

## Detection of the Rotation of the Earth with a Superfluid Gyrometer

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The effect of the rotation of the Earth,  $\Omega_{\oplus}$ , on a superfluid resonator equipped with a 4.0 cm<sup>2</sup> rotation pickup loop and with a microaperture is reported. The velocity circulation induced in the loop by the rotation is detected by phase-slippage techniques. The magnitude of  $\Omega_{\oplus}$  is measured to better than 1%, and the north direction to  $\pm 0.5^\circ$  for a 10 h observation time. This experiment is the superfluid counterpart of interferometric measurements based on the Sagnac effect. [S0031-9007(97)03211-0]

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From early astronomical observations to Foucault's pendulum experiment to Gravity Probe B [1], increasingly sophisticated gyrometric mechanical devices have been used to monitor the rotation of the Earth  $\Omega_{\oplus}$ . Interferometric techniques exploiting the wave-mechanical nature of light beams—the Sagnac effect [2]—of neutron [3] and electron [4] beams have been devised in order both to demonstrate the general laws of inertia and to improve gyrometric techniques [5]. We describe below an experiment [6] which has been considered by a number of authors [7–11] and in which the effect of  $\Omega_{\oplus}$  on a loop of superfluid <sup>4</sup>He is measured with the help of quantum phase slippage [12] to a resolution of better than  $10^{-2}$ .

The principle of this experiment, illustrated in Fig. 1, rests on the possibility to measure with great accuracy the change of the circulation of the superfluid velocity around a loop. Dual points of view may be taken to describe the effect of imposing a rotation  $\Omega$  on the superfluid-filled closed conduit which forms this loop.

The first approach is wave-mechanical. We can follow, for instance, Dresden and Yang [13] in their analysis of the effect of rotation on the neutron interferometer of Ref. [3]: rotation is viewed as causing a Doppler shift of the interfering waves due to the motion of the source, interferometer walls (here, the conduit), and detector (here, the microaperture in the conduit) with respect to the frame of inertia. The phase shift imposed on the beam of particles circulating around a loop  $\Gamma$  with velocity  $v$  and wave number  $k$  is shown to be  $\delta\varphi = (2k/v)\vec{\Omega} \cdot \vec{A}$ ,  $\vec{A}$  being the area spanned by the (oriented) contour  $\Gamma$ . This result, which can be obtained in different ways [2–5], implicitly assumes that the beam trajectory lies on a geometrically sharply defined contour. It can be written, with the help of de Broglie's relation  $\hbar k = mv$ , as [3]:

$$\delta\varphi = (2m/\hbar)\vec{\Omega} \cdot \vec{A}. \quad (1)$$

Equation (1) holds in superfluid helium as the quantum phase of the condensate wave function is shifted by the motion of the gyrometer walls in the same manner as for a coherent beam of particles.

Equation (1) also stems directly from superfluid hydrodynamics. This second approach provides a more suitable framework for the discussion of the present experiment. The circulation of the superfluid velocity  $\vec{v}_s$  around a contour  $\Gamma$  in the inertial frame is  $\kappa = \oint \vec{v}_s \cdot d\vec{l}$ . In the rotating frame, which is the laboratory frame here, the velocity at point  $\vec{r}$  is transformed according to  $\vec{v}'_s = \vec{v}_s + \vec{\Omega} \times \vec{r}$ ; the circulation along the same contour reads  $\kappa' = \kappa + 2\vec{\Omega} \cdot \vec{A}$ . The change of circulation due to rotation exactly corresponds to Eq. (1)

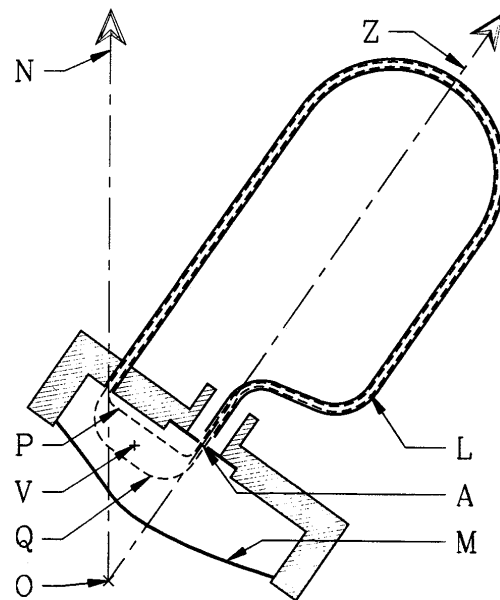


FIG. 1. Sketch of the superfluid resonator. Point O is the center of the Earth. The rotation axis points to the north N. The cryostat can be oriented about the local vertical axis Z. Two superfluid contours threading the superfluid loop L are shown: P not including the trapped vortex V, and Q including it. Pickup loop L has two turns, not shown for clarity. The microaperture A and the membrane M constitute, together with L, the superfluid resonator. The “fat” space between the membrane and the resonator wall is grossly exaggerated on the sketch above, the actual gap being only  $\sim 50 \mu\text{m}$ .

because the superfluid velocity  $v_s$  is related to the quantum-mechanical phase by  $\vec{v}_s = (\hbar/m_4)\nabla\varphi$  [14].

