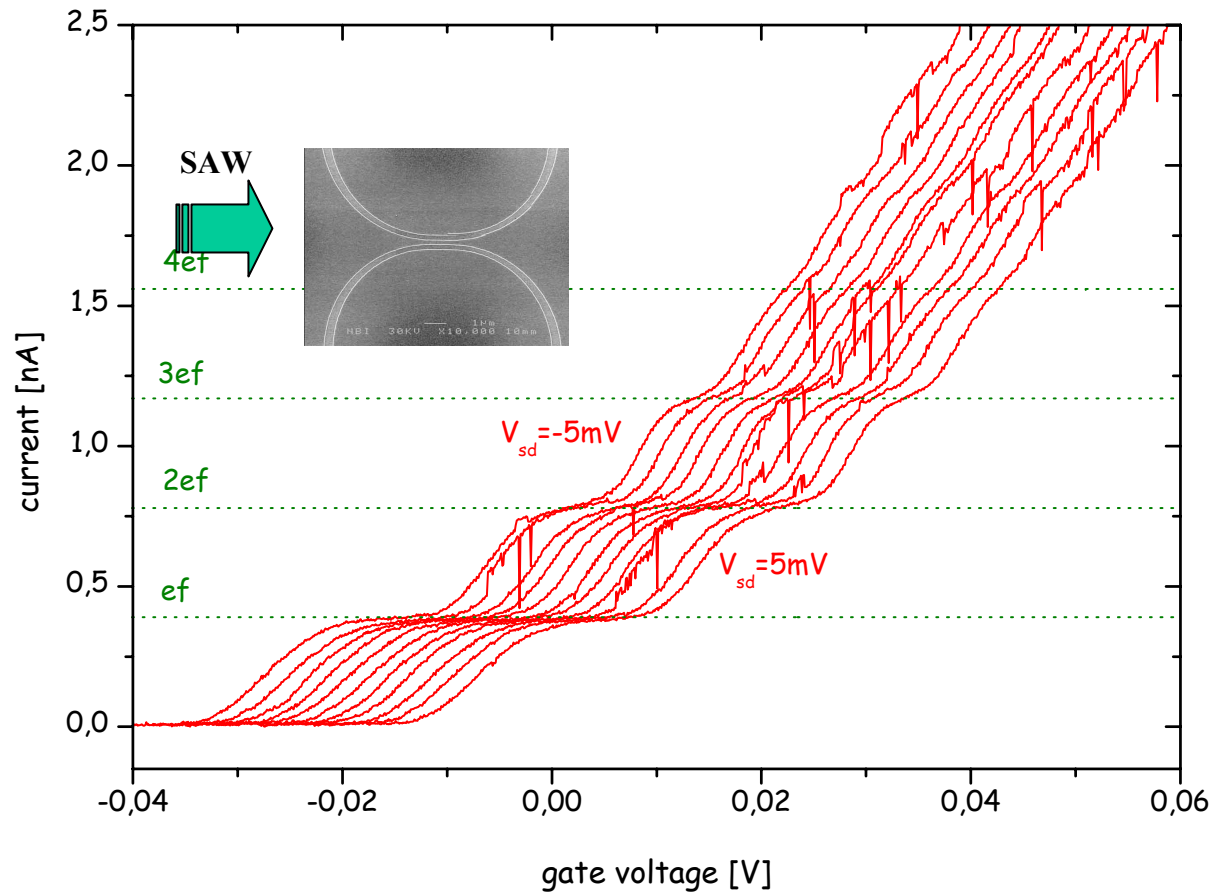


Result of analysis: $C=3$ fF
& JJ active radius= $1.6 \mu\text{m}$

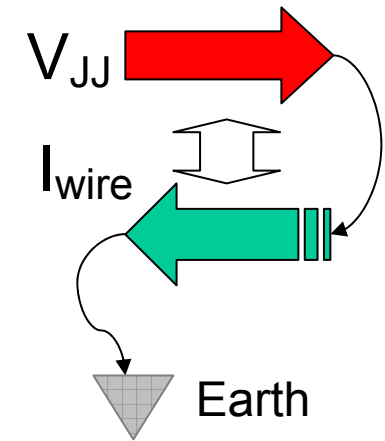
S.E.Andresen,
P.E.Lindelof, in Towards
the Controllable Quantum
States, (Eds. Takayanagi,
Nitta, World Scientific
2003), pp. 295

