

how far can one pressurize a liquid before it crystallizes ?

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*for references and files, including video sequences,
go to <http://www.lps.ens.fr/~balibar/>*

abstract

an equivalent question:

*what is the threshold for homogeneous nucleation
of crystals in a pressurized liquid phase ?*

a related question :

how far can one supercool liquid water ? why - 40 °C ?

helium is pure and simple

the surface tension of solid helium is accurately known

eliminate the influence of impurities walls and defects

acoustic techniques: acoustic cavitation and acoustic crystallization

test an intrinsic stability limit of the liquid state of matter

and a few other problems related to superfluidity at high density

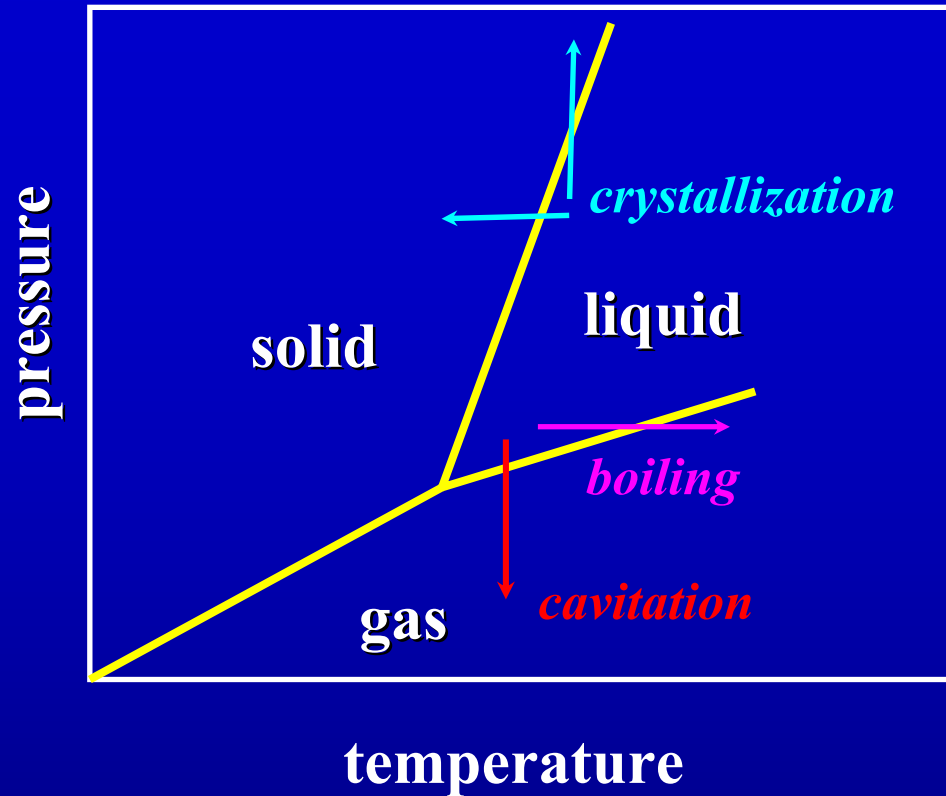
metastable liquids

*liquid-gas or liquid-solid:
first order phase transitions
⇒ metastability is possible*

*energy barriers against the nucleation
of either the solid or the gas phase*

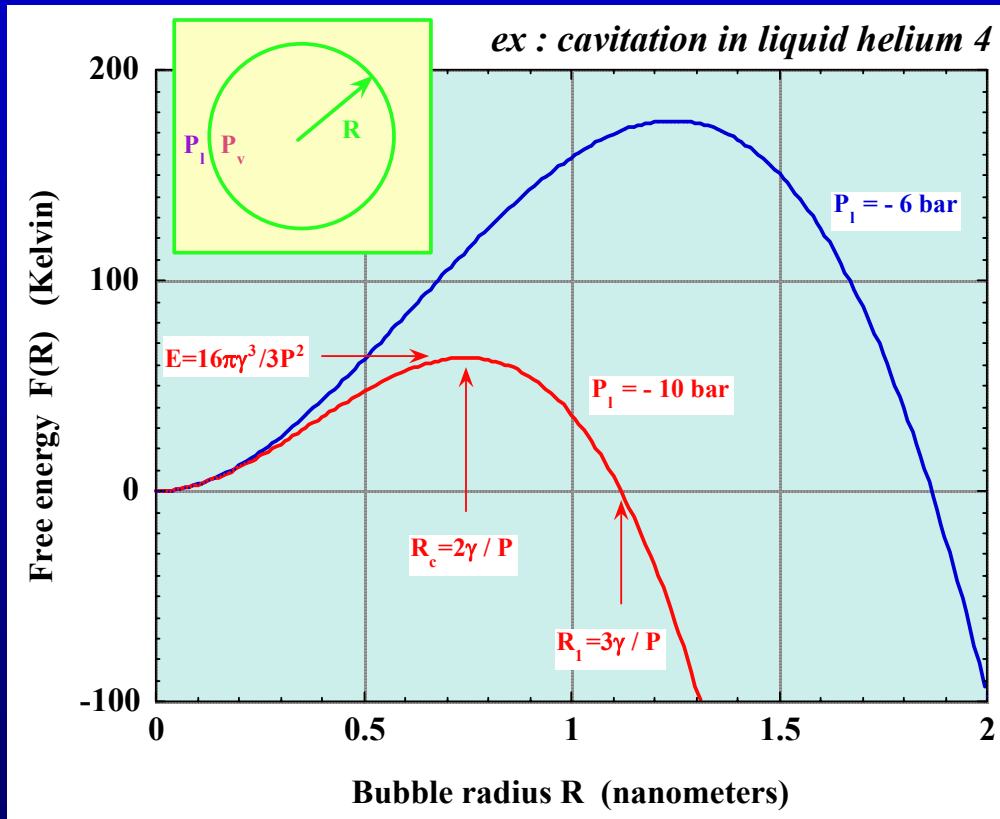
example:

liquid water to - 40 °C or + 200°C at 1 bar, or - 1400 bar at +35 °C



the barrier against nucleation is due to the surface energy

Standard nucleation theory (Landau and Lifshitz, Stat. Phys. p553):



a spherical nucleus with **radius R**
and **surface energy γ** (the macroscopic surface tension)

$$F(R) = 4\pi R^2 \gamma - 4/3 \pi R^3 \Delta P$$

ΔP : difference in free energy per unit volume between the 2 phases

Critical radius : $R_c = 2\gamma/\Delta P$

Activation energy : $E = (16\pi\gamma^3)/(3\Delta P^2)$

$R > R_c \Rightarrow$ growth

The critical nucleus is in *unstable equilibrium*

$$\Rightarrow \Delta P = (1 - \rho_v/\rho_l)(P_{eq} - P)$$

nucleation rate per unit time and volume : $\Gamma = \Gamma_0 \exp(-E/T)$

Γ_0 : attempt frequency x density of independent sites

supercooling water: Taborek 's experiment

(*Phys. Rev. B* 32, 5902, 1985)

*avoid heterogeneous nucleation on defects,
impurities or walls:*

- *divide the sample into micro-droplets*
- *minimize surface effects (STS not STO)*

*Regulate T : the heating power P increases
exponentially with time*

The time constant $\tau = 1/VJ$

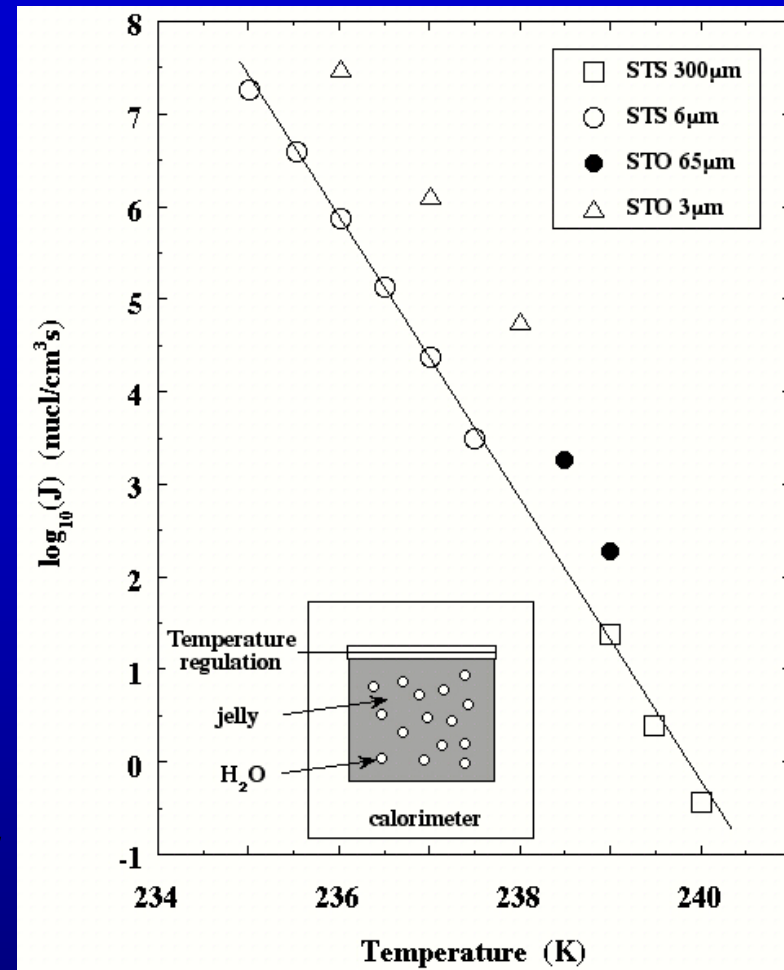
The nucleation rate J varies exponentially with T

*Compare with standard theory of homogeneous
nucleation :*

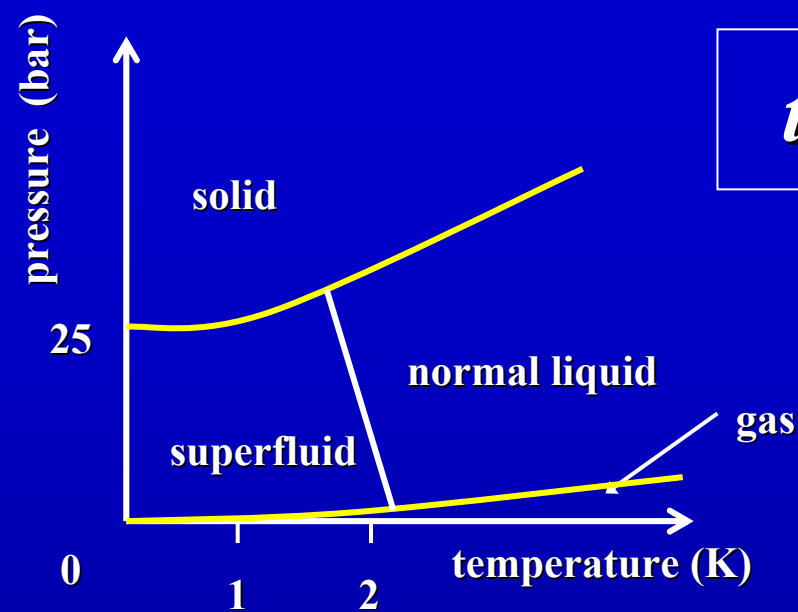
*Taborek used his nucleation experiment to measure the (unknown)
tension of the ice/water interface : it is 28.3 erg/cm² at 236 K*

(see also Seidel and Maris 1986 for H₂ crystals)

the surface tension of helium 4 crystals is accurately known



the surface of helium crystals



model systems

*for very general properties of crystal surfaces
for ex: the roughening transitions*

*unusual growth dynamics of "rough" surfaces
due to quantum properties
for ex: crystallization waves*