This convergence of points of view can also be justified, as noted by a number of authors [7,8,10], by the analogy between charged and neutral superfluids. As discussed by Nozières [14], a rotation field  $2\vec{\Omega}$  applied to a toroidal container of helium can be viewed as analogous to a magnetic field  $\vec{B}$  applied to a superconducting annulus, the rotation induced velocity  $\vec{\Omega} \times \vec{r}$  as analogous to the vector potential  $\vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$ , and the quantization of the circulation of  $\vec{v}'_s - \vec{\Omega} \times \vec{r}$  corresponding to that of the fluxoid [15]. It is worth pointing out in this context that the superfluid two-hole resonator used here to detect small rotations constitutes a direct analog of the electrodynamic rf SQUID used to measure small magnetic fluxes [7].

Equation (1) expresses a very general result. Its application to the actual experimental cell sketched in Fig. 1, however, runs into difficulties: (1) the conduit constraining the superfluid to a loop is not infinitely thin and contains fat sections around the flexible membrane and the microaperture; this leads to an indeterminacy in the choice of  $\Gamma$ , hence in the geometrical area  $\vec{A}$ ; (2) if a trapped vorticity is present, as is most likely found in superfluid  $^4\text{He}$ , two different paths threading the conduit might enclose a different number of trapped vortices, leading to different quantum states of circulation; (3) the conduit is open into the external bath of superfluid into which the cell is immersed, yielding the possibility of path leakage in and out. We have attempted to address these difficulties both from a basic standpoint and by careful layout of the cell.

Superflow around the conduit [16] is given by the solution of Laplace's equation which satisfies the moving-wall boundary conditions. The various contours  $\Gamma$  can be taken to follow the streamlines and to be bunched up into infinitesimal streamtubes, each carrying infinitesimal flow  $\delta J$ . To each of these contours  $\Gamma$  is attached a geometrical area  $\vec{A}_\Gamma$ . The resulting global area which the conduit spans is the average of  $\vec{A}_\Gamma$  over the conduit cross section:  $\vec{A} = (1/J) \int \int \vec{A}_\Gamma dJ$ . The pickup loop consists of a two-turn coil of 0.4 mm i.d. capillary, 135 mm in length, and of the fat portions in the cell itself (which are reduced to the feasible minimum). The full pickup loop area as computed from its geometry is 4.0 cm<sup>2</sup>, with a possible indeterminacy  $\lesssim 2\%$ .

The whole resonator is immersed in a superfluid bath cooled by a dilution refrigerator. The opening to this outer bath provides possible path leakage. After preliminary experiments, described below, we have adopted the reentrant, axially symmetric geometry shown in Fig. 1 which guards the entrance of the conduit against external influences.

Trapped vorticity, reduced for simplicity to a single trapped vortex as sketched in Fig. 1, gives rise to phase differences of  $2\pi$  between contours (filamentary streamtubes) which include and do not include it. If a fraction  $f$  of the flow includes it, the net apparent phase differ-

ence around the conduit due to this trapped vortex will be  $f \times 2\pi$ . In our experimental setup, this mostly random bias phase, or bias circulation  $\kappa_b = f\kappa_4$ ,  $\kappa_4$  being the quantum of circulation in  $^4\text{He}$ , is always present. It can be changed during the run by (gently) tapping on the cryostat and/or by boosting the driving power of the resonator by 2 orders of magnitude or more. During normal operation of the cell, the bias remains constant, even when phase slips by large multiples of  $2\pi$  are taking place [17], and also when the temperature is (slowly) swept from 12 to 350 mK. The addition of the pickup loop, which contains more than twice as much fluid as the rest of the cell, had no detrimental effect on the stability of the bias: the bias shifts only when the cell is deliberately perturbed.