A surface acoustic wave driven QPC

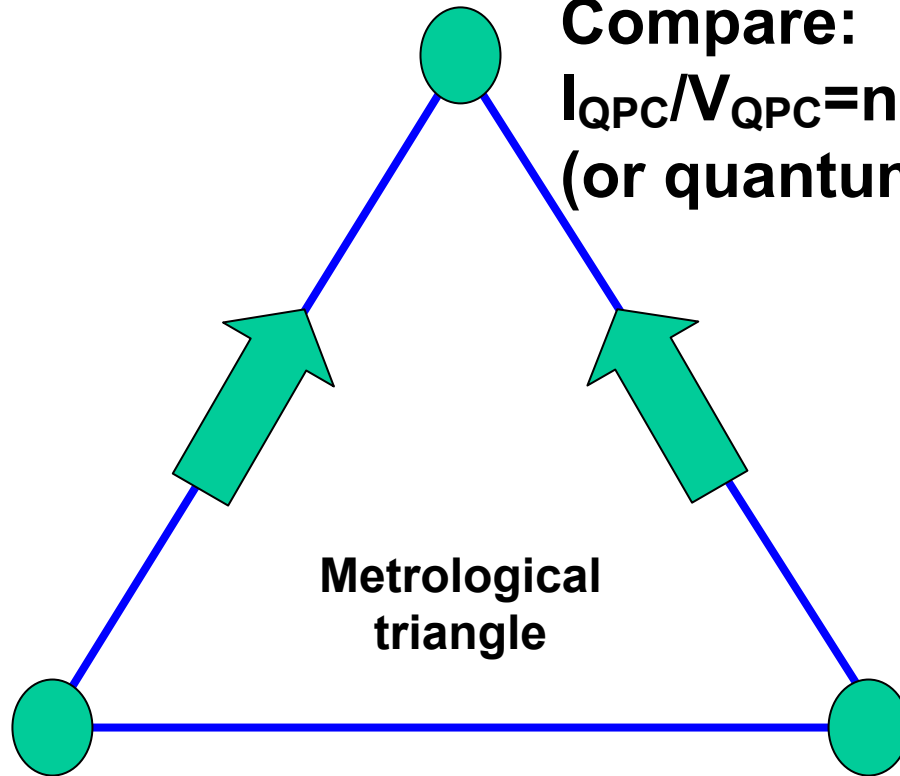
$f_{\text{saw}} = 3 \text{ GHz}$



New Device. $h/2e^2$ Injection locking?



$V_{JJ}/I_{\text{wire}} = h/2e^2$ (12.9 k Ω)
Compare:
 $I_{\text{QPC}}/V_{\text{QPC}} = n2e^2/h$
(or quantum Hall effect)



$V_{JJ} = h\nu/2e$
(20 μV)

$I_{\text{wire}} = e\nu$
(1.6 nA)

1Dim.-wire or quantum dot turnstile
+ Josephson junction microbridge

CONCLUSIONS

1. Superconducting microbridges emit Josephson radiation, which can be detected in the near field limit only (due to low impedance)
2. Superconducting microbridges is only well understood in the zero length limit. The non-equilibrium and Andreev processes needs more study
3. The ideal QPC is well understood (Glazman et al, Büttiker). The maximum subband splitting is 20 meV. Conductance quantization in units of $2e^2/h$.
4. 0.7 structure in QPCs and the activation behaviour is still not microscopically explained.
5. QPCs is often dominated by impurity scattering and show strong non-linearities (resonances) which lead to rectification of a.c. signals.
6. Rectification of the AC Josephson effect in a non-linear QPC (or possibly even a quantum dot) in the close vicinity has an interesting scope. If the Josephson frequency could be locked to a single electron transfer (similar to the SAWSET device), a new type of $h/2e^2$ metrological mesoscopic device emerges.

Nordic Cultural Foundation stipends 1969 organized by Prof. Olli V. Lounasmaa

Stipendiats and their planned activities:

Tore Eriksson, Sverige: Mössbauer effect, with Toivo Katila

Jan Ivar Ivarsson, Sverige, He3 and paramagnetic demagnetization cryostats, with George Pickett