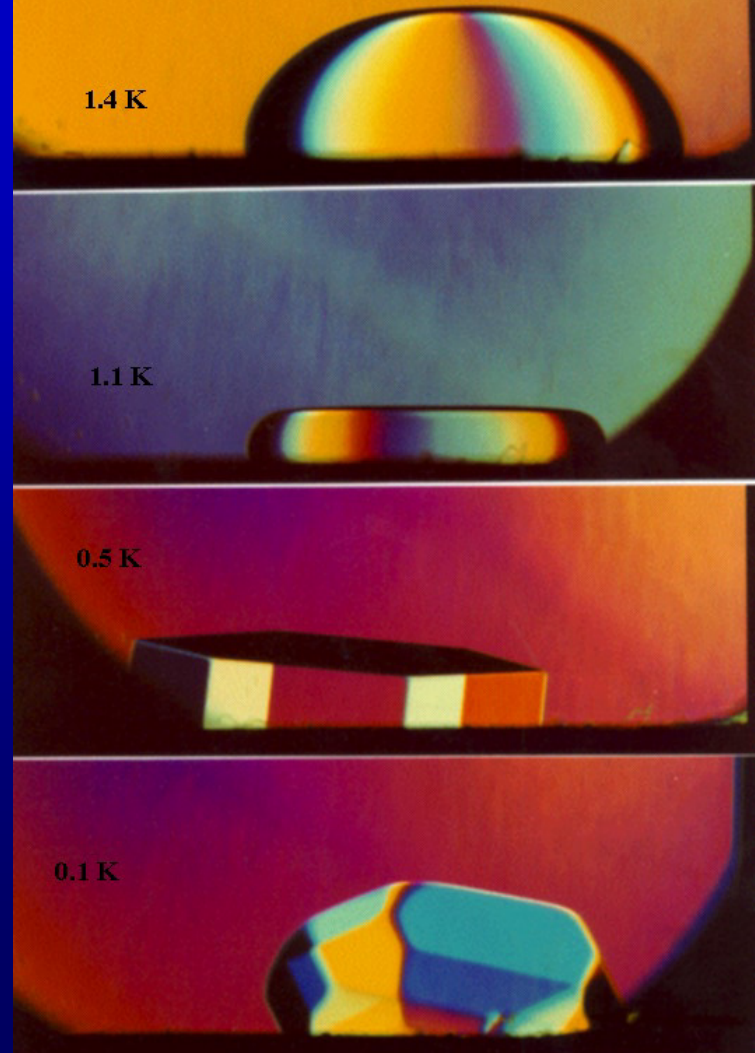
for review articles, see:

S. Balibar and P. Nozières, Sol. State Comm. 92, 19 (1994)

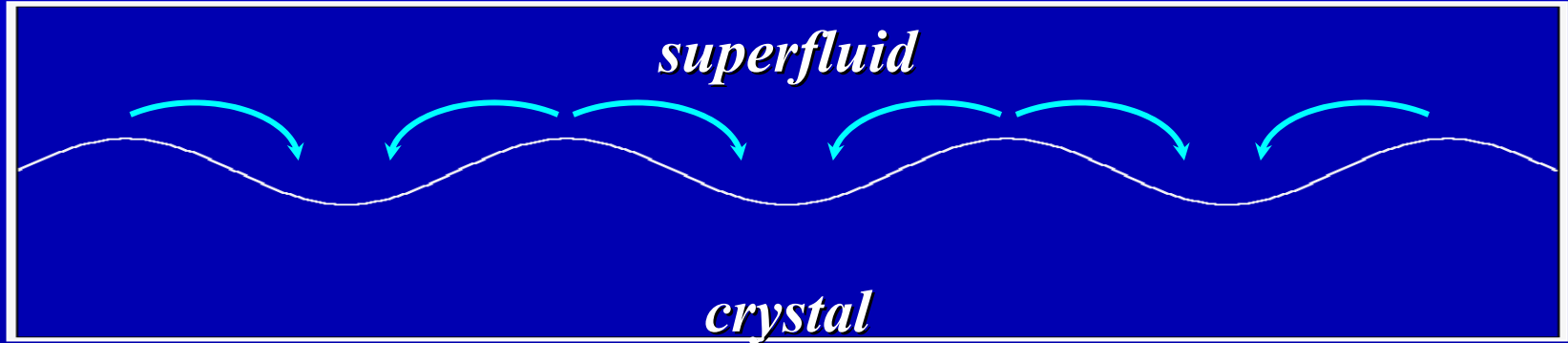
S. Balibar, H. Alles and A. Ya. Parshin, to be published in Rev. Mod. Phys. (2004).

S.BALIBAR, C.GUTHMANN nad E.ROLLEY (ENS PARIS)

THE ROUGHENING TRANSITIONS
OF HELIUM 4 CRYSTALS



crystallization waves



*helium crystals can grow and melt so fast that **crystallization waves propagate at their surfaces as if they were liquids.***

same restoring forces :

-surface tension γ

(more precisely the "surface stiffness" γ)

- gravity g

inertia : mass flow in the liquid ($\rho_C > \rho_L$)

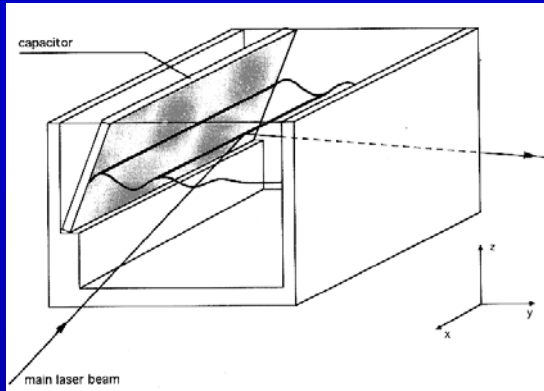
$$\omega^2 = \frac{\rho_L}{(\rho_C - \rho_L)^2} [\gamma q^3 + (\rho_C - \rho_L) g q]$$

\Rightarrow accurate measurement of the surface stiffness γ

video waves

QuickTime™ et un
décompresseur Animation
sont requis pour visionner cette image.

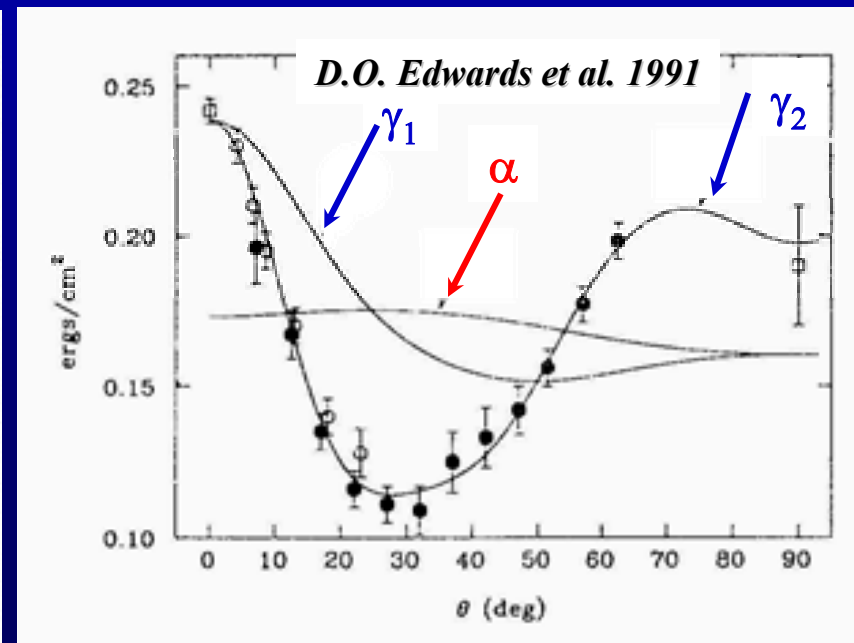
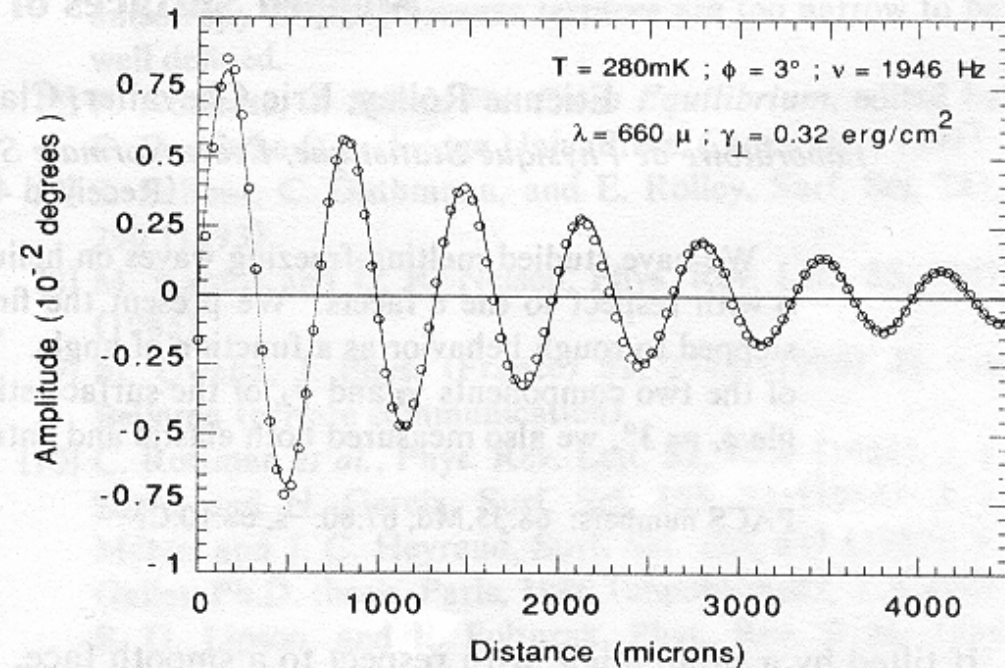
surface stiffness measurements



E. Rolley, S. Balibar and C. Guthmann
PRL 72, 872, 1994 and *J. Low Temp. Phys.* 99, 851, 1995

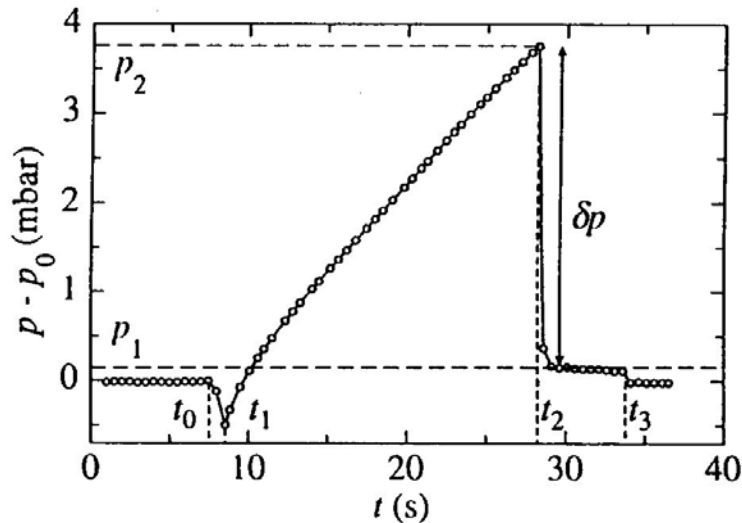
the **surface tension α** is anisotropic
 the anisotropy of the **surface stiffness**
 $\gamma = \alpha + \partial^2 \alpha / \partial \theta^2$ is even larger.