The bias  $\kappa_b$  is the key quantity in these experiments because it contains the contribution of  $\Omega_\oplus$  to the overall circulation around the conduit. At the latitude of Saclay (48°43') where the experiment is conducted, the expected full-span variation of  $\kappa_b$  due to  $\Omega_\oplus$  amounts to  $0.7722\kappa_4$  for the 4.0 cm<sup>2</sup> loop. This contribution can be separated from that of trapped vorticity by orienting the cell with respect to the local vertical axis [18] as sketched in Fig. 1, assuming that the trapped vorticity remains fixed with respect to the cell (which is not necessarily the case—see below). The cell orientation, specified in the laboratory by angle  $\beta$ , is varied between measurements by rotating about the local vertical the cryostat insert which carries the dilution refrigerator unit and the superfluid resonator. The Dewar itself, very rigidly secured to a vibration-insulated platform [19], remains steady in the laboratory. Rotation is effected by a step motor with a high step-down gear ratio by  $\pm 170^\circ$  at a rotation speed of  $2^\circ/\text{min}$ . Up to the maximum rotation speed of  $18^\circ/\text{min}$ ,  $\kappa_b$  repeats with angle  $\beta$  to the resolution of the measurements: the cryostat rotation drive is smooth and does not knock pinned vortices off their pinning sites.

The cell itself is very similar to the two-hole resonator used in previous work [20]. Its operation is described in detail in Ref. [17], as well as data acquisition and analysis procedures. The raw experimental quantity under observation is the flexible membrane deflection  $A(t)$  output by a SQUID displacement gauge [11,12,17]. This quantity is computer analyzed in real time to obtain the positive going and negative going peak amplitudes  $A$  at each half-cycle of the resonance; the values of  $A$  are recorded. Phase slips are the discontinuous jumps which appear in the time evolution, half-cycle after half-cycle, of  $A$ .

The bias circulation is obtained from the analysis of these peak amplitude data as described in Refs. [11,17]. This analysis rests on the following relation obeyed by the "critical" membrane amplitude  $A_q$  at which slips occur for each given loop quantum state  $q$ :

$$A_q = |\epsilon A_m - (q + \kappa_b/\kappa_4)R\Delta A_1|. \quad (2)$$

A  $2\pi$  phase slip causes an elemental amplitude jump  $\Delta A_1$ . The geometrical parameter  $R$  is the ratio of the hydraulic inductance of the microaperture to that of the

conduit, that is, the fraction of the membrane-induced flow which goes through the conduit over that which goes through the microaperture. The membrane-induced flow direction is indicated by  $\epsilon$  ( $= \pm 1$ ). Equation (2) simply expresses that a circulation present in the loop (the 2nd term on the right-hand side) yields a flow velocity that either adds or subtracts, according to  $\epsilon$ , to the membrane-driven fluid motion. When the combined flow velocity becomes critical in the microaperture, a phase slip takes place:  $A_q$  always has a smaller amplitude than  $A_m$ , the maximum critical amplitude that is reached when both  $\kappa_b$  and  $q$  are zero; phase slips always have a higher probability to occur in the flow direction which results in a reduction of  $|q|$ . Phase slips demolish and reconstruct the quantized states of circulation in the loop in such a way that  $q$  remains clustered around  $q = 0$ , making it possible to repeat measurements of  $A_q$  a large number of times and achieve a high accuracy.

A typical data record comprises between 5000 and 7000 values for  $A_q$ . The resonator frequency is 9.23 Hz. The corresponding recording time is about 30 min. Fitting Eq. (2) to this set of  $A_q$  values yields  $R$  (here 1.025, also obtained by analyzing the rare large multiple slips [17]),  $\kappa_b/\kappa_4$ —the quantity of interest here, and  $A_m$ . A resolution better than  $\sim 10^{-4}$  is achieved on  $A_q$ , and better than  $10^{-2}\kappa_4$  on  $\kappa_b$ . About 95% of the recorded phase slips are single slips, the remaining being mostly by two quanta. This method for determining  $\kappa_b$  is basically independent of the slip multiplicity.

In preliminary runs in which the pickup capillary was opened directly to the outer bath, the expected sinusoidal variation of  $\kappa_b$  with  $\beta$  was visibly distorted and its amplitude was changing by as much as 50% upon warming and cooling through the  $\lambda$  point and tapping on the cryostat. We have attributed these effects to flow admixing from stray currents in the outer bath. Although these currents are not related to the rotation of the Earth, their contribution to  $\kappa_b$  also varies with  $\beta$ . Their influence has been eliminated in the present setup by adopting the reentrant geometry shown in Fig. 1 and by keeping the end of the capillary very close to the orifice.