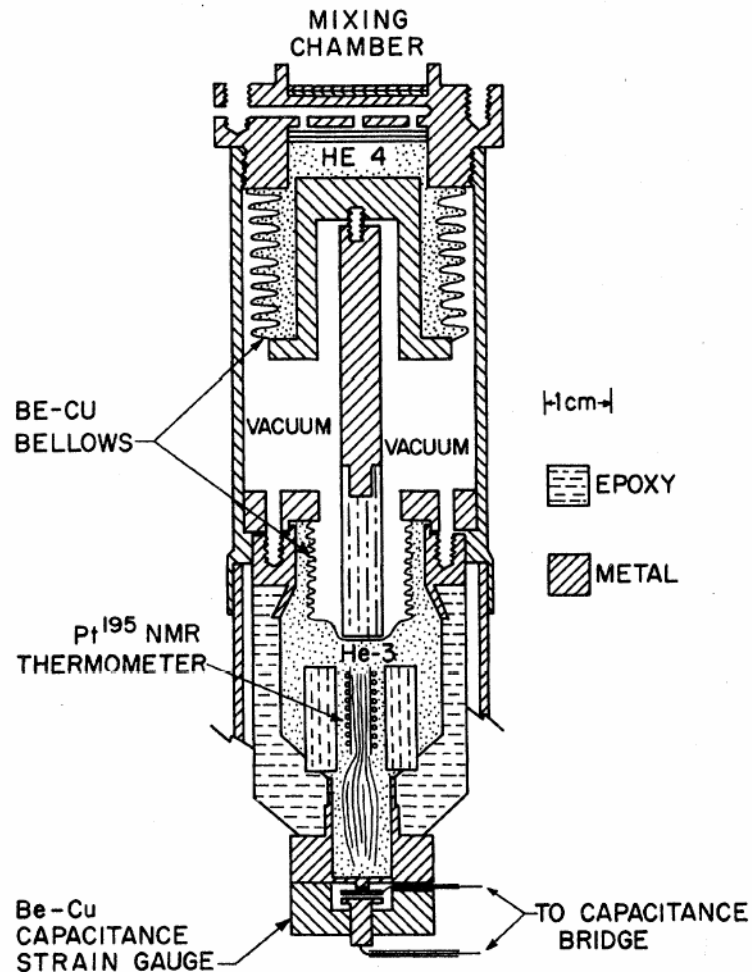
Poul Lindelof, Denmark, Building a Pomeranchuk cooling system, with Olli Lounasmaa and technician Virtanen.

Rolf Søvik, Norge, Nuclear Demagnetization cryostat, with Robert Gylling

Something for a Dane to get use to and then appreciate:

- A strong leader and the founder of "Professorsforbundet" in Finland.
- Hard working scientists and technicians. Full time really means day and night.
- The language and the weekly saunaparties. The many social events.
- The many visiting great scientists from abroad.
- The great science and a world record in low (equilibrium) temperature (0.3 mK)

The Pomeranchuk cell (Anufriev, Wheatley and Osheroff)



D.D. Osheroff et al.,
Phys.Rev.Lett. 28, 885 (1972)

FIG. 1. Pomeranchuk cooling and pressure-measuring apparatus.

Matti Krusius and George Pickett

Great experiments in 1969-70.

The ideas thought out and planned by Matti and George

-and carried out with me as the assistant:

- The nuclear heat capacity can reveal the sign of the electric field gradient at the nucleus.

- The hyperfine enhanced nuclear ordering in praseodymium by studying heat capacity in a strong magnetic field.

- The switching on of the nuclear heat capacity in bismuth by minute doping with tellerium.

-I am still thankful for this learning year in Finland.

Olli V. Lounasmaa heat capacity Era I

1. O. V. Lounasmaa, Specific Heat of Samarium Metal between 0.4 and 4°K, Phys. Rev. 126, 1352-1356 (1962);
2. O. V. Lounasmaa and R. A. Guenther, Specific Heat of Dysprosium Metal between 0.4 and 4°K, Phys. Rev. 126, 1357-1363 (1962);
3. O. V. Lounasmaa and P. R. Roach, Specific Heat of Terbium Metal between 0.37 and 4.2°K, Phys. Rev. 128, 622-626 (1962);
4. O. V. Lounasmaa, Specific Heat of Holmium Metal between 0.38 and 4.2°K, Phys. Rev. 128, 1136-1139 (1962);
5. O. V. Lounasmaa, C. H. Cheng, and P. A. Beck, Nuclear Magnetic Specific Heat of Ferromagnetic Iron Alloys with Sb and Re, Phys. Rev. 128, 2153-2154 (1962);
6. O. V. Lounasmaa, Specific Heat of Gadolinium and Ytterbium Metals between 0.4 and 4°K, Phys. Rev. 129, 2460-2464 (1963);
7. O. V. Lounasmaa, Specific Heat of Praseodymium and Neodymium Metals Between 0.4 and 4°K, Phys. Rev. 133, A211-A218 (1964);
8. O. V. Lounasmaa, Specific Heat of Lutetium Metal Between 0.38 and 4°K, Phys. Rev. 133, A219-A224 (1964)
9. O. V. Lounasmaa, Specific Heat of Cerium and Europium Metals between 0.4 and 4°K, Phys. Rev. 133, A502-A509 (1964)
10. O. V. Lounasmaa, Specific Heat of Thulium Metal Between 0.38 and 3.9°K, Phys. Rev. 134, A1620-A1624 (1964)
11. O. V. Lounasmaa, Specific Heat of Europium and Ytterbium Metals between 3 and 25°K, Phys. Rev. 143, 399-405 (1966)
12. O. V. Lounasmaa and L. J. Sundström, Specific Heat of Gadolinium, Terbium, Dysprosium, Holmium, and Thulium Metals between 3 and 25°K, Phys. Rev. 150, 399-412 (1966)
13. O. V. Lounasmaa and L. J. Sundström, Specific Heat of Lanthanum, Praseodymium, Neodymium, and Samarium Metals between 3 and 25°K, Phys. Rev. 158, 591-600 (1967)