Edwards et al. (1991) from the measurements
 by **O.A. Andreeva and K.O. Keshishev (1990)**
 the surface tension $\alpha = 0.16$ to 0.17 erg/cm²



nucleation of solid helium

pressurizing liquid helium in an ordinary cell:



J.P. Ruutu et al., Helsinki, 1996
consistent with other measurements by
Balibar (1980), Sasaki (1998)

heterogeneous nucleation occurs

~ 3 to 10 mbar above P_m

(Balibar 1980, Ruutu 1996, Sasaki 1998)

Balibar, Mizusaki and Sasaki

(J. Low Temp. Phys. 120, 293, 2000):

it cannot be homogeneous nucleation,

since $E = 16/3 \pi \alpha^3 / \Delta P^2 \approx 10^{10} \text{ K} !$

*heterogeneous nucleation on favorable sites
(graphite dust particles ?)*

\Rightarrow acoustic crystallization : eliminate heterogeneous nucleation ?

heterogeneous nucleation with an electric field

Helium 4 Crystals

Sébastien Balibar, Claude Guthmann and Etienne Rolley

Laboratoire de Physique Statistique
Ecole Normale Supérieure

Paris, 1994

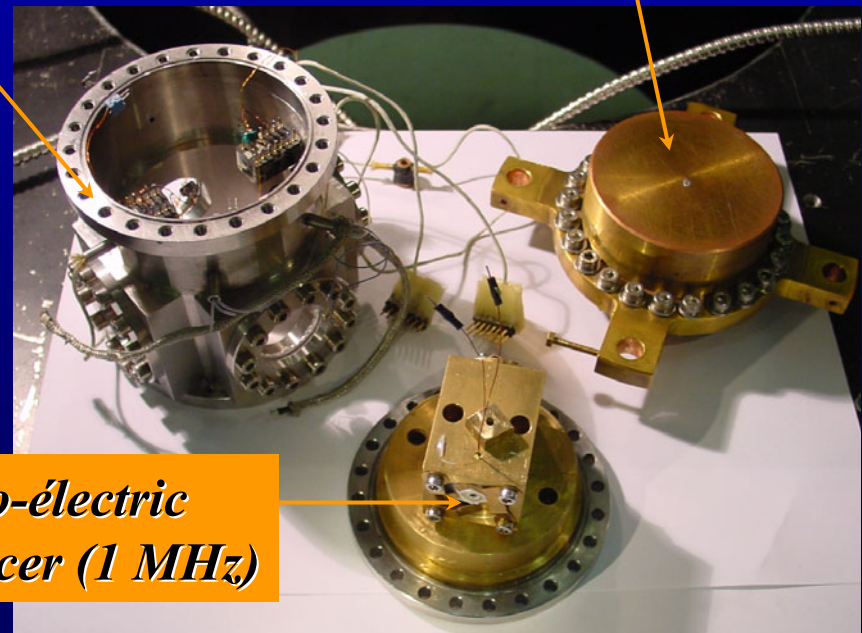
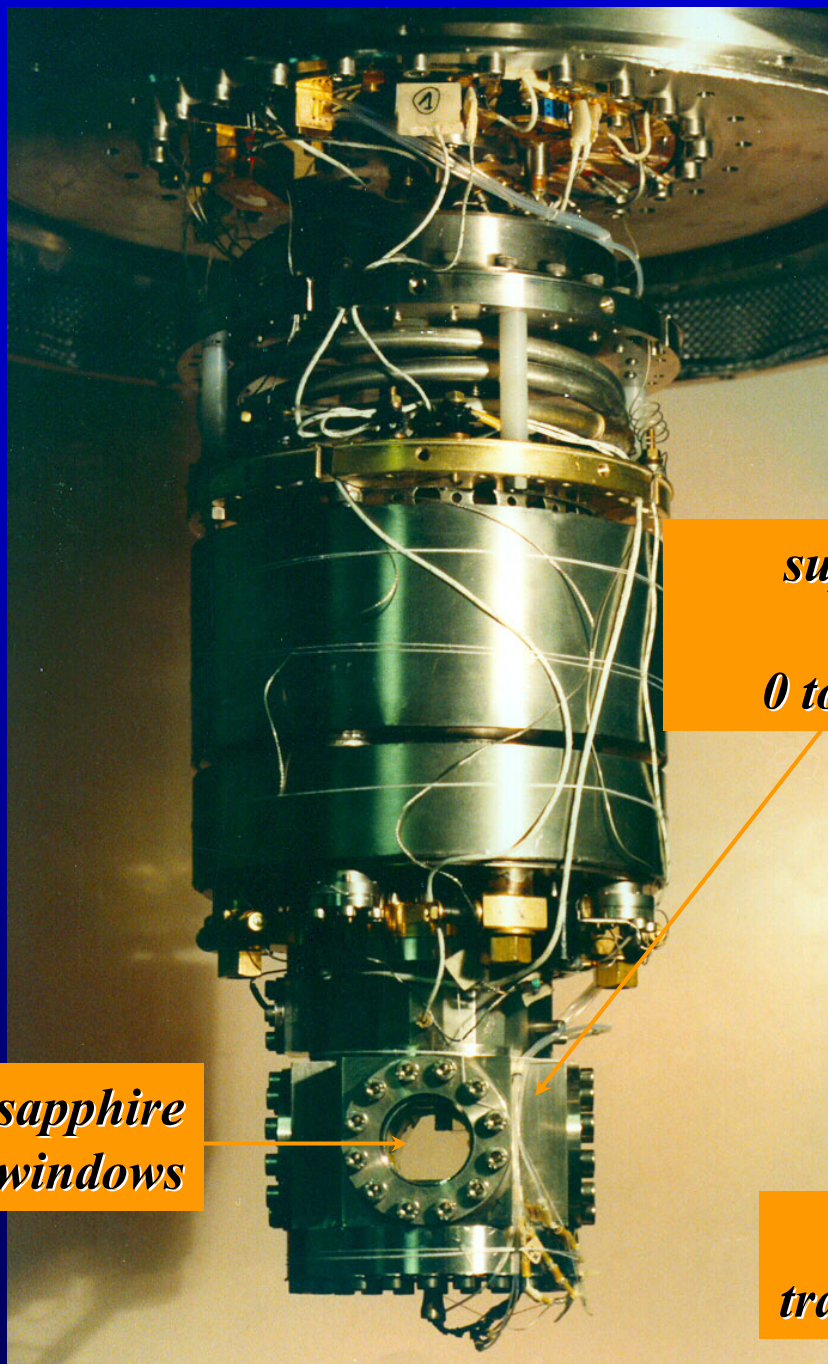
*the optical
refrigerator
at ENS-Paris*

*superfluid helium cell :
300 cm³
0 to 25 bar ; 0.02 to 1.4 K*

heat exchangers

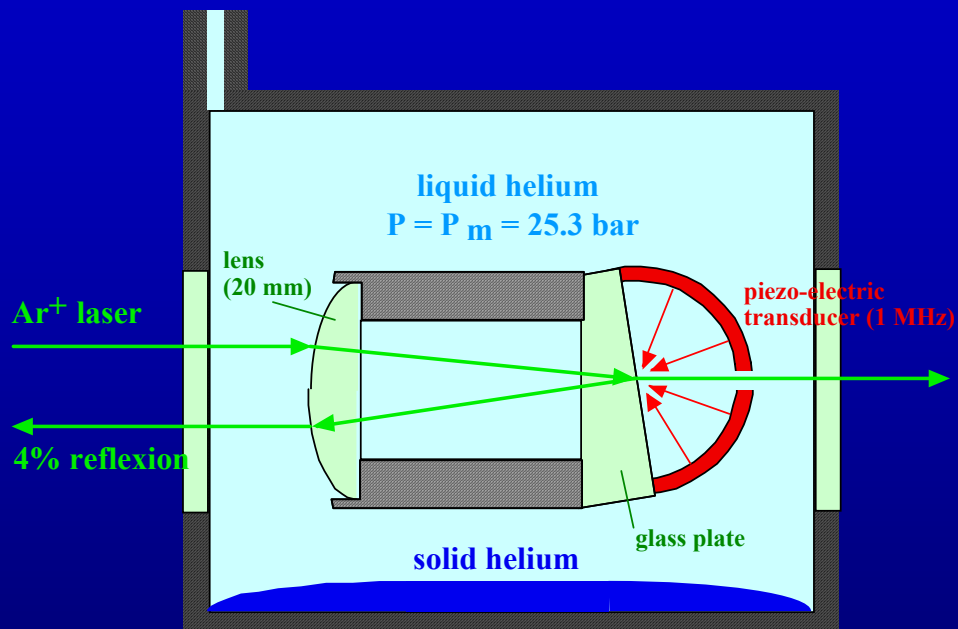
*sapphire
windows*

*piezo-électrique
transducer (1 MHz)*

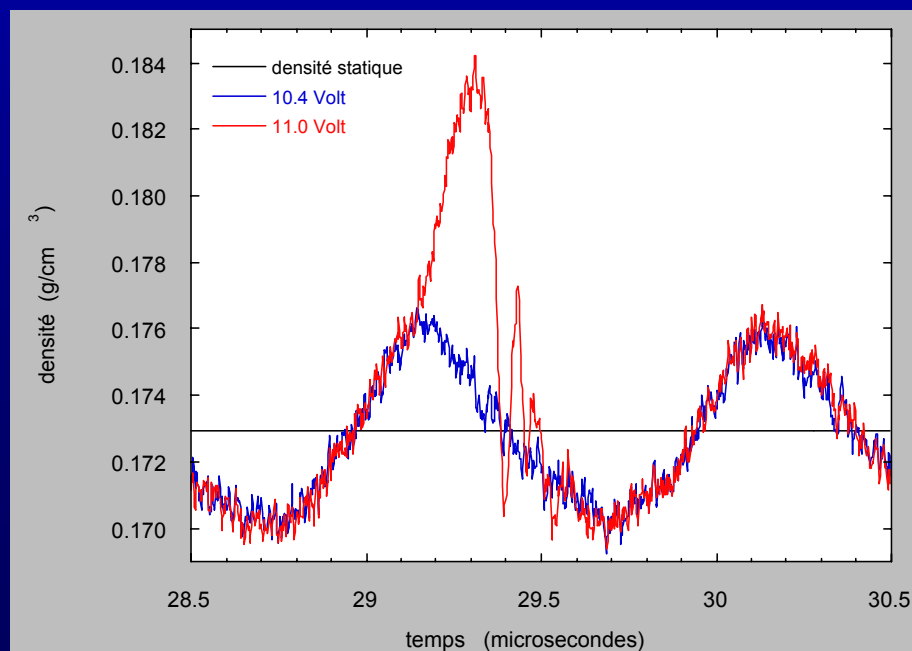
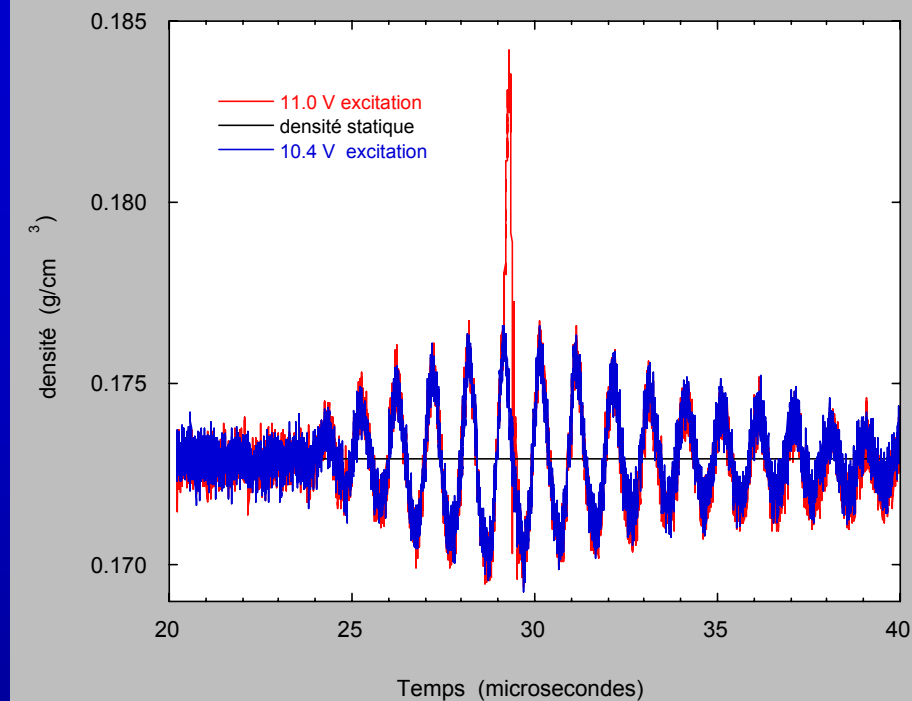