However, a small bias-dependent distortion of  $\sim 2\%$  remained unabated. Its origin has now been traced to the slip statistics and its dependence on the bias. When the bias is zero or  $\kappa_4/2$ , both slip directions are equally probable. They are treated on an equal footing and no systematic difference between them is introduced. When  $\kappa_b$  differs from these values, the time spent on the average by the system in the different states  $q = 0, \pm 1, \dots$  before a slip occurs is different for the two flow directions. This asymmetry is not properly taken into account in the present analysis. Equation (2) holds with mean values for  $A_q$  computed with the statistical distribution function pertaining to the actual nucleation process. The computerized analysis uses only the arithmetic mean values of the observed  $A_q$ . This simplified procedure introduces a systematic bias-dependent deviation in the determination of  $\kappa_b$ .

This systematic deviation is corrected by using the vortex nucleation model of Ref. [19] in the following way. Surrogate data incorporating the stochastic aspects of nucleation are produced by numerical simulations of the cell operation [21]. Analysis of these simulated data for which the “true” bias value is known yields the correction function needed to correct the systematic deviation. This correction is seen in the inset of Fig. 2 to be quite significant. It is an odd function of  $\kappa_b$  since the cell operation is symmetric with respect to flow reversal (except for a weak asymmetry in the synchronization of the applied drive), as described by Eq. (2). It accounts for most of the error on the determination of  $\Omega_\oplus$  reported in Ref. [6].

The outcome of measurements at fixed angles  $\beta$  differing by  $20^\circ$  is shown in Fig. 2 for different trapped vorticities obtained by strongly perturbing the cell. These data display the expected sinusoidal dependence in  $\beta$  independently of the mean trapped circulation. They yield the magnitude of  $\Omega_\oplus$  with a resolution better than  $10^{-2}$ , and the true north direction to better than  $\pm 30'$ . The effect of  $\Omega_\oplus$  on  $\kappa_b$  averaged over 10 full ten-hours sweeps of  $\beta$  is found to be  $0.7750 \pm 0.0036$  in units of  $\kappa_4$ , which differs from the computed value by only 0.36%.

These experiments were conducted with nominal purity  $^4\text{He}$ , containing approximately 0.1 ppm of  $^3\text{He}$  cooled at  $\sim 12$  mK under a pressure of 0.6 bar. The peak amplitude, expressed as the critical winding number of the phase across the microaperture  $\mathcal{N}_c = A_m/[\Delta A_1(1 + R)]$  is close to 60 with an rms scatter of  $\sim 0.2$ . This corresponds to a scatter on  $A_q$  of the order of  $0.4\Delta A_1$ . The noise is due to the stochastic nature of the (quantum) nucleation process. This noise limits the present resolution and is much larger than vibration noise, so that the pickup loop area can be further enlarged. It can be reduced further by lowering the temperature, increasing the hydrostatic

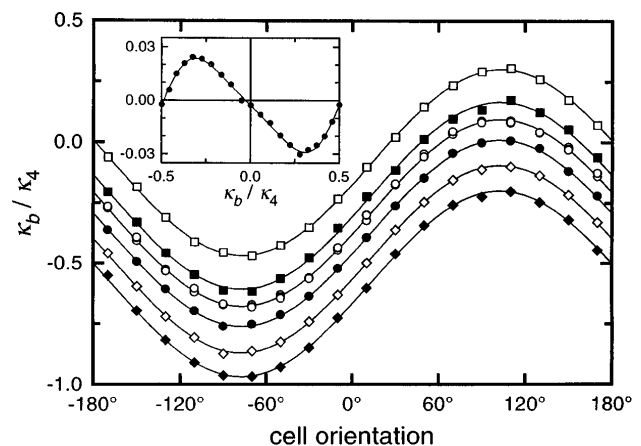


FIG. 2. Velocity circulation, in units of  $\kappa_4$ , as a function of the cell orientation about the local vertical axis. The ordinates are defined modulo 1. The  $0^\circ$  orientation of the plane of the pickup coil has been fixed at the cell assembly and is offset by  $12.5^\circ$  with respect to the north. The inset shows the correction discussed in the text, in units of  $\kappa_4$ . The sine curves are fit to the data; the solid line in the inset is a guide for the eye.

pressure [17], and increasing the  $^3\text{He}$  content [22]. The volume and geometry of the present cell have been greatly altered with respect to those of previous work with no apparent change in the stability of the pinned vorticity on which the success of these experiments crucially depends. These findings leave room for significant improvement of the sensitivity of the superfluid resonator as a rotation sensor [23]. They also give a clear illustration of how phase coherence in the superfluid can be put to work to measure extremely small velocity circulation changes.

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