Olli V. Lounasmaa heat capacity Era II

14. A. C. Anderson, B. Holmström, and M. Krusius, Calorimetric Investigation of Hyperfine Interactions in Rare-Earth Metals, *Phys. Rev. Lett.* 20, 154-157 (1968).
15. B. Holmström, A. C. Anderson, and M. Krusius, Calorimetric Investigation of Hyperfine Interactions in Metallic Praseodymium and Thulium, *Phys. Rev.* 188, 888-892 (1969).
16. A. C. Anderson, B. Holmström, M. Krusius, and G. R. Pickett, Calorimetric Investigation of the Hyperfine Interactions in Metallic Nd, Sm, and Dy, *Phys. Rev.* 183, 546-552 (1969).
17. M. Krusius, A. C. Anderson, and B. Holmström, Calorimetric Investigation of Hyperfine Interactions in Metallic Ho and Tb, *Phys. Rev.* 177, 910-916 (1969).
18. A. C. Anderson, B. Holmström, M. Krusius, and G. R. Pickett, Calorimetric Investigation of the Hyperfine Interactions in Metallic Nd, Sm, and Dy, *Phys. Rev.* 183, 546-552 (1969).
19. H. K. Collan, M. Krusius, and G. R. Pickett, Suppression of the Nuclear Heat Capacity in Bismuth Metal by Very Slow Spin-Lattice Relaxation, and a New Value for the Electronic Specific Heat, *Phys. Rev. Lett.* 23, 11-13 (1969).
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21. M. Krusius, and G.R. Pickett, Calorimetric determination of nuclear quadrupole interaction in Arsenic, *Solid State Commun.* 9, 1917 (1971)
22. P. E. Gregers-Hansen, M. Krusius, and G. R. Pickett, Sign of the Nuclear Quadrupole Interaction in Rhenium Metal, *Phys. Rev. Lett.* 27, 38-41 (1971)
23. P. E. Gregers-Hansen, M. Krusius, and G. R. Pickett, Magnetic Properties of Praseodymium in Magnetic Fields Determined from the Nuclear Heat Capacity and Applied to Nuclear Cooling, *Phys. Rev. Lett.* 29, 420-423 (1972)
24. P.E. Gregers-Hansen, M. Krusius, G.R. Pickett, Calorimetric measurements of nuclear quadrupole interactions in tellurium-doped bismuth, *J. Low Temp. Phys.* 12, 309 (1973)
25. M. Krusius, G.R. Pickett, and M.C. Veuro, Hyperfine heat-capacity of metallic europium and erbium, *Solid State Commun.* 14, 191 (1974)
26. P. E. Lindelof, I. E. Miller, and G. R. Pickett, Nuclear Cooperative Ordering in Single-Crystal Praseodymium at Low Temperatures, *Phys. Rev. Lett.* 35, 1297-1299 (1975)

Lounasmaa, the concerned head of lab..

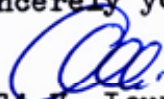
Dear Paul:

George Pickett is presently visiting us for a few days and I took the opportunity to discuss with him and with Matti Kruusius the writing and publication of the remaining papers from the ADM cryostat.

As you know, George has worked here for 5 years, including several lean ones when the low temperature know-how was introduced to my laboratory and when the cryostats were being built. Matti has spent the last 15 months doing nothing but writing papers on the data obtained before you and George left. Thus, taking into account the amount of labor that each person has contributed, we came to the conclusion that if you want to be included as an author of the "Bismuth doped with tellurium" and the "Praseodymium in a magnetic field" -papers you will have to do the job of writing the drafts and the final versions of these papers, in consultation with George and Matti, of course. It is also important that the papers get published quickly; I thus hope that you can devote your energies to them to such an extent that the papers are ready by the end of March, 1972. As you know they are both quite short.

Please let me or Matti know how you feel about our proposal.

Best wishes,
Sincerely yours,



Olli V. Lounasmaa