acoustic crystallization on a clean glass plate

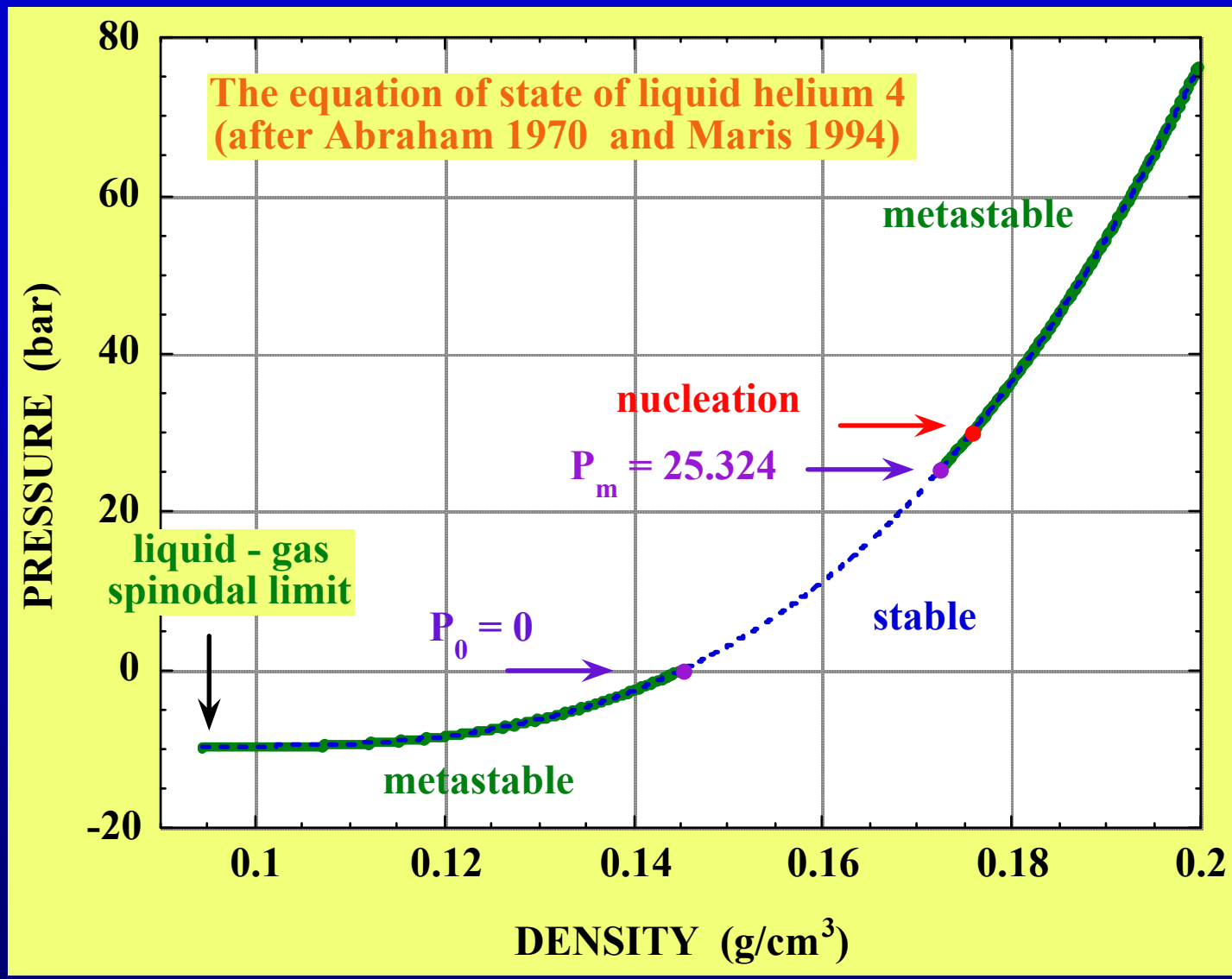
X. Chavanne, S. Balibar and F. Caupin
Phys. Rev. Lett. 86, 5506 (2001)



acoustic bursts (6 oscillations, rep. rate ~ 2 Hz)
wave amplitude at the crystallization threshold:
 $\pm 3.1 \cdot 10^{-3} \text{ g/cm}^3$ ($\sim 2\%$ of ρ_m),
i.e. ± 4.3 bar according to the eq. of state



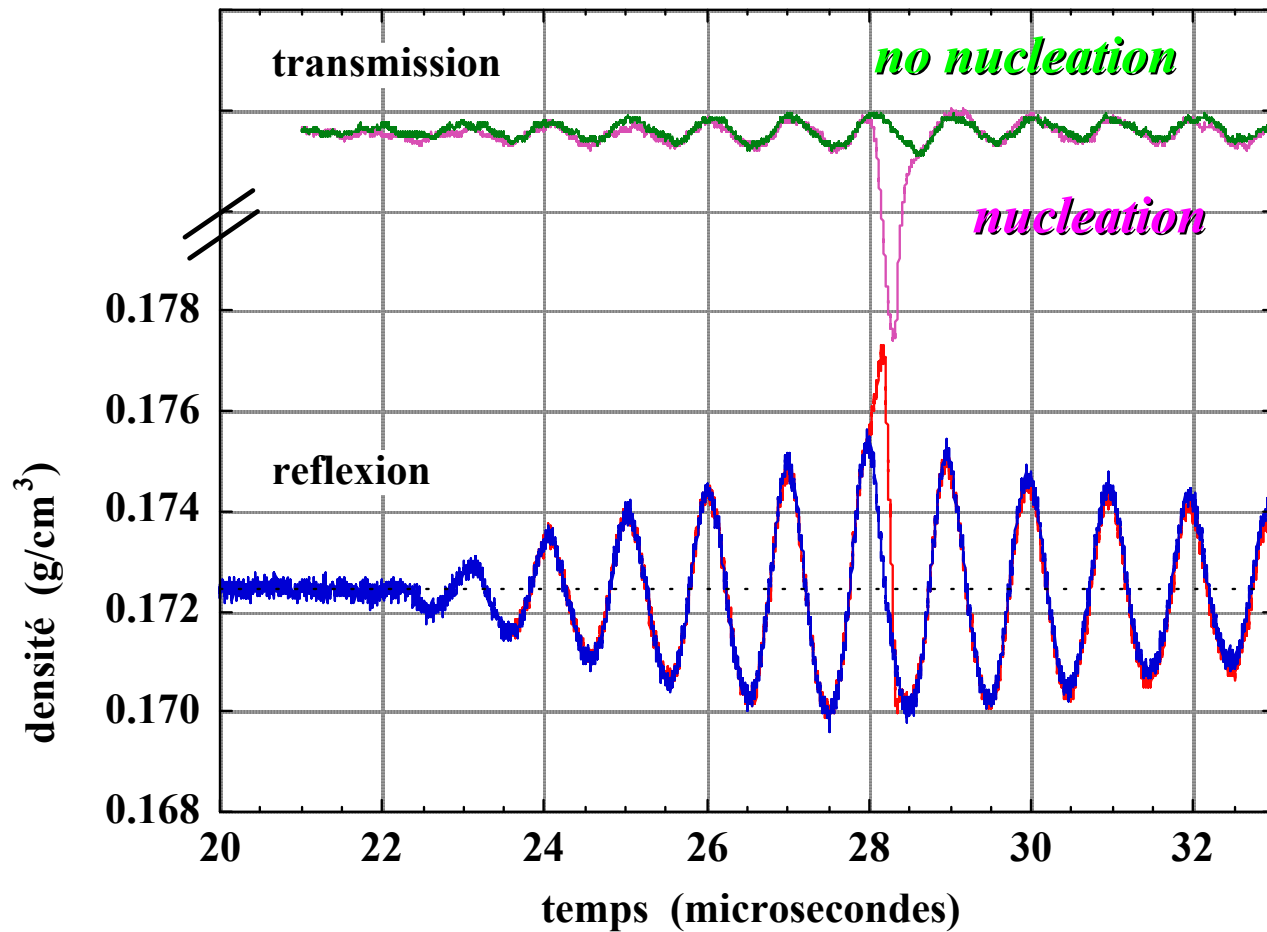
*the
equation
of state
of liquid
helium 4*



a rather well established cubic law (Maris 1991)

$$P - P_{sp} = a (\rho - \rho_{sp})^3$$

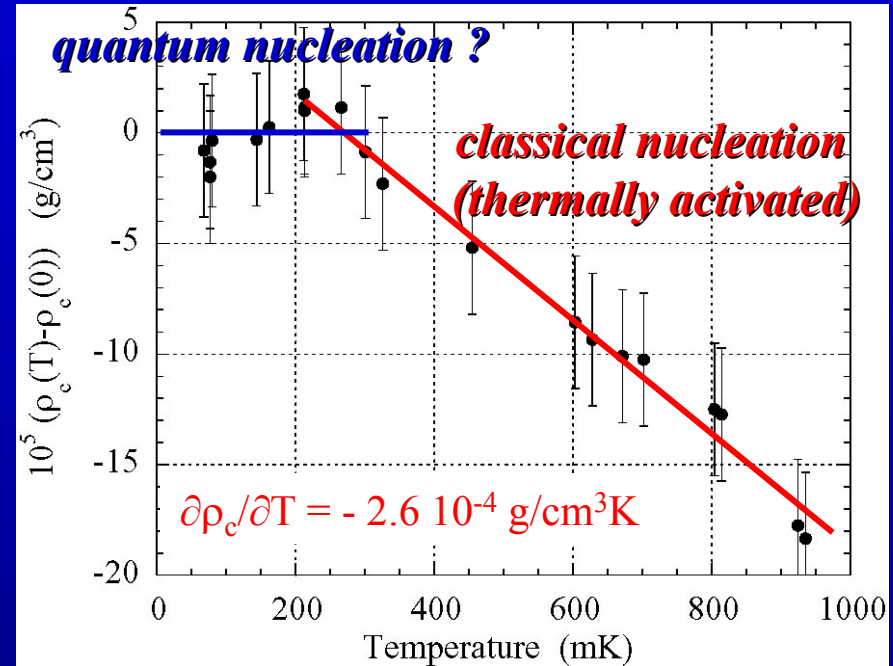
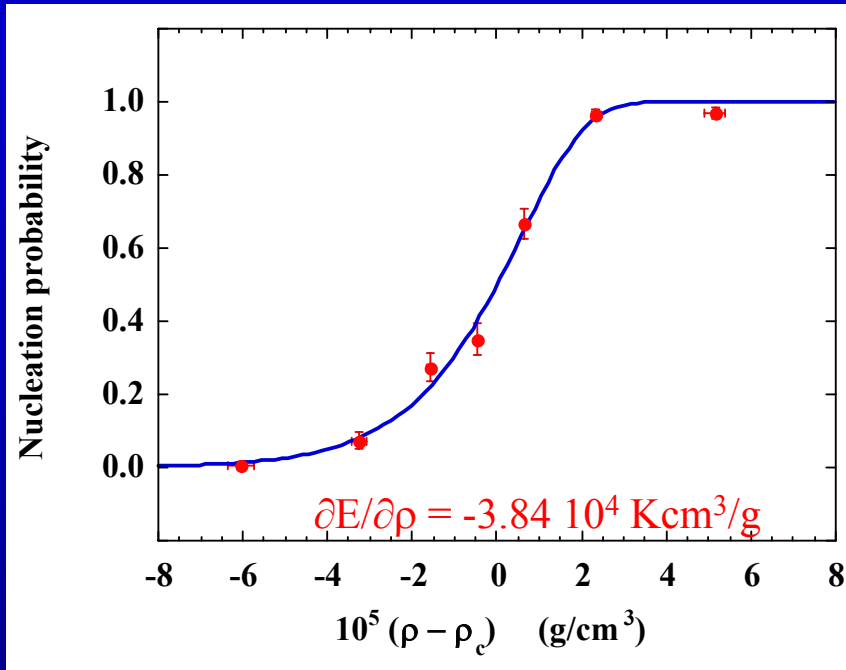
nucleation is stochastic



transmission signals are not averaged, so that the nucleation probability is easily obtained by counting events

a selective averaging is made on reflexion signals, in order to measure the wave amplitude at the nucleation threshold

on a clean glass plate, nucleation of solid He is still heterogeneous



the nucleation probability Σ increases continuously from 0 to 1 in a small density interval, as expected for nucleation due to thermal or quantum fluctuations. This is the usual "asymmetric S-shape curve":

$$\Sigma = 1 - \exp(-\Gamma_0 V \tau \exp(-E/T)) = 1 - \exp\{-\ln 2 \exp[-(1/T)(\partial E / \partial \rho)(\rho - \rho_c)]\}$$

*from $\Sigma(\rho)$ and $\rho_c(T)$, we obtain the activation energy $E = T \cdot \partial E / \partial \rho \cdot \partial \rho_c(T) / \partial T = 6 T$
 \Rightarrow heterogeneous nucleation on the glass (~ 1 preferential site)
 (at $P_m + 4$ bar the homogeneous nucleation barrier would be ~ 3000 K)*

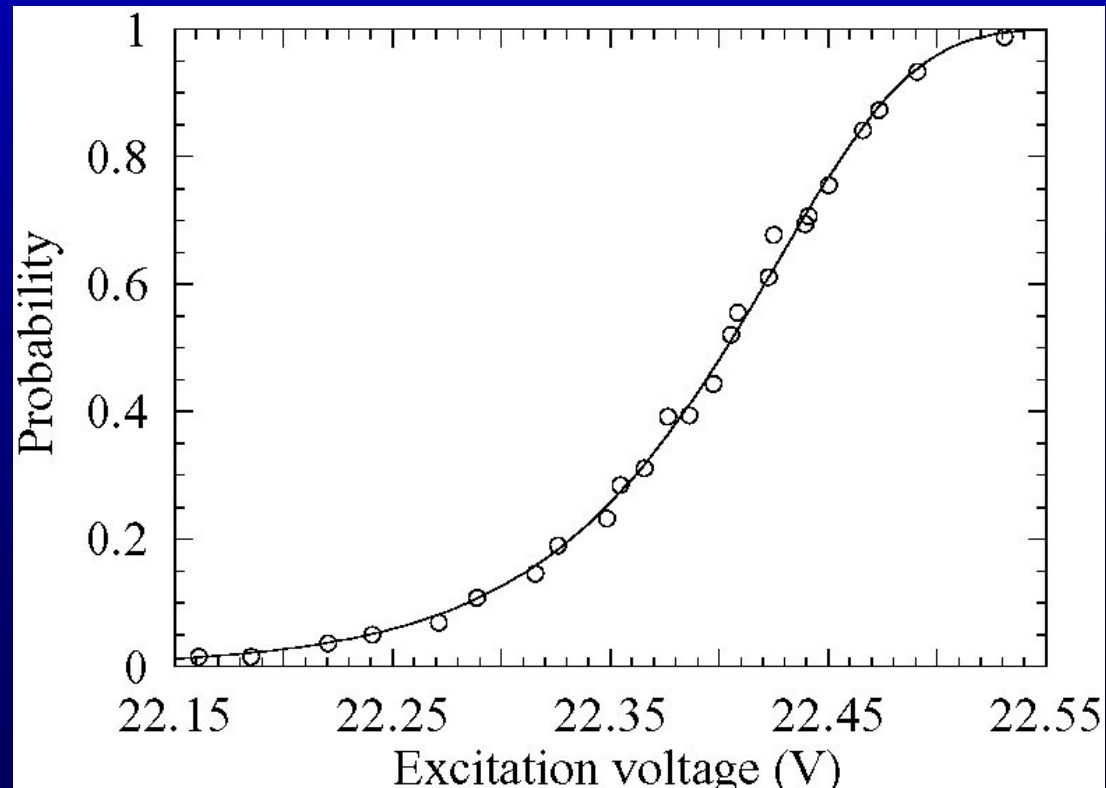
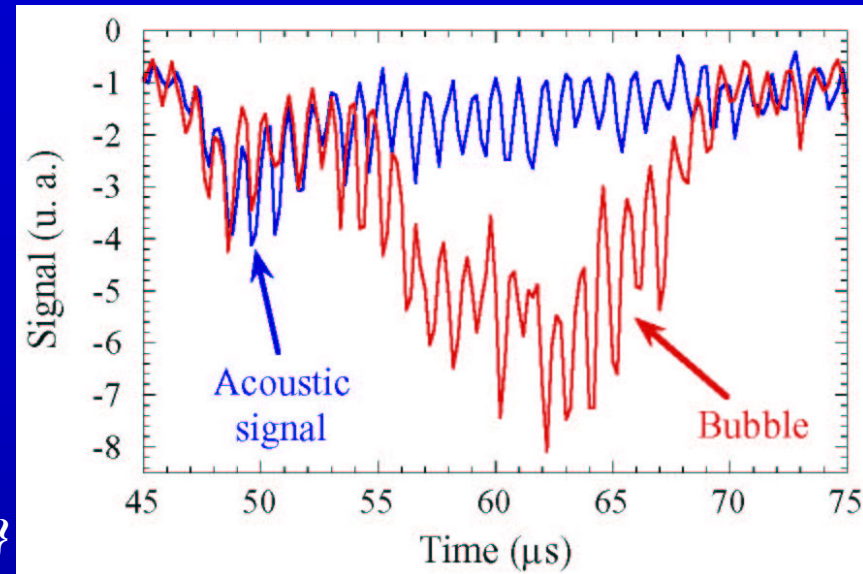
cavitation in helium 3

*same "asymmetric S-shape" law
for the nucleation probability:*

$$\Sigma = 1 - \exp(-\Gamma_0 V \tau \exp(-E/T))$$

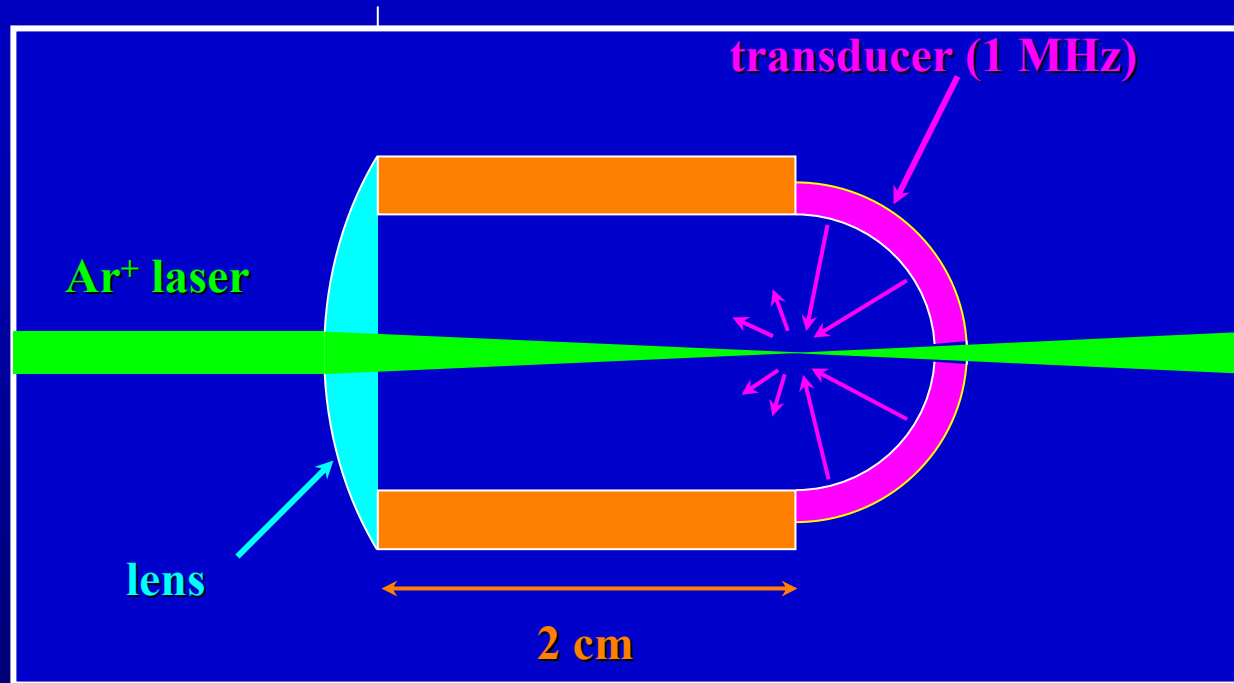
$$= 1 - \exp\{-\ln 2 \exp[-(1/T)(dE/d\phi)](\phi - \phi_c)\}$$

*F. Caupin and S. Balibar,
Phys. Rev. B 64, 064507 (2001)*



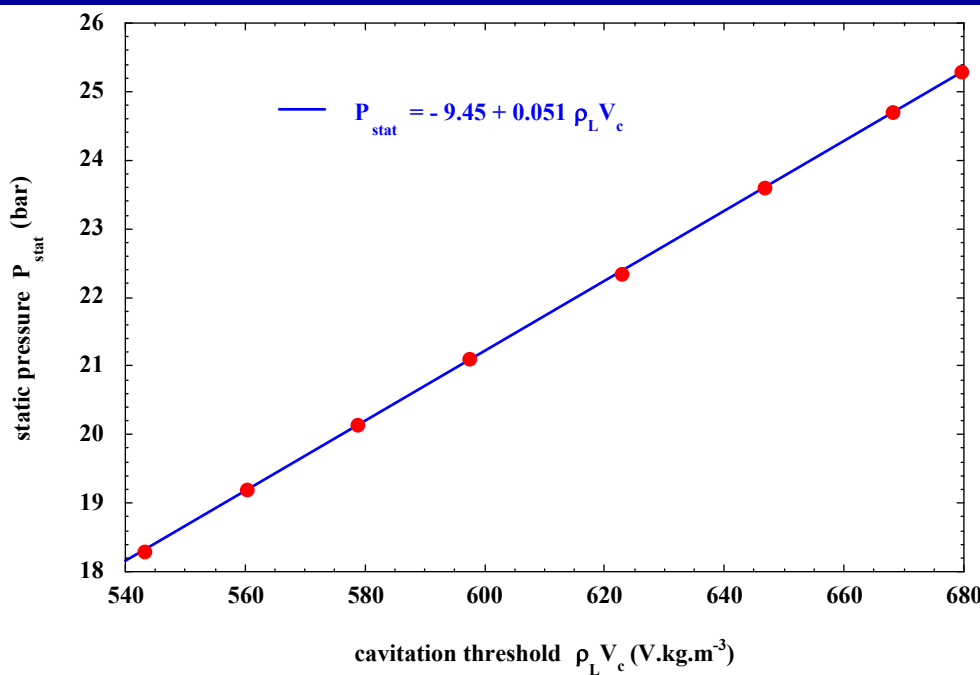
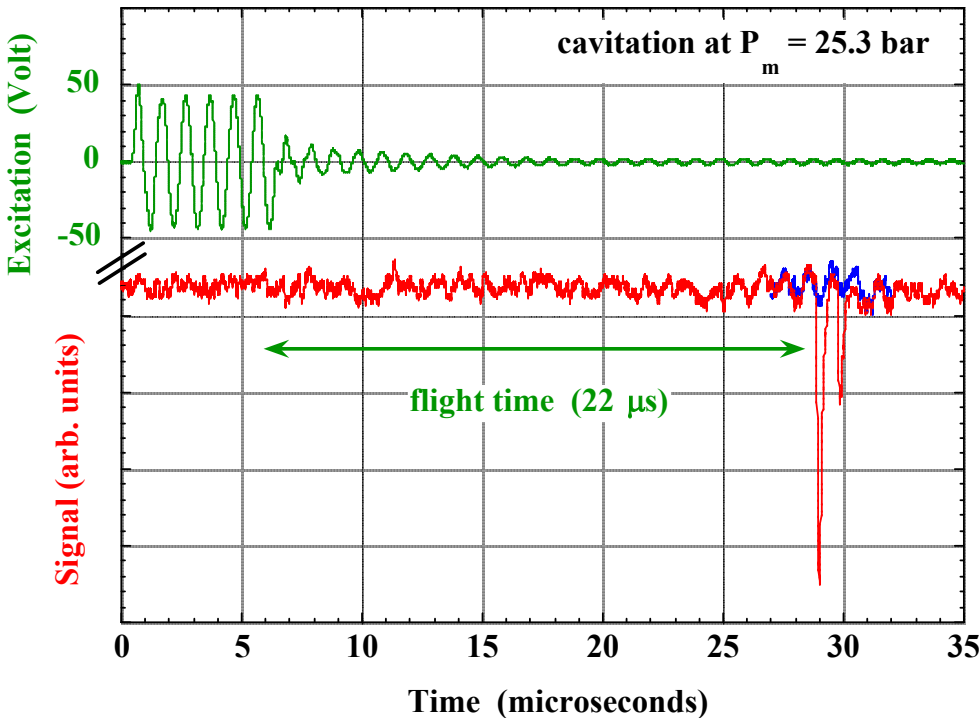
search for homogeneous nucleation of solid helium with acoustic waves

F. Werner, G. Beaume, C. Herrmann, A. Hobeika, S. Nascimbene, F. Caupin and S. Balibar (submitted to J. Low Temp. Phys. dec. 2003)



remove the glass plate

increase the amplitude of the acoustic wave

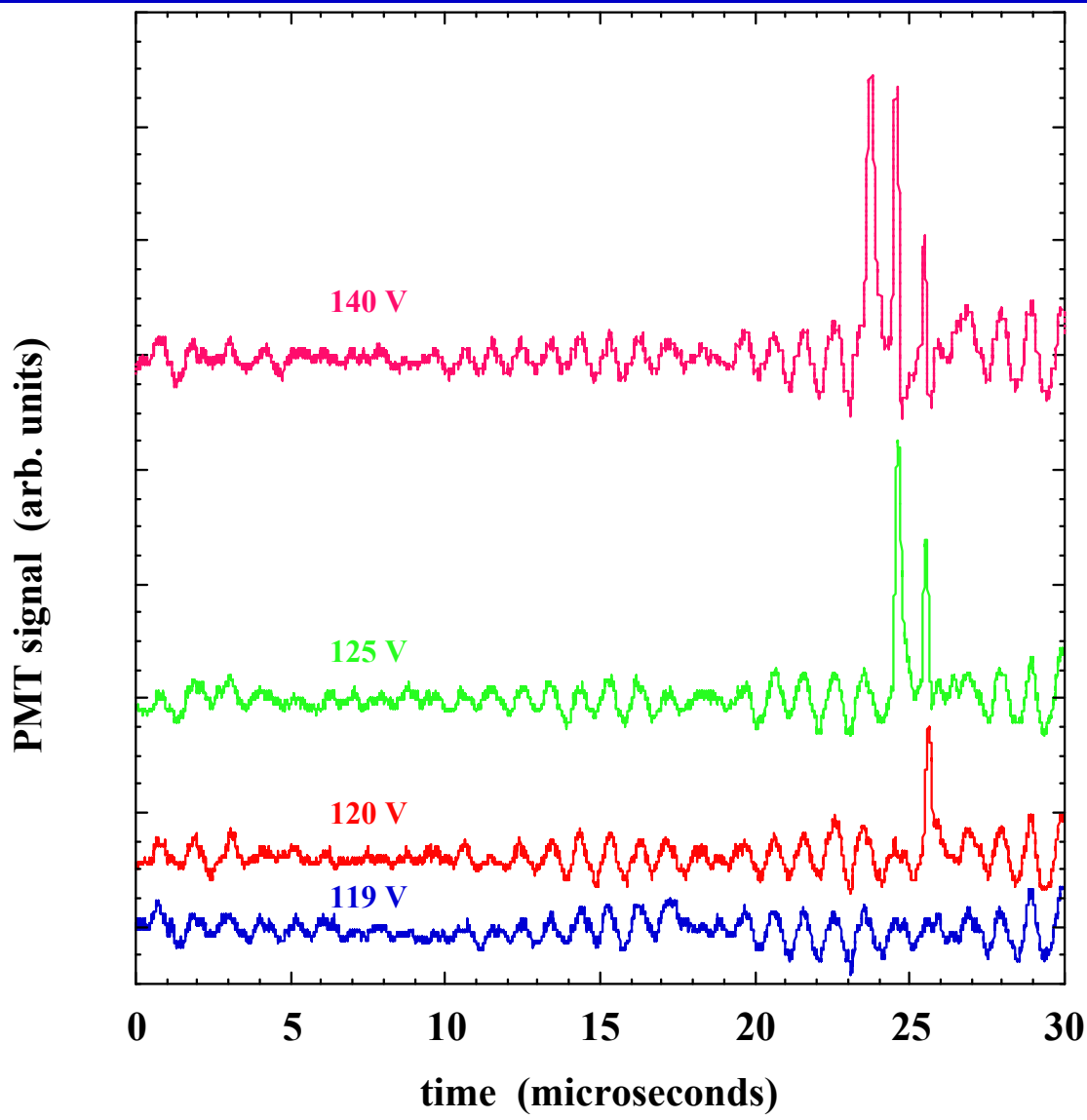


acoustic cavitation in liquid 4He at high pressure

- *the cavitation threshold voltage V_c (more precisely the product $\rho_L V_c$) varies linearly with the pressure in the cell P_{stat}*
- *agreement with the linear approximation for the amplitude of the wave at the focus:*

$$\delta P \approx R \omega^2 \rho_L V$$
- *in our hemispherical geometry, non-linear effects must be small.*
- *extrapolation \Rightarrow cavitation occurs at -9.45 bar, in excellent agreement with theory (0.2 bar above the spinodal limit at -9.65 bar)*
- \Rightarrow *a calibration method for the wave*

increasing the acoustic amplitude



** as one increases the excitation voltage, cavitation occurs on earlier and earlier oscillations. This is due to the finite Q factor of the transducer*

(we measured $Q = 53$)

** here, for bursts of 3 oscillations and at 25 bar, 55 mK:*

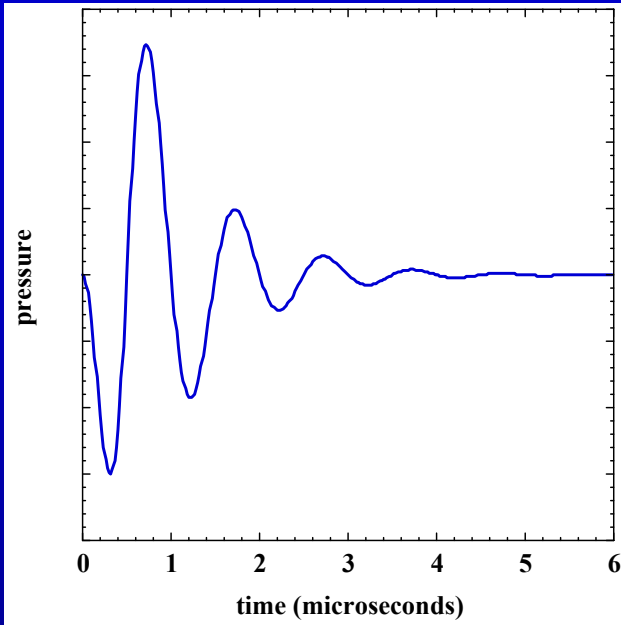
- no cavitation at 119 V

- cavitation on third oscillation at 120 V

- on second oscillation at 125 V

- on first oscillation at 140 V

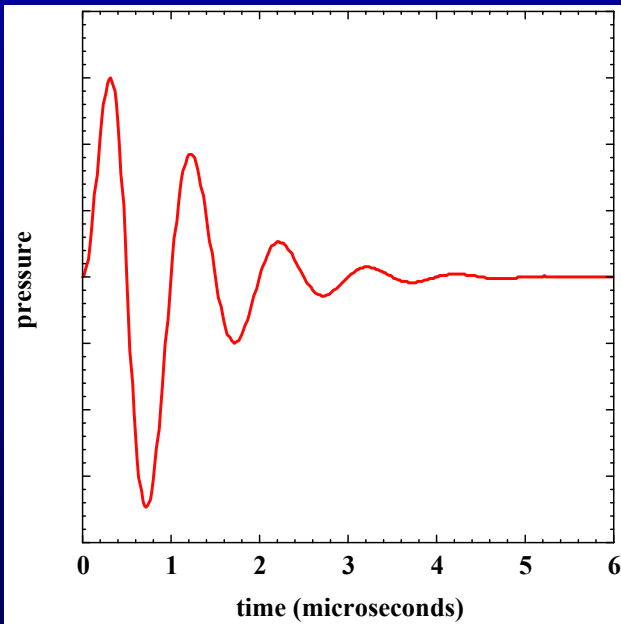
principle of the experiment



*In liquid helium at 25 bar,
we emit a sound pulse, which starts with a
negative pressure swing
cavitation is observed for a threshold voltage V_c
when the pressure reaches - 9.45 bar
at the acoustic focus at time $\tau_{flight} + 0.25 \mu s$.*

⇒ calibration:

V_c corresponds to a $25 + 9.45 = 34.45$ bar amplitude



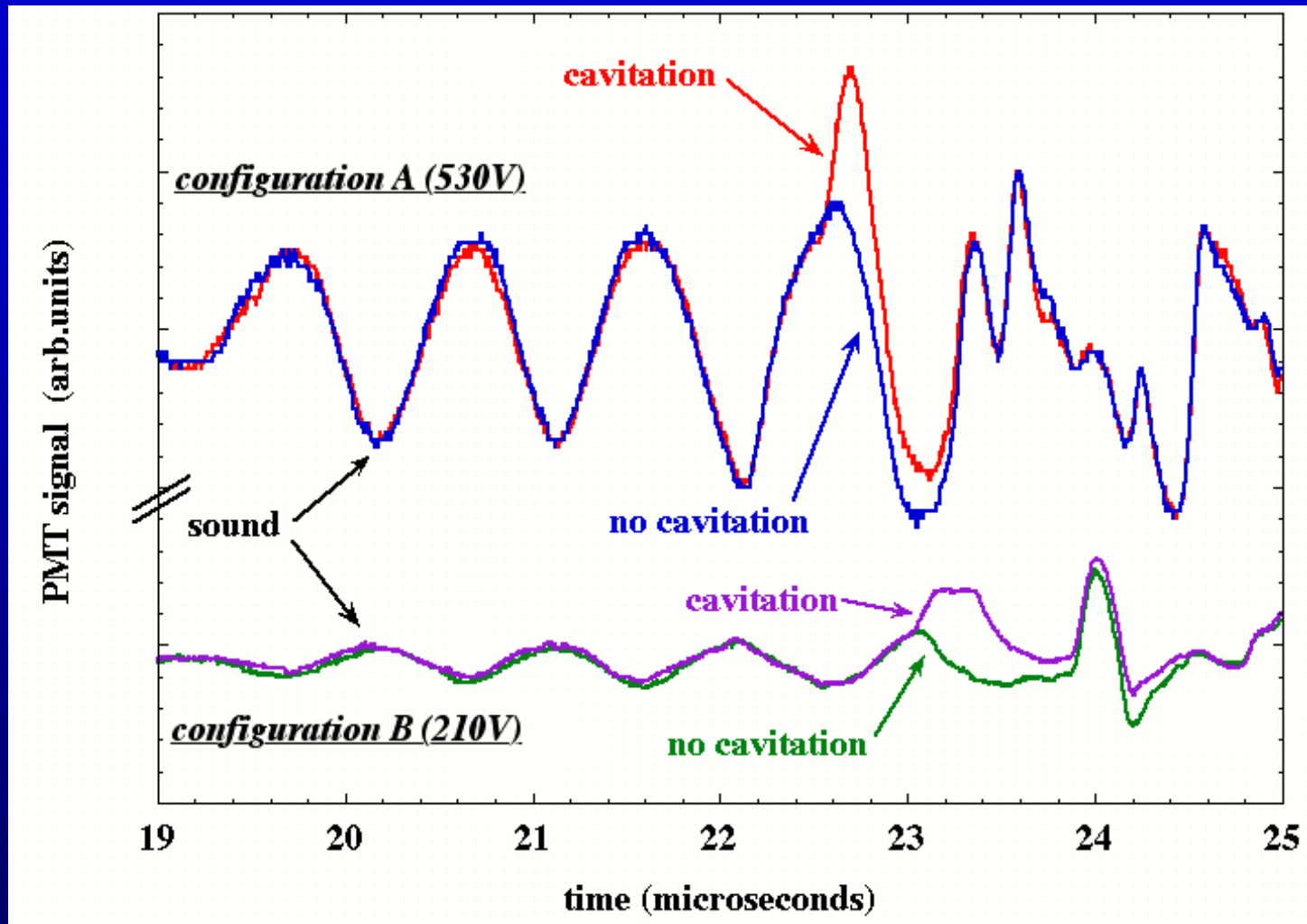
*We reverse the voltage applied to the transducer.
We increase this voltage V as much as possible,
cavitation occurs at time $\tau_{flight} + 0.75 \mu s$
we look for nucleation of crystals before that,
at time $\tau_{flight} + 0.25 \mu s$.*

A maximum positive pressure

$P_{max} = 25 + 34.45(V/V_c)$ bar

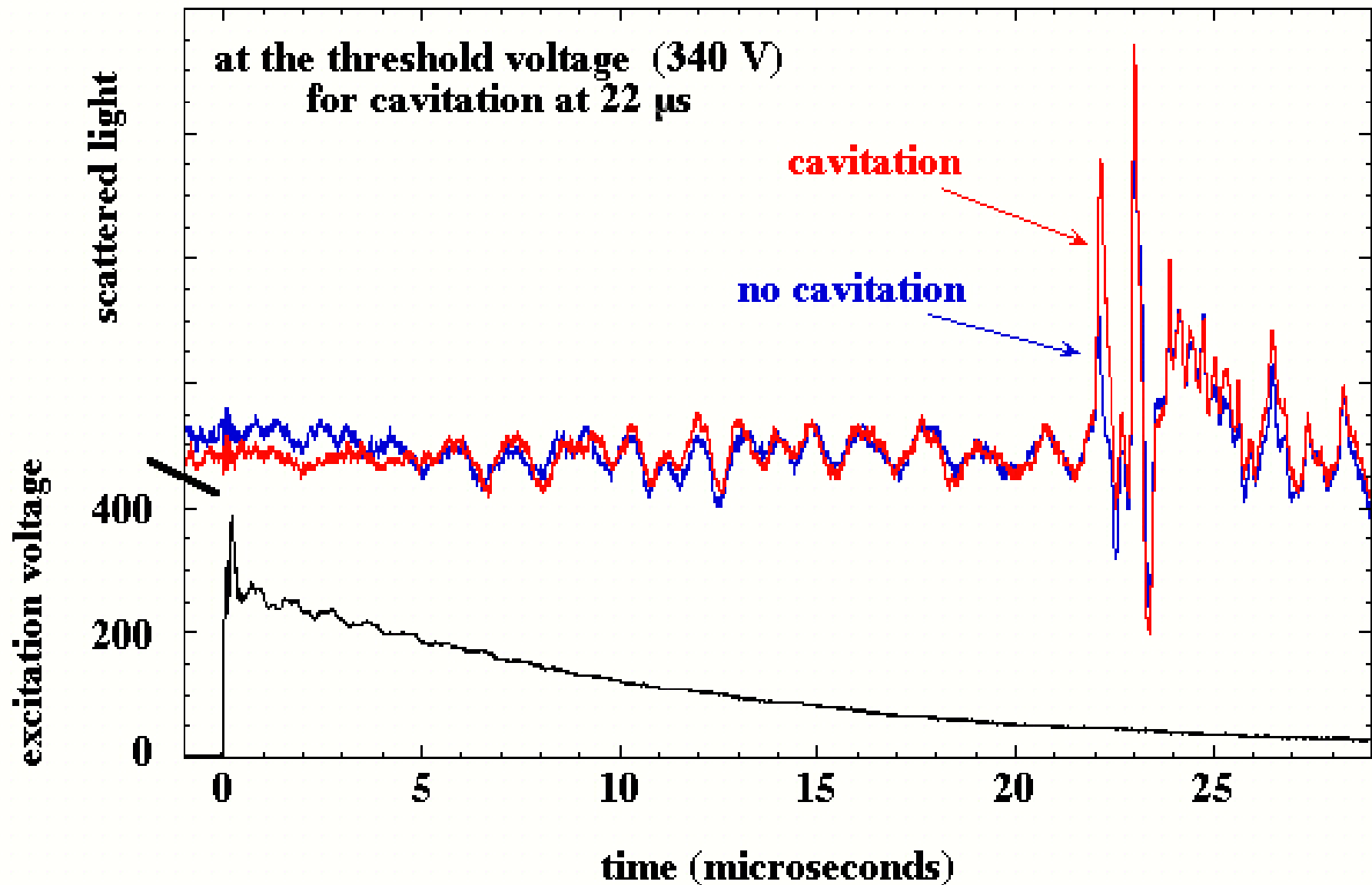
is reached at time $\tau_{flight} + 0.25 \mu s$

reversing the phase in a real experiment

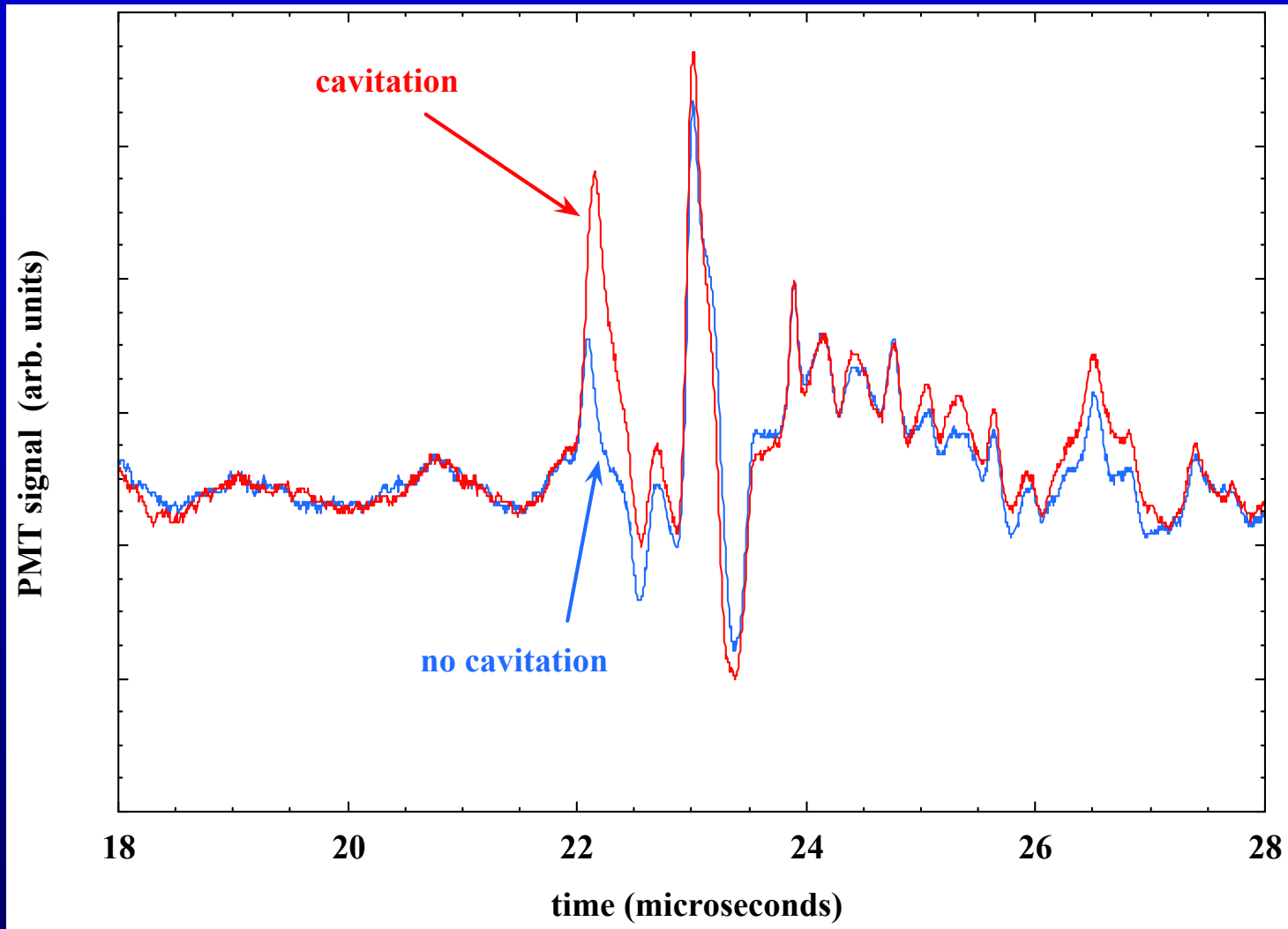


*changing from configuration A to B,
the cavitation signal shifts by half a period ($0.5 \mu\text{s}$)*

exciting the transducer with a simple pulse

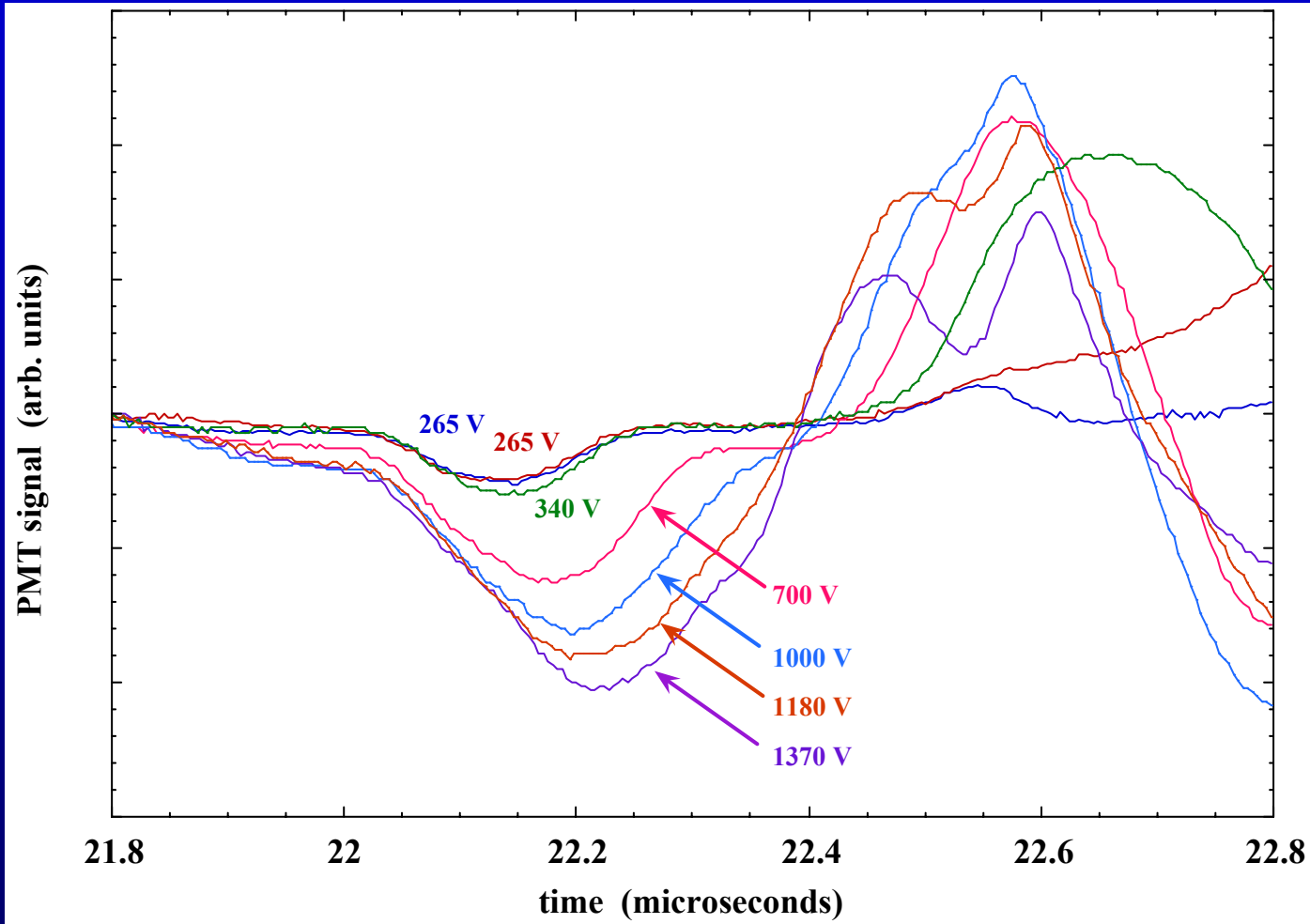


at the threshold voltage (340 V)



random cavitation at time 22 μ s

liquid helium 4 up to 163 bar



*after reversing the excitation voltage, no nucleation of crystals up to 1370 Volt.
this sound amplitude corresponds to a maximum pressure*

$$P_{max} = 25 + 34.45 (1370/340) = 163 \text{ bar}$$

some comments

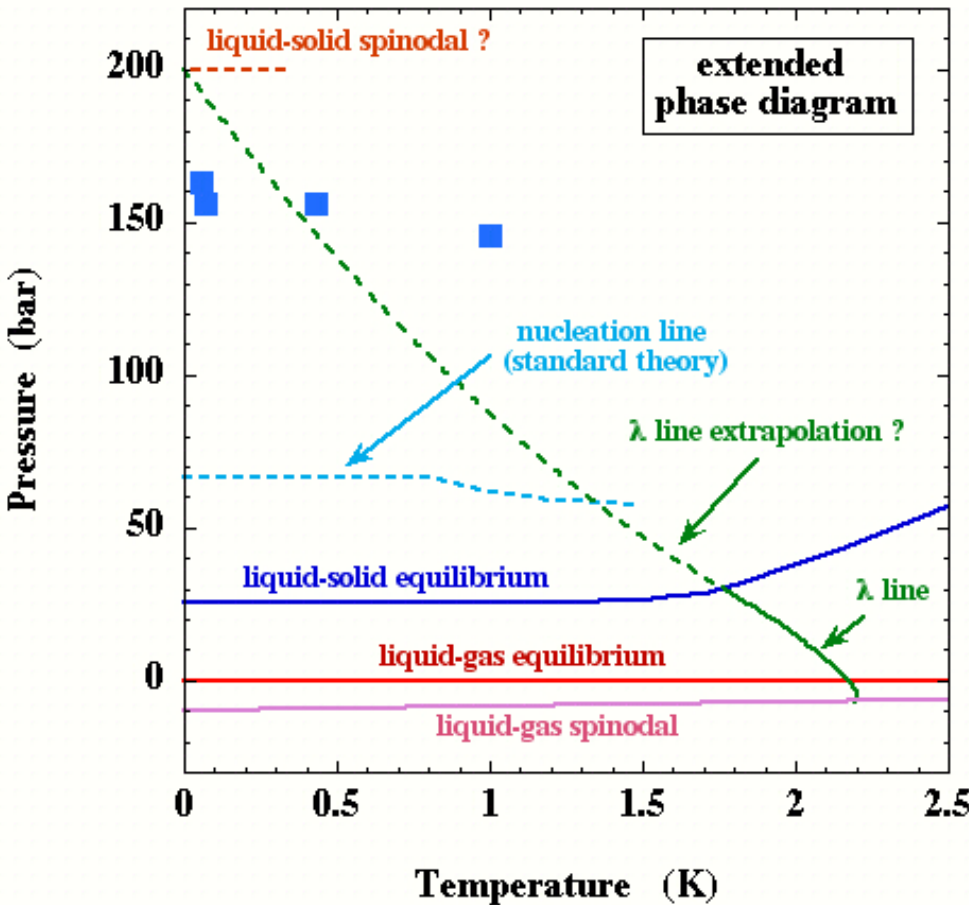
the standard nucleation theory fails

the standard theory predicts homogeneous nucleation at 65 bar.

It assumes a pressure independent surface tension, but this assumption was criticized by Maris and Caupin (*J. Low Temp. Phys.* 131, 145, 2003)

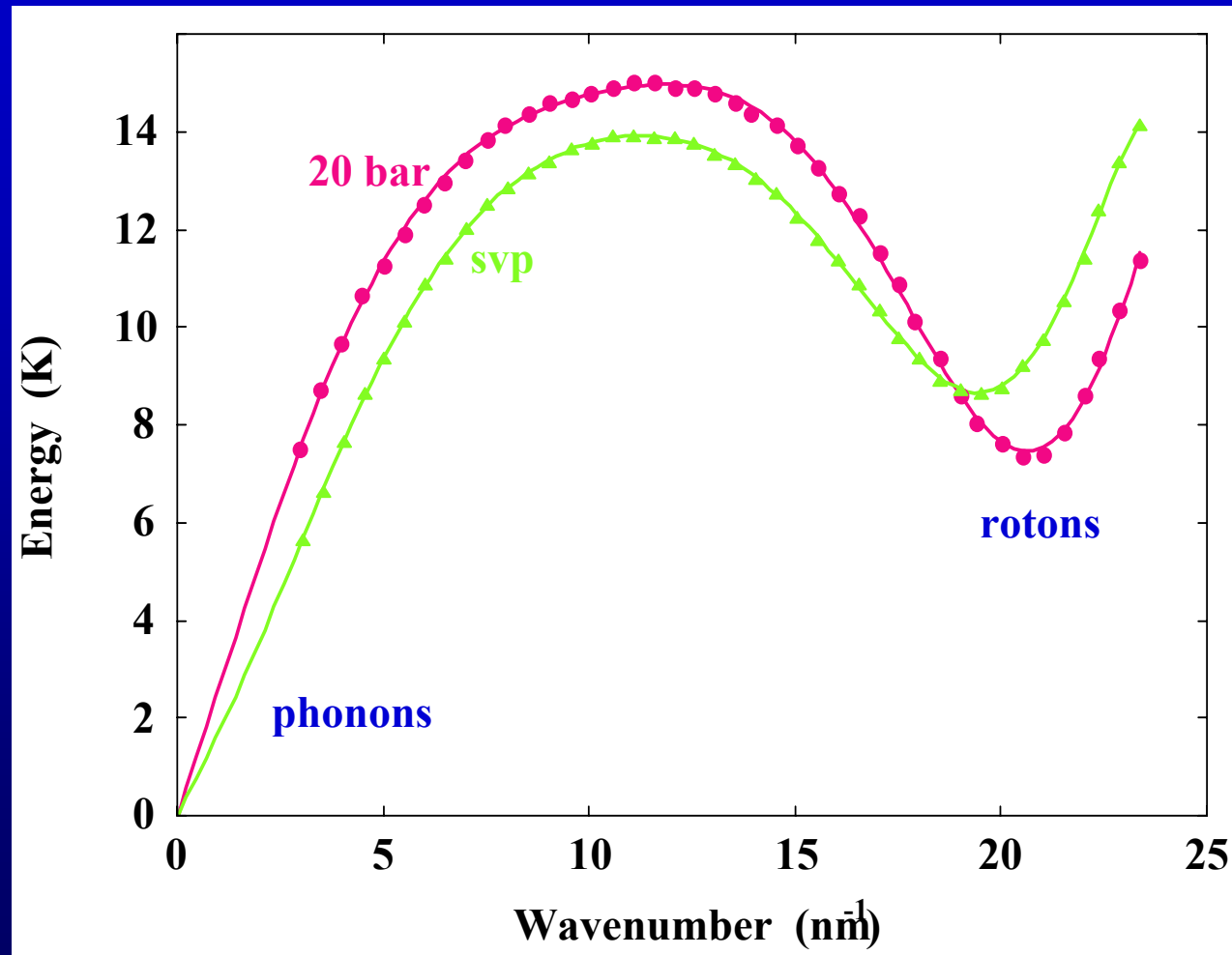
superfluidity at 163 bar ?

It is unlikely that crystals nucleated but were not detected, since they should grow even faster at 163 bar than at 29.6 bar, except if liquid helium is no longer superfluid ($\rho_L \sim 0.227 \text{ gcm}^{-3}$, much more than $\rho_L = 0.172$ or $\rho_C = 0.191$ at 25 bar). The extrapolation of the λ line is not precisely known, but it should reach $T = 0$ at 200 bar, where the roton gap vanishes according to H.J. Maris, and where the liquid should become unstable (Schneider and Enz, PRL 27, 1186, 1971).

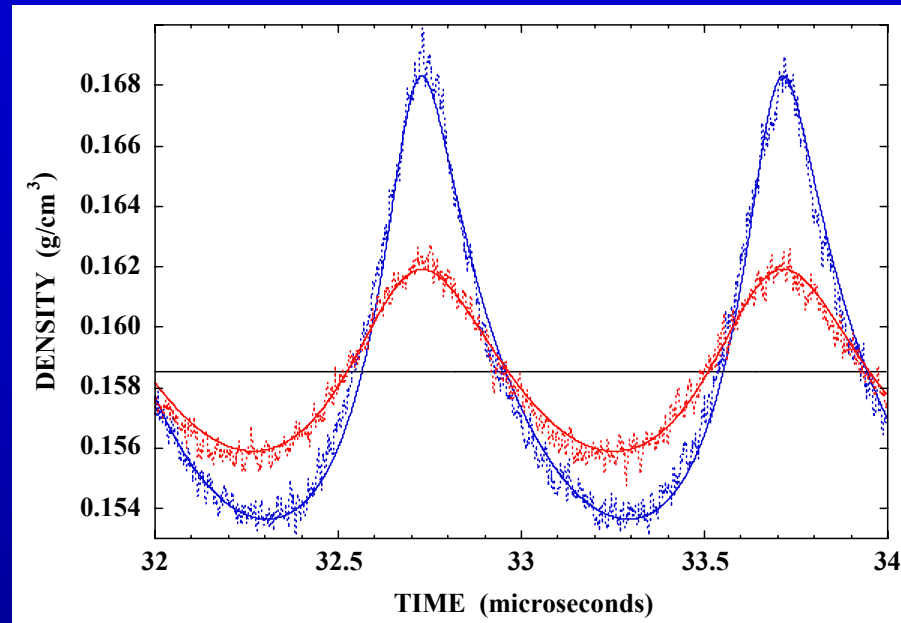
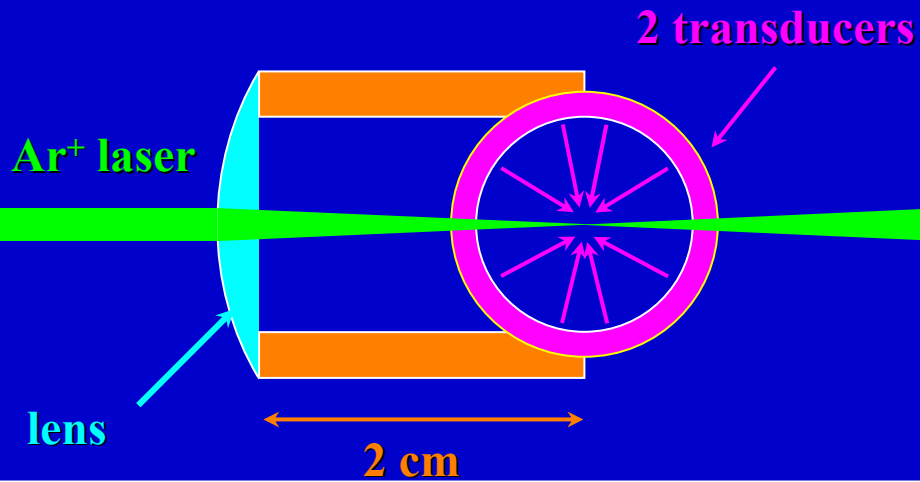


an instability at 200 bar ?

H.J. Maris noticed that, according to the density functional form of Dalfovo et al., the roton gap vanishes around 200 bar where the density reaches 0.237 g/cm^3 . If true, this "soft mode" at finite wave vector could imply an instability towards a periodic (i.e. crystalline ?) phase (Schneider andENZ PRL 27, 1186, 1971)



*future experiments:
reach 200 bar or more*



use 2 transducers (full spherical geometry)

due to non-linear effects, positive swings are larger than negative swings

easy to reach + 200 bar

difficult to calibrate the amplitude

improve numerical calculations of the sound amplitude

*(see C. Appert , C. Tenaud, X. Chavanne, S. Balibar, F. Caupin, and D. d'Humières
Euro. Phys. Journal B 35, 531, 2003)*